

thm_2Elift_1eee_2Eis__finite__finite
(TMG6rP4BjPps1FssU4Fw28Mby2US7SdWuk7)

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Definition 1 We define `c_2Emin_2E_3D` to be $\lambda A. \lambda x \in A. \lambda y \in A. inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define `c_2Ebool_2E_21` to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V 0x \in 2.V 0x)) (\lambda V 1x \in 2.V 1x))$

Definition 3 We define `c_2Ebool_2E_21` to be $\lambda A_27a : \iota. (\lambda V 0P \in (2^{A_27a}). (ap (ap (c_2Emin_2E_3D (2^{A_27a})))$

Definition 4 We define `c_2Ebool_2E_21` to be $(ap (c_2Ebool_2E_21 2) (\lambda V 0t \in 2.V 0t))$.

Let `ty_2Erealax_2Ereal` : ι be given. Assume the following.

$$nonempty\ ty_2Erealax_2Ereal \tag{1}$$

Let `ty_2Ebool_2Eitself` : $\iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A 0. nonempty\ A 0 \Rightarrow nonempty\ (ty_2Ebool_2Eitself\ A 0) \tag{2}$$

Let `c_2Ebool_2Ethe_value` : $\iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty\ A_27a \Rightarrow c_2Ebool_2Ethe_value\ A_27a \in (ty_2Ebool_2Eitself\ A_27a) \tag{3}$$

Let `ty_2Enum_2Enum` : ι be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \tag{4}$$

Let `c_2Efc_2Edimindex` : $\iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty\ A_27a \Rightarrow c_2Efc_2Edimindex\ A_27a \in (ty_2Enum_2Enum^{(ty_2Ebool_2Eitself\ A_27a)}) \tag{5}$$

Let `c_2Enum_2ZERO_REP` : ι be given. Assume the following.

$$c_2Enum_2ZERO_REP \in omega \tag{6}$$

Let `c_2Enum_2EABS_num` : ι be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{omega}) \tag{7}$$

Definition 5 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Definition 6 We define $c_2Earithmetic_2EZERO$ to be c_2Enum_2E0 .

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \quad (8)$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \quad (9)$$

Definition 7 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap\ c_2Enum_2EABS_num\ c_2Enum_2ESUC_REP)$.

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum)^{ty_2Enum_2Enum})^{ty_2Enum_2Enum} \quad (10)$$

Definition 8 We define $c_2Earithmetic_2EBIT2$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap\ (ap\ c_2Earithmetic_2E_2B\ n)\ c_2Enum_2E0)$.

Definition 9 We define $c_2Earithmetic_2ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Let $c_2Ereal_2Ereal_of_num : \iota$ be given. Assume the following.

$$c_2Ereal_2Ereal_of_num \in (ty_2Erealax_2Ereal)^{ty_2Enum_2Enum} \quad (11)$$

Let $c_2Ereal_2Epow : \iota$ be given. Assume the following.

$$c_2Ereal_2Epow \in ((ty_2Erealax_2Ereal)^{ty_2Enum_2Enum})^{ty_2Erealax_2Ereal} \quad (12)$$

Let $ty_2EfcP_2Ecart : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2EfcP_2Ecart\ A0\ A1) \quad (13)$$

Let $ty_2Ebinary_ieee_2Efloat : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Ebinary_ieee_2Efloat\ A0\ A1) \quad (14)$$

Let $c_2Ebinary_ieee_2Efloat_Significand : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27t.nonempty\ A_27t \Rightarrow \forall A_27w.nonempty\ A_27w \Rightarrow c_2Ebinary_ieee_2Efloat_Significand\ A_27t\ A_27w \in ((ty_2EfcP_2Ecart\ 2\ A_27t)^{(ty_2Ebinary_ieee_2Efloat\ A_27t\ A_27w)}) \quad (15)$$

Let $ty_2EfcP_2Efinite_image : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2EfcP_2Efinite_image\ A0) \quad (16)$$

Definition 10 We define $c_Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 11 We define $c_Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_Emin_2E_3D_3D_3E V0t) c_Ebool_2E_7E$

Definition 12 We define $c_Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_Ebool_2E_21) 2) (\lambda V2t \in 2.$

Definition 13 We define $c_Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x))$ then (the $(\lambda x.x \in A \wedge P x)$ of type $\iota \Rightarrow \iota$).

Definition 14 We define $c_Ebool_2E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap V0P (ap (c_Emin_2E_40$

Definition 15 We define $c_Eprim_rec_2E_3C$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Definition 16 We define $c_Ebool_2E_3F_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap c_Ebool_2E_2F_5C$

Definition 17 We define $c_Efcp_2Efinite_index$ to be $\lambda A_27a : \iota.(ap (c_Emin_2E_40 (A_27a^{ty_2Enum_2Enum}$

Let $c_Efcp_2Edest_cart : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_Efcp_2Edest_cart \\ & A_27a A_27b \in ((A_27a^{(ty_2Efc_2Efinite_image A_27b)})^{(ty_2Efc_2Ecart A_27a A_27b)}) \end{aligned} \quad (17)$$

Definition 18 We define $c_Efcp_2Efcp_index$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in (ty_2Efc_2Ecart A_27a$

Let $c_Earithmetic_2EEXP : \iota$ be given. Assume the following.

$$c_Earithmetic_2EEXP \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (18)$$

Definition 19 We define c_Ebool_2ECOND to be $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a.(\lambda V3t3 \in 2.$

Definition 20 We define c_Ebit_2ESBIT to be $\lambda V0b \in 2.\lambda V1n \in ty_2Enum_2Enum.(ap (ap (ap (c_Ebool_2E_7E$

Let $c_Esum_num_2ESUM : \iota$ be given. Assume the following.

$$c_Esum_num_2ESUM \in ((ty_2Enum_2Enum^{(ty_2Enum_2Enum^{ty_2Enum_2Enum})})^{ty_2Enum_2Enum}) \quad (19)$$

Definition 21 We define c_Ewords_2Ew2n to be $\lambda A_27a : \iota.\lambda V0w \in (ty_2Efc_2Ecart 2 A_27a).(ap (ap c_Ebool_2E_7E$

Let $ty_2Ehreal_2Ehreal : \iota$ be given. Assume the following.

$$nonempty ty_2Ehreal_2Ehreal \quad (20)$$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Epair_2Eprod \\ & A0 A1) \end{aligned} \quad (21)$$

Let $c_Erealax_2Ereal_REP_CLASS : \iota$ be given. Assume the following.

$$c_Erealax_2Ereal_REP_CLASS \in ((2^{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)})^{ty_2Erealax_2Ereal_REP_CLASS}) \quad (22)$$

Definition 22 We define $c_2Erealax_2Ereal_REP$ to be $\lambda V0a \in ty_2Erealax_2Ereal.(ap (c_2Emin_2E40 (t$
Let $c_2Erealax_2Etreal_inv : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreal_inv \in ((ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)) \quad (23)$$

Let $c_2Erealax_2Etreal_eq : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreal_eq \in ((2^{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)})(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)) \quad (24)$$

Let $c_2Erealax_2Ereal_ABS_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_ABS_CLASS \in (ty_2Erealax_2Ereal^{(2^{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)}})) \quad (25)$$

Definition 23 We define $c_2Erealax_2Ereal_ABS$ to be $\lambda V0r \in (ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)$

Definition 24 We define $c_2Erealax_2Einv$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.(ap c_2Erealax_2Ereal_ABS$

Let $c_2Erealax_2Etreal_mul : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreal_mul \in (((ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal))^{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)}) \quad (26)$$

Definition 25 We define $c_2Erealax_2Ereal_mul$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal$

Definition 26 We define c_2Ereal_2E2F to be $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1y \in ty_2Erealax_2Ereal.($

Definition 27 We define $c_2Earithmetic_2EBIT1$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap (ap c_2Earithmetic$

Let $c_2Erealax_2Etreal_add : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreal_add \in (((ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal))^{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)}) \quad (27)$$

Definition 28 We define $c_2Erealax_2Ereal_add$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal$

Let $c_2Ewords_2EINT_MAX : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Ewords_2EINT_MAX A_27a \in (ty_2Enum_2Enum^{(ty_2Ebool_2Eitself A_27a)}) \quad (28)$$

Let $c_2Ebinary_ieee_2Efloat_Exponent : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27t.nonempty A_27t \Rightarrow \forall A_27w.nonempty A_27w \Rightarrow c_2Ebinary_ieee_2Efloat_Exponent A_27t A_27w \in ((ty_2EfcP_2Ecart 2 A_27w)^{(ty_2Ebinary_ieee_2Efloat A_27t A_27w)}) \quad (29)$$

Let $ty_2Eone_2Eone : \iota$ be given. Assume the following.

$$nonempty\ ty_2Eone_2Eone \quad (30)$$

Let $c_2Ebinary_ieee_2Efloat_Sign : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27t.nonempty\ A_27t \Rightarrow \forall A_27w.nonempty\ A_27w \Rightarrow c_2Ebinary_ieee_2Efloat_Sign \\ & A_27t\ A_27w \in ((ty_2Efc_2Ecart\ 2\ ty_2Eone_2Eone)^{(ty_2Ebinary_ieee_2Efloat\ A_27t\ A_27w)}) \end{aligned} \quad (31)$$

Let $c_2Erealax_2Etreax_neg : \iota$ be given. Assume the following.

$$\begin{aligned} & c_2Erealax_2Etreax_neg \in ((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal \\ & ty_2Ehreal_2Ehreal)^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)}) \end{aligned} \quad (32)$$

Definition 29 We define $c_2Erealax_2Ereal_neg$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.(ap\ c_2Erealax_2Ereal$

Let $c_2Earithmetic_2EDIV : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EDIV \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (33)$$

Definition 30 We define $c_2Ebit_2EDIV_2EXP$ to be $\lambda V0x \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Let $c_2Earithmetic_2E_2D : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2D \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (34)$$

Let $c_2Earithmetic_2EMOD : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EMOD \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (35)$$

Definition 31 We define $c_2Ebit_2EMOD_2EXP$ to be $\lambda V0x \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Definition 32 We define c_2Ebit_2EBITS to be $\lambda V0h \in ty_2Enum_2Enum.\lambda V1l \in ty_2Enum_2Enum.\lambda V$

Definition 33 We define c_2Ebit_2EBIT to be $\lambda V0b \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum.(ap$

Definition 34 We define c_2Efc_2EFCP to be $\lambda A_27a : \iota.\lambda A_27b : \iota.(\lambda V0g \in (A_27a^{ty_2Enum_2Enum}).(ap$

Definition 35 We define $c_2Ewords_2En2w$ to be $\lambda A_27a : \iota.\lambda V0n \in ty_2Enum_2Enum.(ap\ (c_2Efc_2EFCP$

Definition 36 We define $c_2Ebinary_ieee_2Efloat_to_real$ to be $\lambda A_27t : \iota.\lambda A_27w : \iota.\lambda V0x \in (ty_2Ebinar$

Let $ty_2Ebinary_ieee_2Efloat_value : \iota$ be given. Assume the following.

$$nonempty\ ty_2Ebinary_ieee_2Efloat_value \quad (36)$$

Let $c_2Ebinary_ieee_2Efloat : \iota$ be given. Assume the following.

$$c_2Ebinary_ieee_2Efloat \in (ty_2Ebinary_ieee_2Efloat_value^{ty_2Erealax_2Ereal}) \quad (37)$$

Let $c_2Ebinary_ieeee_2ENaN : \iota$ be given. Assume the following.

$$c_2Ebinary_ieeee_2ENaN \in ty_2Ebinary_ieeee_2Efloat_value \quad (38)$$

Let $c_2Ebinary_ieeee_2EInfinity : \iota$ be given. Assume the following.

$$c_2Ebinary_ieeee_2EInfinity \in ty_2Ebinary_ieeee_2Efloat_value \quad (39)$$

Let $c_2Ewords_2EUINT_MAX : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow c_2Ewords_2EUINT_MAX \ A_27a \in (\quad (40)$$

$$ty_2Enum_2Enum^{(ty_2Ebool_2Eitself \ A_27a)})$$

Definition 37 We define $c_2Ewords_2Eword_T$ to be $\lambda A_27a : \iota.(ap \ (c_2Ewords_2En2w \ A_27a) \ (ap \ (c_2Ew$

Definition 38 We define $c_2Ebinary_ieeee_2Efloat_value$ to be $\lambda A_27t : \iota.\lambda A_27w : \iota.\lambda V0x \in (ty_2Ebinary_$

Let $c_2Ebinary_ieeee_2Efloat_value_CASE : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow c_2Ebinary_ieeee_2Efloat_value_CASE$$

$$A_27a \in (((A_27a^{A_27a})^{A_27a})^{(A_27a^{ty_2Erealax_2Ereal})})^{ty_2Ebinary_ieeee_2Efloat_value}) \quad (41)$$

Definition 39 We define $c_2Ebinary_ieeee_2Efloat_is_finite$ to be $\lambda A_27t : \iota.\lambda A_27w : \iota.\lambda V0x \in (ty_2Ebinary$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow \forall A_27b.nonempty \ A_27b \Rightarrow c_2Epair_2EABS_prod$$

$$A_27a \ A_27b \in ((ty_2Epair_2Eprod \ A_27a \ A_27b)^{(2^{A_27b})^{A_27a}}) \quad (42)$$

Definition 40 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap \ (c_2$

Let $c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow \forall A_27b.nonempty \ A_27b \Rightarrow c_2Epred_set_2EGSPEC$$

$$A_27a \ A_27b \in ((2^{A_27a})^{((ty_2Epair_2Eprod \ A_27a \ 2)^{A_27b})}) \quad (43)$$

Definition 41 We define $c_2Ecombin_2ES$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.(\lambda V0f \in ((A_27c^{A_27b})^{A_27a})$

Definition 42 We define $c_2Epred_set_2EUNIV$ to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.c_2Ebool_2ET)$.

Definition 43 We define c_2Ebool_2EIN to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.(\lambda V1f \in (2^{A_27a}).(ap \ V1f \ V0x)))$

Definition 44 We define $c_2Epred_set_2ESUBSET$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(ap \ ($

Definition 45 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap \ (c_2Ebool_2E_21 \ 2) \ (\lambda V2t \in$

Definition 46 We define $c_2Epred_set_2EINSERT$ to be $\lambda A_27a : \iota.\lambda V0x \in A_27a.\lambda V1s \in (2^{A_27a}).(ap \ (c_2$

Definition 47 We define $c_2Epred_set_2EEMPTY$ to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.c_2Ebool_2EF)$.

Definition 48 We define $c_2Epred_set_2EFINITE$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).(ap (c_2Ebool_2E_21 (2$

Assume the following.

$$True \quad (44)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \quad (45)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A_27a.(p V0t)) \Leftrightarrow (p V0t))) \quad (46)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge ((p V0t) \Rightarrow False) \Leftrightarrow \neg(p V0t)))))) \quad (47)$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \wedge ((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \quad (48)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \quad (49)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (50)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow \neg(p V0t)) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow \neg(p V0t)))))) \quad (51)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0Q \in 2.(\forall V1P \in (2^{A_27a}).((\forall V2x \in A_27a.((p (ap V1P V2x)) \vee (p V0Q))) \Leftrightarrow ((\forall V3x \in A_27a.(p (ap V1P V3x)) \vee (p V0Q)))))) \quad (52)$$

Assume the following.

$$\begin{aligned} & \forall A_{.27t}. \text{nonempty } A_{.27t} \Rightarrow \forall A_{.27w}. \text{nonempty } A_{.27w} \Rightarrow (\\ & p (ap (c_2Epred_set_2EFINITE (ty_2Ebinary_ieee_2Efloat A_{.27t} \\ & A_{.27w})) (c_2Epred_set_2EUNIV (ty_2Ebinary_ieee_2Efloat A_{.27t} \\ & A_{.27w})))) \end{aligned} \quad (53)$$

Assume the following.

$$\begin{aligned} & \forall A_{.27a}. \text{nonempty } A_{.27a} \Rightarrow (\forall V0s \in (2^{A_{.27a}}). (p (ap (\\ & ap (c_2Epred_set_2ESUBSET A_{.27a} V0s) (c_2Epred_set_2EUNIV \\ & A_{.27a})))))) \end{aligned} \quad (54)$$

Assume the following.

$$\begin{aligned} & \forall A_{.27a}. \text{nonempty } A_{.27a} \Rightarrow (\forall V0s \in (2^{A_{.27a}}). ((p (ap \\ & (c_2Epred_set_2EFINITE A_{.27a} V0s)) \Rightarrow (\forall V1t \in (2^{A_{.27a}}). \\ & ((p (ap (ap (c_2Epred_set_2ESUBSET A_{.27a} V1t) V0s)) \Rightarrow (p (ap (c_2Epred_set_2EFINITE \\ & A_{.27a} V1t))))))) \end{aligned} \quad (55)$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \quad (56)$$

Assume the following.

$$(\forall V0A \in 2. ((p V0A) \Rightarrow ((\neg(p V0A)) \Rightarrow \text{False}))) \quad (57)$$

Assume the following.

$$\begin{aligned} & (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p V0A) \vee (p V1B))) \Rightarrow \text{False}) \Leftrightarrow \\ & (((p V0A) \Rightarrow \text{False}) \Rightarrow ((\neg(p V1B)) \Rightarrow \text{False})))))) \end{aligned} \quad (58)$$

Assume the following.

$$\begin{aligned} & (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((\neg(p V0A)) \vee (p V1B))) \Rightarrow \text{False}) \Leftrightarrow \\ & ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow \text{False})))))) \end{aligned} \quad (59)$$

Assume the following.

$$(\forall V0A \in 2. (((\neg(p V0A)) \Rightarrow \text{False}) \Rightarrow (((p V0A) \Rightarrow \text{False}) \Rightarrow \text{False}))) \quad (60)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (\\ & (p V1q) \Leftrightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee ((\neg \\ & p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee ((\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee \\ & ((\neg(p V1q)) \vee (\neg(p V0p)))))))))) \end{aligned} \quad (61)$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \ V0p) \Leftrightarrow (\\
& (p \ V1q) \vee (p \ V2r))) \Leftrightarrow (((p \ V0p) \vee \neg(p \ V1q)) \wedge ((p \ V0p) \vee \neg(p \ V2r))) \wedge \\
& ((p \ V1q) \vee ((p \ V2r) \vee \neg(p \ V0p))))))))))
\end{aligned} \tag{62}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \ V0p) \Leftrightarrow (\\
& (p \ V1q) \Rightarrow (p \ V2r))) \Leftrightarrow (((p \ V0p) \vee (p \ V1q)) \wedge ((p \ V0p) \vee \neg(p \ V2r))) \wedge (\\
& \neg(p \ V1q) \vee ((p \ V2r) \vee \neg(p \ V0p))))))))))
\end{aligned} \tag{63}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (((p \ V0p) \Leftrightarrow \neg(p \ V1q)) \Leftrightarrow (((p \ V0p) \vee \\
& (p \ V1q)) \wedge (\neg(p \ V1q) \vee \neg(p \ V0p))))))
\end{aligned} \tag{64}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\neg((p \ V0p) \Rightarrow (p \ V1q))) \Rightarrow (p \ V0p))) \tag{65}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\neg((p \ V0p) \Rightarrow (p \ V1q))) \Rightarrow \neg(p \ V1q))) \tag{66}$$

Theorem 1

$$\begin{aligned}
& \forall A.27a.nonempty \ A.27a \Rightarrow \forall A.27b.nonempty \ A.27b \Rightarrow (\\
& p \ (ap \ (c.2Epred_set.2EFINITE \ (ty.2Ebinary_ieee.2Efloat \ A.27a \\
& A.27b)) \ (ap \ (c.2Epred_set.2EGSPEC \ (ty.2Ebinary_ieee.2Efloat \\
& A.27a \ A.27b) \ (ty.2Ebinary_ieee.2Efloat \ A.27a \ A.27b)) \ (\lambda V0a \in \\
& (ty.2Ebinary_ieee.2Efloat \ A.27a \ A.27b). (ap \ (ap \ (c.2Epair.2E.2C \\
& (ty.2Ebinary_ieee.2Efloat \ A.27a \ A.27b) \ 2) \ V0a) \ (ap \ (c.2Ebinary_ieee.2Efloat_is_finite \\
& A.27a \ A.27b) \ V0a))))))
\end{aligned}$$