

thm\_2Elift\_2ieee\_2Enon\_2representable\_2float\_2is\_2zero  
(TMc5RPfS98anRC2tH8dfAqfRpFCjCZGPCyy)

October 26, 2020

Let  $ty\_2Ebool\_2Eitself : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Ebool\_2Eitself\ A0) \quad (1)$$

Let  $c\_2Ebool\_2Ethe\_2value : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ebool\_2Ethe\_2value\ A\_27a \in (ty\_2Ebool\_2Eitself\ A\_27a) \quad (2)$$

Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \quad (3)$$

Let  $c\_2Efc\_2Edimindex : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Efc\_2Edimindex\ A\_27a \in (ty\_2Enum\_2Enum^{(ty\_2Ebool\_2Eitself\ A\_27a)}) \quad (4)$$

Let  $c\_2Enum\_2EZERO\_2REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2EZERO\_2REP \in \omega \quad (5)$$

Let  $c\_2Enum\_2EABS\_2num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_2num \in (ty\_2Enum\_2Enum^{\omega}) \quad (6)$$

**Definition 1** We define  $c\_2Emin\_2E3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_2o\ (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Enum\_2E0$  to be  $(ap\ c\_2Enum\_2EABS\_2num\ c\_2Enum\_2EZERO\_2REP)$ .

**Definition 3** We define  $c\_2Earithmic\_2EZERO$  to be  $c\_2Enum\_2E0$ .

Let  $c\_2Enum\_2EREP\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EREP\_num \in (\omega^{ty\_2Enum\_2Enum}) \quad (7)$$

Let  $c\_2Enum\_2ESUC\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2ESUC\_REP \in (\omega^{\omega}) \quad (8)$$

**Definition 4** We define  $c\_2Ebool\_2ET$  to be  $(\lambda x. (\lambda y. (\lambda z. (c\_2Emin\_2E\_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))))$

**Definition 5** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A\_27a : \iota. (\lambda V0P \in (2^{A\_27a}). (\lambda x. (c\_2Emin\_2E\_3D (2^{A\_27a}))))$

**Definition 6** We define  $c\_2Enum\_2ESUC$  to be  $\lambda V0m \in ty\_2Enum\_2Enum. (\lambda x. (c\_2Enum\_2EABS\_num (x)))$

Let  $c\_2Earithmetic\_2E\_2B : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum)^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum} \quad (9)$$

**Definition 7** We define  $c\_2Earithmetic\_2EBIT2$  to be  $\lambda V0n \in ty\_2Enum\_2Enum. (\lambda x. (c\_2Earithmetic\_2E\_2B (x)))$

**Definition 8** We define  $c\_2Earithmetic\_2ENUMERAL$  to be  $\lambda V0x \in ty\_2Enum\_2Enum.V0x$ .

Let  $ty\_2Erealax\_2Ereal : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Erealax\_2Ereal \quad (10)$$

Let  $c\_2Ereal\_2Ereal\_of\_num : \iota$  be given. Assume the following.

$$c\_2Ereal\_2Ereal\_of\_num \in (ty\_2Erealax\_2Ereal)^{ty\_2Enum\_2Enum} \quad (11)$$

Let  $c\_2Ereal\_2Epow : \iota$  be given. Assume the following.

$$c\_2Ereal\_2Epow \in ((ty\_2Erealax\_2Ereal)^{ty\_2Enum\_2Enum})^{ty\_2Erealax\_2Ereal} \quad (12)$$

Let  $ty\_2EfcP\_2Ecart : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2EfcP\_2Ecart\ A0\ A1) \quad (13)$$

Let  $ty\_2Ebinary\_ieee\_2Efloat : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Ebinary\_ieee\_2Efloat\ A0\ A1) \quad (14)$$

Let  $c\_2Ebinary\_ieee\_2Efloat\_Significand : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27t.nonempty\ A\_27t \Rightarrow \forall A\_27w.nonempty\ A\_27w \Rightarrow c\_2Ebinary\_ieee\_2Efloat\_Significand\ A\_27t\ A\_27w \in ((ty\_2EfcP\_2Ecart\ 2\ A\_27t)^{(ty\_2Ebinary\_ieee\_2Efloat\ A\_27t\ A\_27w)}) \quad (15)$$

Let  $ty\_2EfcP\_2Efinite\_image : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2EfcP\_2Efinite\_image\ A0) \quad (16)$$

**Definition 9** We define  $c\_2Ebool\_2EF$  to be  $(ap (c\_2Ebool\_2E\_21) 2) (\lambda V0t \in 2.V0t)$ .

**Definition 10** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p P \Rightarrow p Q)$  of type  $\iota$ .

**Definition 11** We define  $c\_2Ebool\_2E\_7E$  to be  $(\lambda V0t \in 2.(ap (ap c\_2Emin\_2E\_3D\_3D\_3E V0t) c\_2Ebool\_2E\_21))$

**Definition 12** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21) 2) (\lambda V2t \in 2.V2t)))$

**Definition 13** We define  $c\_2Emin\_2E\_40$  to be  $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x))$  then  $(the (\lambda x.x \in A \wedge P x))$  of type  $\iota \Rightarrow \iota$ .

**Definition 14** We define  $c\_2Ebool\_2E\_3F$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap V0P (ap (c\_2Emin\_2E\_40 A\_27a))))$

**Definition 15** We define  $c\_2Eprim\_rec\_2E\_3C$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum$

**Definition 16** We define  $c\_2Ebool\_2E\_3F\_21$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap (ap c\_2Ebool\_2E\_2F\_5C A\_27a))))$

**Definition 17** We define  $c\_2Efcp\_2Efinite\_index$  to be  $\lambda A\_27a : \iota.(ap (c\_2Emin\_2E\_40 (A\_27a^{ty\_2Enum\_2Enum})))$

Let  $c\_2Efcp\_2Edest\_cart : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow c\_2Efcp\_2Edest\_cart A\_27a A\_27b \in ((A\_27a^{(ty\_2Efcp\_2Efinite\_image A\_27b)})^{(ty\_2Efcp\_2Ecart A\_27a A\_27b)}) \quad (17)$$

**Definition 18** We define  $c\_2Efcp\_2Efcp\_index$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in (ty\_2Efcp\_2Ecart A\_27a A\_27b)$

Let  $c\_2Earithmetic\_2EEXP : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2EEXP \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (18)$$

**Definition 19** We define  $c\_2Ebool\_2ECOND$  to be  $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.(ap (ap c\_2Ebool\_2E\_21) 2) (\lambda V3t3 \in 2.V3t3))))$

**Definition 20** We define  $c\_2Ebit\_2ESBIT$  to be  $\lambda V0b \in 2.\lambda V1n \in ty\_2Enum\_2Enum.(ap (ap (ap (c\_2Ebool\_2E\_21) 2) (\lambda V2t \in 2.V2t))))$

Let  $c\_2Esum\_num\_2ESUM : \iota$  be given. Assume the following.

$$c\_2Esum\_num\_2ESUM \in ((ty\_2Enum\_2Enum^{(ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})})^{ty\_2Enum\_2Enum}) \quad (19)$$

**Definition 21** We define  $c\_2Ewords\_2Ew2n$  to be  $\lambda A\_27a : \iota.\lambda V0w \in (ty\_2Efcp\_2Ecart 2 A\_27a).(ap (ap c\_2Ebool\_2E\_21) 2) (\lambda V1t \in 2.V1t)$

Let  $ty\_2Ehreal\_2Ehreal : \iota$  be given. Assume the following.

$$nonempty ty\_2Ehreal\_2Ehreal \quad (20)$$

Let  $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty\_2Epair\_2Eprod A0 A1) \quad (21)$$

Let  $c\_2Erealax\_2Ereal\_REP\_CLASS : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Ereal\_REP\_CLASS \in ((2^{(ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty\_2Ehreal\_2Ehreal)})^{ty\_2Erealax\_2Ereal\_REP\_CLASS}) \quad (22)$$

**Definition 22** We define  $c\_2Erealax\_2Ereal\_REP$  to be  $\lambda V0a \in ty\_2Erealax\_2Ereal.(ap (c\_2Emin\_2E40 (t$   
Let  $c\_2Erealax\_2Etreal\_inv : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etreal\_inv \in ((ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty\_2Ehreal\_2Ehreal)(ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty\_2Ehreal\_2Ehreal)) \quad (23)$$

Let  $c\_2Erealax\_2Etreal\_eq : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etreal\_eq \in ((2^{(ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty\_2Ehreal\_2Ehreal)})(ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty\_2Ehreal\_2Ehreal)) \quad (24)$$

Let  $c\_2Erealax\_2Ereal\_ABS\_CLASS : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Ereal\_ABS\_CLASS \in (ty\_2Erealax\_2Ereal^{(2^{(ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty\_2Ehreal\_2Ehreal)}})) \quad (25)$$

**Definition 23** We define  $c\_2Erealax\_2Ereal\_ABS$  to be  $\lambda V0r \in (ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty\_2Ehreal\_2Ehreal)$

**Definition 24** We define  $c\_2Erealax\_2Einv$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.(ap c\_2Erealax\_2Ereal\_ABS$

Let  $c\_2Erealax\_2Etreal\_mul : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etreal\_mul \in (((ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty\_2Ehreal\_2Ehreal)(ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty\_2Ehreal\_2Ehreal))^{(ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty\_2Ehreal\_2Ehreal)}) \quad (26)$$

**Definition 25** We define  $c\_2Erealax\_2Ereal\_mul$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Erealax\_2Ereal$

**Definition 26** We define  $c\_2Ereal\_2E2F$  to be  $\lambda V0x \in ty\_2Erealax\_2Ereal.\lambda V1y \in ty\_2Erealax\_2Ereal.($

**Definition 27** We define  $c\_2Earithmetic\_2EBIT1$  to be  $\lambda V0n \in ty\_2Enum\_2Enum.(ap (ap c\_2Earithmetic$

Let  $c\_2Erealax\_2Etreal\_add : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etreal\_add \in (((ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty\_2Ehreal\_2Ehreal)(ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty\_2Ehreal\_2Ehreal))^{(ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty\_2Ehreal\_2Ehreal)}) \quad (27)$$

**Definition 28** We define  $c\_2Erealax\_2Ereal\_add$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Erealax\_2Ereal$

Let  $c\_2Ewords\_2EINT\_MAX : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_2Ewords\_2EINT\_MAX A\_27a \in (ty\_2Enum\_2Enum^{(ty\_2Ebool\_2Eitself A\_27a)}) \quad (28)$$

Let  $c\_2Ebinary\_ieee\_2Efloat\_Exponent : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27t.nonempty A\_27t \Rightarrow \forall A\_27w.nonempty A\_27w \Rightarrow c\_2Ebinary\_ieee\_2Efloat\_Exponent A\_27t A\_27w \in ((ty\_2Efc\_2Ecart 2 A\_27w)^{(ty\_2Ebinary\_ieee\_2Efloat A\_27t A\_27w)}) \quad (29)$$

Let  $ty\_2Eone\_2Eone : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Eone\_2Eone \quad (30)$$

Let  $c\_2Ebinary\_ieee\_2Efloat\_sign : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} & \forall A\_27t.nonempty\ A\_27t \Rightarrow \forall A\_27w.nonempty\ A\_27w \Rightarrow c\_2Ebinary\_ieee\_2Efloat\_sign \\ & A\_27t\ A\_27w \in ((ty\_2Efc\_2Ecart\ 2\ ty\_2Eone\_2Eone)^{(ty\_2Ebinary\_ieee\_2Efloat\ A\_27t\ A\_27w)}) \end{aligned} \quad (31)$$

Let  $c\_2Erealax\_2Etreax\_neg : \iota$  be given. Assume the following.

$$\begin{aligned} & c\_2Erealax\_2Etreax\_neg \in ((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal \\ & ty\_2Ehreal\_2Ehreal)^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)}) \end{aligned} \quad (32)$$

**Definition 29** We define  $c\_2Erealax\_2Ereal\_neg$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.(ap\ c\_2Erealax\_2Ereal$

Let  $c\_2Earithmetic\_2EDIV : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2EDIV \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (33)$$

**Definition 30** We define  $c\_2Ebit\_2EDIV\_2EXP$  to be  $\lambda V0x \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum$

Let  $c\_2Earithmetic\_2E\_2D : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2D \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (34)$$

Let  $c\_2Earithmetic\_2EMOD : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2EMOD \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (35)$$

**Definition 31** We define  $c\_2Ebit\_2EMOD\_2EXP$  to be  $\lambda V0x \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum$

**Definition 32** We define  $c\_2Ebit\_2EBITS$  to be  $\lambda V0h \in ty\_2Enum\_2Enum.\lambda V1l \in ty\_2Enum\_2Enum.\lambda V$

**Definition 33** We define  $c\_2Ebit\_2EBIT$  to be  $\lambda V0b \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum.(ap$

**Definition 34** We define  $c\_2Efc\_2EFCP$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.(\lambda V0g \in (A\_27a^{ty\_2Enum\_2Enum}).(ap$

**Definition 35** We define  $c\_2Ewords\_2En2w$  to be  $\lambda A\_27a : \iota.\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (c\_2Efc\_2EFCP$

**Definition 36** We define  $c\_2Ebinary\_ieee\_2Efloat\_to\_real$  to be  $\lambda A\_27t : \iota.\lambda A\_27w : \iota.\lambda V0x \in (ty\_2Ebinar$

Let  $c\_2Ebinary\_ieee\_2Efloat\_minus\_zero : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} & \forall A\_27t.nonempty\ A\_27t \Rightarrow \forall A\_27w.nonempty\ A\_27w \Rightarrow c\_2Ebinary\_ieee\_2Efloat\_minus\_zero \\ & A\_27t\ A\_27w \in ((ty\_2Ebinary\_ieee\_2Efloat\ A\_27t\ A\_27w)^{(ty\_2Ebool\_2Eitself\ (ty\_2Epair\_2Eprod\ A\_27t\ A\_27w)}) \end{aligned} \quad (36)$$

Let  $c\_2Ebinary\_ieee\_2Efloat\_plus\_zero : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27t.nonempty\ A\_27t \Rightarrow \forall A\_27w.nonempty\ A\_27w \Rightarrow c\_2Ebinary\_ieee\_2Efloat\_plus\_zero\ A\_27t\ A\_27w \in ((ty\_2Ebinary\_ieee\_2Efloat\ A\_27t\ A\_27w)^{(ty\_2Ebool\_2Eitself\ (ty\_2Epair\_2Eprod\ A\_27t\ A\_27w))}) \quad (37)$$

Let  $ty\_2Ebinary\_ieee\_2ERounding : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Ebinary\_ieee\_2ERounding \quad (38)$$

Let  $c\_2Ebinary\_ieee\_2ERoundTiesToEven : \iota$  be given. Assume the following.

$$c\_2Ebinary\_ieee\_2ERoundTiesToEven \in ty\_2Ebinary\_ieee\_2ERounding \quad (39)$$

Let  $c\_2Ebinary\_ieee\_2Elargest : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27t.nonempty\ A\_27t \Rightarrow \forall A\_27w.nonempty\ A\_27w \Rightarrow c\_2Ebinary\_ieee\_2Elargest\ A\_27t\ A\_27w \in (ty\_2Erealax\_2Ereal)^{(ty\_2Ebool\_2Eitself\ (ty\_2Epair\_2Eprod\ A\_27t\ A\_27w))}) \quad (40)$$

Let  $c\_2Erealax\_2Ereal\_lt : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Ereal\_lt \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal)}) \quad (41)$$

**Definition 37** We define  $c\_2Erealax\_2Ereal\_lt$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Erealax\_2Ereal$ .

**Definition 38** We define  $c\_2Ereal\_2Ereal\_lte$  to be  $\lambda V0x \in ty\_2Erealax\_2Ereal.\lambda V1y \in ty\_2Erealax\_2Ereal$ .

**Definition 39** We define  $c\_2Ereal\_2Eabs$  to be  $\lambda V0x \in ty\_2Erealax\_2Ereal.(ap\ (ap\ (ap\ (c\_2Ebool\_2ECONV))))$ .

Let  $ty\_2Ebinary\_ieee\_2Efloat\_value : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Ebinary\_ieee\_2Efloat\_value \quad (42)$$

Let  $c\_2Ebinary\_ieee\_2EFfloat : \iota$  be given. Assume the following.

$$c\_2Ebinary\_ieee\_2EFfloat \in (ty\_2Ebinary\_ieee\_2Efloat\_value)^{ty\_2Erealax\_2Ereal} \quad (43)$$

Let  $c\_2Ebinary\_ieee\_2ENaN : \iota$  be given. Assume the following.

$$c\_2Ebinary\_ieee\_2ENaN \in ty\_2Ebinary\_ieee\_2Efloat\_value \quad (44)$$

Let  $c\_2Ebinary\_ieee\_2EInfinity : \iota$  be given. Assume the following.

$$c\_2Ebinary\_ieee\_2EInfinity \in ty\_2Ebinary\_ieee\_2Efloat\_value \quad (45)$$

Let  $c\_2Ewords\_2EUINT\_MAX : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ewords\_2EUINT\_MAX\ A\_27a \in (ty\_2Enum\_2Enum)^{(ty\_2Ebool\_2Eitself\ A\_27a)} \quad (46)$$

**Definition 40** We define  $c\_2Ewords\_2Eword\_T$  to be  $\lambda A\_27a : \iota.(ap (c\_2Ewords\_2En2w A\_27a) (ap (c\_2Ew$

**Definition 41** We define  $c\_2Ebinary\_ieee\_2Efloat\_value$  to be  $\lambda A\_27t : \iota.\lambda A\_27w : \iota.\lambda V0x \in (ty\_2Ebinary\_$

Let  $c\_2Ebinary\_ieee\_2Efloat\_value\_CASE : \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty A\_27a \Rightarrow c\_2Ebinary\_ieee\_2Efloat\_value\_CASE \\ A\_27a \in & (((A\_27a^{A\_27a})^{A\_27a})^{(A\_27a^{ty\_2Erealax\_2Ereal})})^{ty\_2Ebinary\_ieee\_2Efloat\_value}) \end{aligned} \quad (47)$$

**Definition 42** We define  $c\_2Ebinary\_ieee\_2Efloat\_is\_finite$  to be  $\lambda A\_27t : \iota.\lambda A\_27w : \iota.\lambda V0x \in (ty\_2Ebina$

Let  $c\_2Epair\_2EABS\_prod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow c\_2Epair\_2EABS\_prod \\ A\_27a A\_27b \in & ((ty\_2Epair\_2Eprod A\_27a A\_27b)^{(2^{A\_27b})^{A\_27a}}) \end{aligned} \quad (48)$$

**Definition 43** We define  $c\_2Epair\_2E\_2C$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap (c\_2$

Let  $c\_2Epred\_set\_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow c\_2Epred\_set\_2EGSPEC \\ A\_27a A\_27b \in & ((2^{A\_27a})^{(ty\_2Epair\_2Eprod A\_27a 2)^{A\_27b}}) \end{aligned} \quad (49)$$

**Definition 44** We define  $c\_2Ecombin\_2EK$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.(\lambda V0x \in A\_27a.(\lambda V1y \in A\_27b.V0x)$

**Definition 45** We define  $c\_2Ereal\_2Ereal\_sub$  to be  $\lambda V0x \in ty\_2Erealax\_2Ereal.\lambda V1y \in ty\_2Erealax\_2E$

**Definition 46** We define  $c\_2Ebool\_2EIN$  to be  $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.(\lambda V1f \in (2^{A\_27a}).(ap V1f V0x))$

**Definition 47** We define  $c\_2Ebinary\_ieee\_2Eis\_closest$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0s \in (2^{(ty\_2Ebina$

**Definition 48** We define  $c\_2Ebinary\_ieee\_2Eclosest\_such$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0p \in (2^{(ty\_2Ebina$

**Definition 49** We define  $c\_2Ebinary\_ieee\_2Eclosest$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.(ap (c\_2Ebina$

Let  $c\_2Ebinary\_ieee\_2Efloat\_top : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} & \forall A\_27t.nonempty A\_27t \Rightarrow \forall A\_27w.nonempty A\_27w \Rightarrow c\_2Ebina$$

**Definition 50** We define  $c\_2Ereal\_2Ereal\_gt$  to be  $\lambda V0x \in ty\_2Erealax\_2Ereal.\lambda V1y \in ty\_2Erealax\_2E$

Let  $c\_2Ebina$

$$\begin{aligned} & \forall A\_27t.nonempty A\_27t \Rightarrow \forall A\_27w.nonempty A\_27w \Rightarrow c\_2Ebina$$

**Definition 51** We define  $c\_Ebool\_ELET$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.(\lambda V0f \in (A\_27b^{A\_27a}).(\lambda V1x \in A\_27$

Let  $c\_Ebinary\_ieee\_Efloat\_minus\_infinity : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27t.nonempty\ A\_27t \Rightarrow \forall A\_27w.nonempty\ A\_27w \Rightarrow c\_Ebinary\_ieee\_Efloat\_minus\_infinity\ A\_27t\ A\_27w \in ((ty\_Ebinary\_ieee\_Efloat\ A\_27t\ A\_27w)^{(ty\_Ebool\_Eitself\ (ty\_Epair\_Eprod\ A\_27t\ A\_27w))}) \quad (52)$$

**Definition 52** We define  $c\_Ereal\_Ereal\_ge$  to be  $\lambda V0x \in ty\_Erealax\_Ereal.\lambda V1y \in ty\_Erealax\_Ereal$

Let  $c\_Ebinary\_ieee\_Efloat\_plus\_infinity : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27t.nonempty\ A\_27t \Rightarrow \forall A\_27w.nonempty\ A\_27w \Rightarrow c\_Ebinary\_ieee\_Efloat\_plus\_infinity\ A\_27t\ A\_27w \in ((ty\_Ebinary\_ieee\_Efloat\ A\_27t\ A\_27w)^{(ty\_Ebool\_Eitself\ (ty\_Epair\_Eprod\ A\_27t\ A\_27w))}) \quad (53)$$

Let  $c\_Ebinary\_ieee\_Ethreshold : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27t.nonempty\ A\_27t \Rightarrow \forall A\_27w.nonempty\ A\_27w \Rightarrow c\_Ebinary\_ieee\_Ethreshold\ A\_27t\ A\_27w \in (ty\_Erealax\_Ereal^{(ty\_Ebool\_Eitself\ (ty\_Epair\_Eprod\ A\_27t\ A\_27w))}) \quad (54)$$

**Definition 53** We define  $c\_Ewords\_Eword\_lsb$  to be  $\lambda A\_27a : \iota.\lambda V0w \in (ty\_Efcpcart\ 2\ A\_27a).(ap$

Let  $c\_Ebinary\_ieee\_ERounding2num : \iota$  be given. Assume the following.

$$c\_Ebinary\_ieee\_ERounding2num \in (ty\_Eenum\_Eenum^{ty\_Ebinary\_ieee\_ERounding}) \quad (55)$$

**Definition 54** We define  $c\_Ebinary\_ieee\_ERounding\_CASE$  to be  $\lambda A\_27a : \iota.\lambda V0x \in ty\_Ebinary\_ieee\_E$

**Definition 55** We define  $c\_Ebinary\_ieee\_ERound$  to be  $\lambda A\_27t : \iota.\lambda A\_27w : \iota.\lambda V0mode \in ty\_Ebinary\_ie$

**Definition 56** We define  $c\_Ebinary\_ieee\_Efloat\_is\_zero$  to be  $\lambda A\_27t : \iota.\lambda A\_27w : \iota.\lambda V0x \in (ty\_Ebinar$

**Definition 57** We define  $c\_Ebinary\_ieee\_Efloat\_round$  to be  $\lambda A\_27t : \iota.\lambda A\_27w : \iota.\lambda V0mode \in ty\_Ebin$

Let  $c\_Ebinary\_ieee\_Eulp : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27t.nonempty\ A\_27t \Rightarrow \forall A\_27w.nonempty\ A\_27w \Rightarrow c\_Ebinary\_ieee\_Eulp\ A\_27t\ A\_27w \in (ty\_Erealax\_Ereal^{(ty\_Ebool\_Eitself\ (ty\_Epair\_Eprod\ A\_27t\ A\_27w))}) \quad (56)$$

Assume the following.

$$\begin{aligned} & \forall A\_27t.nonempty\ A\_27t \Rightarrow \forall A\_27w.nonempty\ A\_27w \Rightarrow ( \\ & ((ap\ (c\_Ebinary\_ieee\_Efloat\_to\_real\ A\_27t\ A\_27w)\ (ap\ (c\_Ebinary\_ieee\_Efloat\_plus\_zero \\ & \quad A\_27t\ A\_27w)\ (c\_Ebool\_Ethe\_value\ (ty\_Epair\_Eprod\ A\_27t \\ & \quad A\_27w)))) = (ap\ c\_Ereal\_Ereal\_of\_num\ c\_Eenum\_E0)) \wedge ((ap \\ & (c\_Ebinary\_ieee\_Efloat\_to\_real\ A\_27t\ A\_27w)\ (ap\ (c\_Ebinary\_ieee\_Efloat\_minus\_zero \\ & \quad A\_27t\ A\_27w)\ (c\_Ebool\_Ethe\_value\ (ty\_Epair\_Eprod\ A\_27t \\ & \quad A\_27w)))) = (ap\ c\_Ereal\_Ereal\_of\_num\ c\_Eenum\_E0))) \end{aligned} \quad (57)$$



Assume the following.

$$\begin{aligned}
& \forall A\_27t.nonempty\ A\_27t \Rightarrow \forall A\_27w.nonempty\ A\_27w \Rightarrow ( \\
& \quad \forall V0x \in ty\_2Erealax\_2Ereal. ((p\ (ap\ (ap\ c\_2Ereal\_2Ereal\_lte \\
& \quad (ap\ (ap\ c\_2Erealax\_2Ereal\_mul\ (ap\ c\_2Ereal\_2Ereal\_of\_num \\
& \quad (ap\ c\_2Earithmetic\_2ENUMERAL\ (ap\ c\_2Earithmetic\_2EBIT2\ c\_2Earithmetic\_2EZERO)))) \\
& \quad (ap\ c\_2Ereal\_2Eabs\ V0x)))\ (ap\ (c\_2Ebinary\_ieee\_2Eulp\ A\_27t\ A\_27w) \\
& \quad (c\_2Ebool\_2Ethe\_value\ (ty\_2Epair\_2Eprod\ A\_27t\ A\_27w)))))) \Rightarrow \\
& ((ap\ (ap\ (ap\ (c\_2Ebinary\_ieee\_2Efloat\_round\ A\_27t\ A\_27w)\ c\_2Ebinary\_ieee\_2EroundTiesToEven) \\
& \quad c\_2Ebool\_2ET)\ V0x) = (ap\ (c\_2Ebinary\_ieee\_2Efloat\_minus\_zero \\
& \quad A\_27t\ A\_27w)\ (c\_2Ebool\_2Ethe\_value\ (ty\_2Epair\_2Eprod\ A\_27t \\
& \quad A\_27w))))))
\end{aligned} \tag{58}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27t.nonempty\ A\_27t \Rightarrow \forall A\_27w.nonempty\ A\_27w \Rightarrow ( \\
& \quad \forall V0x \in ty\_2Erealax\_2Ereal. ((p\ (ap\ (ap\ c\_2Ereal\_2Ereal\_lte \\
& \quad (ap\ (ap\ c\_2Erealax\_2Ereal\_mul\ (ap\ c\_2Ereal\_2Ereal\_of\_num \\
& \quad (ap\ c\_2Earithmetic\_2ENUMERAL\ (ap\ c\_2Earithmetic\_2EBIT2\ c\_2Earithmetic\_2EZERO)))) \\
& \quad (ap\ c\_2Ereal\_2Eabs\ V0x)))\ (ap\ (c\_2Ebinary\_ieee\_2Eulp\ A\_27t\ A\_27w) \\
& \quad (c\_2Ebool\_2Ethe\_value\ (ty\_2Epair\_2Eprod\ A\_27t\ A\_27w)))))) \Rightarrow \\
& ((ap\ (ap\ (ap\ (c\_2Ebinary\_ieee\_2Efloat\_round\ A\_27t\ A\_27w)\ c\_2Ebinary\_ieee\_2EroundTiesToEven) \\
& \quad c\_2Ebool\_2EF)\ V0x) = (ap\ (c\_2Ebinary\_ieee\_2Efloat\_plus\_zero \\
& \quad A\_27t\ A\_27w)\ (c\_2Ebool\_2Ethe\_value\ (ty\_2Epair\_2Eprod\ A\_27t \\
& \quad A\_27w))))))
\end{aligned} \tag{59}$$

Assume the following.

$$True \tag{60}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a. ((V0x = V0x) \Leftrightarrow True)) \tag{61}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2. (((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow \\
& (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow \neg(p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow \neg( \\
& \quad p\ V0t))))))
\end{aligned} \tag{62}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty\_2Enum\_2Enum. (\forall V1m \in ty\_2Enum\_2Enum. ( \\
& \quad (((ap\ c\_2Ereal\_2Ereal\_of\_num\ V0n) = (ap\ c\_2Ereal\_2Ereal\_of\_num \\
& \quad V1m)) \Leftrightarrow (V0n = V1m)) \wedge (((ap\ c\_2Erealax\_2Ereal\_neg\ (ap\ c\_2Ereal\_2Ereal\_of\_num \\
& \quad V0n)) = (ap\ c\_2Ereal\_2Ereal\_of\_num\ V1m)) \Leftrightarrow ((V0n = c\_2Enum\_2E0) \wedge \\
& \quad (V1m = c\_2Enum\_2E0))) \wedge (((ap\ c\_2Ereal\_2Ereal\_of\_num\ V0n) = \\
& \quad (ap\ c\_2Erealax\_2Ereal\_neg\ (ap\ c\_2Ereal\_2Ereal\_of\_num\ V1m))) \Leftrightarrow \\
& \quad ((V0n = c\_2Enum\_2E0) \wedge (V1m = c\_2Enum\_2E0))) \wedge (((ap\ c\_2Erealax\_2Ereal\_neg \\
& \quad (ap\ c\_2Ereal\_2Ereal\_of\_num\ V0n)) = (ap\ c\_2Erealax\_2Ereal\_neg \\
& \quad (ap\ c\_2Ereal\_2Ereal\_of\_num\ V1m))) \Leftrightarrow (V0n = V1m)))))) \\
& \hspace{15em} (63)
\end{aligned}$$

**Theorem 1**

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\
& \quad \forall V0ff \in ty\_2Erealax\_2Ereal. (\forall V1P \in 2. ((p\ (ap\ (ap \\
& \quad c\_2Ereal\_2Ereal\_lte\ (ap\ (ap\ c\_2Erealax\_2Ereal\_mul\ (ap\ c\_2Ereal\_2Ereal\_of\_num \\
& \quad (ap\ c\_2Earithmetic\_2ENUMERAL\ (ap\ c\_2Earithmetic\_2EBIT2\ c\_2Earithmetic\_2EZERO)))) \\
& \quad (ap\ c\_2Ereal\_2Eabs\ V0ff)))\ (ap\ (c\_2Ebinary\_ieee\_2Eulp\ A\_27a \\
& \quad A\_27b)\ (c\_2Ebool\_2Ethe\_value\ (ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)))))) \Rightarrow \\
& \quad ((ap\ (c\_2Ebinary\_ieee\_2Efloat\_to\_real\ A\_27a\ A\_27b)\ (ap\ (ap \\
& \quad (ap\ (c\_2Ebinary\_ieee\_2Efloat\_round\ A\_27a\ A\_27b)\ c\_2Ebinary\_ieee\_2EroundTiesToEven) \\
& \quad V1P)\ V0ff)) = (ap\ c\_2Ereal\_2Ereal\_of\_num\ c\_2Enum\_2E0))))))
\end{aligned}$$