thm_2Elim_2ECONT__HASSUP (TMWo1AqpVqke7M65BnBgeiqr6E9yAWtuAzK)

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Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1$

Definition 3 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

Definition 4 *We define* c_2Ebool_2EF *to be* $(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 5 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 6 *We define* c_2Ebool_2E_5C_2F *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in 2$

Let $c\_2Enum\_2EZERO\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2EZERO\_\_REP \in omega \tag{1}$$

Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{2}$$

Let $c\_2Enum\_2EABS\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EABS\_\_num \in (ty\_2Enum\_2Enum^{omega}) \tag{3}$$

Definition 7 *We define* c_2Enum_2E0 *to be* $(ap\ c\_2Enum\_2EABS\_\_num\ c\_2Enum\_2EZERO\_\_REP)$.

Let $ty\_2Erealax\_2Ereal : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Erealax\_2Ereal \tag{4}$$

Let $c\_2Ereal\_2Ereal\_\_of\_\_num : \iota$ be given. Assume the following.

$$c\_2Ereal\_2Ereal\_\_of\_\_num \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}) \tag{5}$$

Let $\mathit{ty\_2Ehreal\_2Ehreal} : \iota$ be given. Assume the following.

$$\mathit{nonempty}\ \mathit{ty\_2Ehreal\_2Ehreal} \tag{6}$$

Let $\mathit{ty\_2Epair\_2Eprod} : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.\mathit{nonempty}\ A0 \Rightarrow \forall A1.\mathit{nonempty}\ A1 \Rightarrow \mathit{nonempty}\ (\mathit{ty\_2Epair\_2Eprod}\ A0\ A1) \tag{7}$$

Let $\mathit{c\_2Erealax\_2Ereal\_\_REP\_\_CLASS} : \iota$ be given. Assume the following.

$$\mathit{c\_2Erealax\_2Ereal\_\_REP\_\_CLASS} \in ((2^{(\mathit{ty\_2Epair\_2Eprod}\ \mathit{ty\_2Ehreal\_2Ehreal}\ \mathit{ty\_2Ehreal\_2Ehreal})})^{\mathit{ty\_2Erealax}} \tag{8}$$

Definition 8 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.$**if** $(\exists x \in A.p\ (\mathit{ap}\ P\ x))$ **then** $(\mathit{the}\ (\lambda x.x \in A \wedge p$ *of type* $\iota \Rightarrow \iota$.

Definition 9 *We define* c_2Erealax_2Ereal__REP *to be* $\lambda V0a \in \mathit{ty\_2Erealax\_2Ereal}.(\mathit{ap}$ (c_2Emin_2E_40 $(\mathit{ty}$

Let $\mathit{c\_2Erealax\_2Etreal\_\_add} : \iota$ be given. Assume the following.

$$\mathit{c\_2Erealax\_2Etreal\_\_add} \in (((\mathit{ty\_2Epair\_2Eprod}\ \mathit{ty\_2Ehreal\_2Ehreal}\ \mathit{ty\_2Ehreal\_2Ehreal})^{(\mathit{ty\_2Epair\_2Eprod}\ \mathit{ty\_2Ehreal\_2Ehreal}\ \mathit{ty\_2Ehreal\_2Ehreal})})^{(\mathit{ty\_2Epair\_2Eprod}\ \mathit{ty\_2Ehreal\_2Ehr} \tag{9}$$

Let $\mathit{c\_2Erealax\_2Etreal\_\_eq} : \iota$ be given. Assume the following.

$$\mathit{c\_2Erealax\_2Etreal\_\_eq} \in ((2^{(\mathit{ty\_2Epair\_2Eprod}\ \mathit{ty\_2Ehreal\_2Ehreal}\ \mathit{ty\_2Ehreal\_2Ehreal})})^{(\mathit{ty\_2Epair\_2Eprod}\ \mathit{ty\_2Eh} \tag{10}$$

Let $\mathit{c\_2Erealax\_2Ereal\_\_ABS\_\_CLASS} : \iota$ be given. Assume the following.

$$\mathit{c\_2Erealax\_2Ereal\_\_ABS\_\_CLASS} \in (\mathit{ty\_2Erealax\_2Ereal}^{(2^{(\mathit{ty\_2Epair\_2Eprod}\ \mathit{ty\_2Ehreal\_2Ehreal}\ \mathit{ty\_2Ehreal\_2Ehr} \tag{11}$$

Definition 10 *We define* c_2Erealax_2Ereal__ABS *to be* $\lambda V0r \in (\mathit{ty\_2Epair\_2Eprod}\ \mathit{ty\_2Ehreal\_2Ehreal}\ \mathit{ty}$

Definition 11 *We define* c_2Erealax_2Ereal__add *to be* $\lambda V0T1 \in \mathit{ty\_2Erealax\_2Ereal}.\lambda V1T2 \in \mathit{ty\_2Erealax}$

Let $\mathit{c\_2Erealax\_2Etreal\_\_neg} : \iota$ be given. Assume the following.

$$\mathit{c\_2Erealax\_2Etreal\_\_neg} \in ((\mathit{ty\_2Epair\_2Eprod}\ \mathit{ty\_2Ehreal\_2Ehreal}\ \mathit{ty\_2Ehreal\_2Ehreal})^{(\mathit{ty\_2Epair\_2Eprod}\ \mathit{ty\_2Ehreal\_2Ehreal}\ \mathit{ty\_2Ehreal\_2Ehreal})}) \tag{12}$$

Definition 12 *We define* c_2Erealax_2Ereal__neg *to be* $\lambda V0T1 \in \mathit{ty\_2Erealax\_2Ereal}.(\mathit{ap}$ c_2Erealax_2Ereal

Definition 13 *We define* c_2Ereal_2Ereal__sub *to be* $\lambda V0x \in \mathit{ty\_2Erealax\_2Ereal}.\lambda V1y \in \mathit{ty\_2Erealax\_2E}$

Let $\mathit{c\_2Erealax\_2Etreal\_\_lt} : \iota$ be given. Assume the following.

$$\mathit{c\_2Erealax\_2Etreal\_\_lt} \in ((2^{(\mathit{ty\_2Epair\_2Eprod}\ \mathit{ty\_2Ehreal\_2Ehreal}\ \mathit{ty\_2Ehreal\_2Ehreal})})^{(\mathit{ty\_2Epair\_2Eprod}\ \mathit{ty\_2Ehr} \tag{13}$$

Definition 14 *We define* c_2Erealax_2Ereal__lt *to be* $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Erealax\_$

Definition 15 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap$ c_2Emin_2E_3D_3D_3E $V0t)$ c_2Ebool_2E

Definition 16 *We define* c_2Ereal_2Ereal__lte *to be* $\lambda V0x \in ty\_2Erealax\_2Ereal.\lambda V1y \in ty\_2Erealax\_2Er$

Definition 17 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap$ (c_2Ebool_2E_21 2) $(\lambda V2t \in$

Definition 18 *We define* c_2Ebool_2ECOND *to be* $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.($

Definition 19 *We define* c_2Ereal_2Eabs *to be* $\lambda V0x \in ty\_2Erealax\_2Ereal.(ap\ (ap\ (ap$ (c_2Ebool_2ECONI

Let $c\_2Epair\_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2ESND\ A\_27a\ A\_27b \in (A\_27b^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}) \tag{14}$$

Let $c\_2Epair\_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EFST\ A\_27a\ A\_27b \in (A\_27a^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}) \tag{15}$$

Definition 20 *We define* c_2Epair_2EUNCURRY *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda V0f \in ((A\_27c^{A\_27}$

Let $ty\_2Emetric\_2Emetric : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Emetric\_2Emetric\ A0) \tag{16}$$

Let $c\_2Emetric\_2Emetric : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Emetric\_2Emetric\ A\_27a \in ((ty\_2Emetric\_2Emetric\ A\_27a)^{(ty\_2Erealax\_2Ereal^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27a)})}) \tag{17}$$

Definition 21 *We define* c_2Emetric_2Emr1 *to be* $(ap\ (c\_2Emetric\_2Emetric\ ty\_2Erealax\_2Ereal)\ (ap$ (c

Let $c\_2Epair\_2EABS\_\_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EABS\_\_prod\ A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{((2^{A\_27b})^{A\_27a})}) \tag{18}$$

Definition 22 *We define* c_2Epair_2E_2C *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap\ (c\_2$

Let $c\_2Enets\_2Etendsto : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Enets\_2Etendsto\ A\_27a \in (((2^{A\_27a})^{A\_27a})^{(ty\_2Epair\_2Eprod\ (ty\_2Emetric\_2Em} \tag{19}$$

Let $c\_2Emetric\_2Edist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Emetric\_2Edist\ A\_27a \in ((ty\_2Erealax\_2Ereal^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27} \tag{20}$$

Definition 23 *We define* c_2Ebool_2E_3F *to be* $\lambda A\_27a:\iota.(\lambda V0P \in (2^{A\_27a}).(ap\ V0P\ (ap$ (c_2Emin_2E_40

Let $ty\_2Etopology\_2Etopology : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Etopology\_2Etopology\ A0) \tag{21}$$

Let $c\_2Etopology\_2Etopology : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Etopology\_2Etopology\ A\_27a \in ((ty\_2Etopology\_2Etopology\ A\_27a)^{(2^{(2^{A\_27a})})}) \tag{22}$$

Definition 24 *We define* c_2Emetric_2Emtop *to be* $\lambda A\_27a:\iota.\lambda V0m \in (ty\_2Emetric\_2Emetric\ A\_27a).(ap$

Let $c\_2Enets\_2Etends : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Enets\_2Etends\ A\_27a\ A\_27b \in (((2^{(ty\_2Epair\_2Eprod\ (ty\_2Etopology\_2Etopology\ A\_27a)\ ((2^{A\_27b})^{A\_27b}))})^{A\_27a})^{(A\_27a^{A\_27b})}) \tag{23}$$

Definition 25 *We define* c_2Elim_2Etends__real__real *to be* $\lambda V0f \in (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal})$.

Definition 26 *We define* c_2Elim_2Econtl *to be* $\lambda V0f \in (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}).\lambda V1x \in ty$

Definition 27 *We define* c_2Ereal_2Esup *to be* $\lambda V0P \in (2^{ty\_2Erealax\_2Ereal}).(ap$ (c_2Emin_2E_40 ty_2Ereal

Assume the following.

$$True \tag{24}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \tag{25}$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p\ V0t))) \tag{26}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p\ V0t1) \wedge ((p\ V1t2) \wedge (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \wedge (p\ V2t3)))))) \tag{27}$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \wedge True) \Leftrightarrow (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge ((((p\ V0t) \wedge False) \Leftrightarrow False) \wedge (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t))))))) \tag{28}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow False) \Leftrightarrow (\neg (p\ V0t)))))))) \tag{29}$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p\ V0t)))\Leftrightarrow(p\ V0t)))\wedge(((\neg True)\Leftrightarrow False)\wedge((\neg False)\Leftrightarrow True))) \tag{30}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0x \in A\_27a.((V0x = V0x)\Leftrightarrow True)) \tag{31}$$

Assume the following.

$$(\forall V0t \in 2.(((True\Leftrightarrow(p\ V0t))\Leftrightarrow(p\ V0t))\wedge((((p\ V0t)\Leftrightarrow True)\Leftrightarrow(p\ V0t))\wedge(((False\Leftrightarrow(p\ V0t))\Leftrightarrow(\neg(p\ V0t)))\wedge(((p\ V0t)\Leftrightarrow False)\Leftrightarrow(\neg(p\ V0t))))))) \tag{32}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p\ V0A)\wedge(p\ V1B)))\Leftrightarrow((\neg(p\ V0A))\vee(\neg(p\ V1B))))\wedge((\neg((p\ V0A)\vee(p\ V1B)))\Leftrightarrow((\neg(p\ V0A))\wedge(\neg(p\ V1B))))))) \tag{33}$$

Assume the following.

$$(\forall V0f \in (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}).(\forall V1a \in ty\_2Erealax\_2Ereal.(\forall V2b \in ty\_2Erealax\_2Ereal.(((p\ (ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_lte}\ V1a)\ V2b))\wedge(\forall V3x \in ty\_2Erealax\_2Ereal.(((p\ (ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_lte}\ V1a)\ V3x))\wedge(p\ (ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_lte}\ V3x)\ V2b)))\Rightarrow(p\ (ap\ (ap\ \mathsf{c\_2Elim\_2Econtl}\ V0f)\ V3x)))))\Rightarrow(\exists V4M \in ty\_2Erealax\_2Ereal.(\forall V5x \in ty\_2Erealax\_2Ereal.(((p\ (ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_lte}\ V1a)\ V5x))\wedge(p\ (ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_lte}\ V5x)\ V2b)))\Rightarrow(p\ (ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_lte}\ (ap\ V0f\ V5x))\ V4M))))))))) \tag{34}$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.(\neg(p\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_lt}\ V0x)\ V0x)))) \tag{35}$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.(\forall V1y \in ty\_2Erealax\_2Ereal.((\neg(p\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_lt}\ V0x)\ V1y)))\Leftrightarrow(p\ (ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_lte}\ V1y)\ V0x))))) \tag{36}$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.(p\ (ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_lte}\ V0x)\ V0x))) \tag{37}$$

Assume the following.

$$(\forall V0P \in (2^{ty\_2Erealax\_2Ereal}).(((\exists V1x \in ty\_2Erealax\_2Ereal.(p\ (ap\ V0P\ V1x)))\wedge(\exists V2z \in ty\_2Erealax\_2Ereal.(\forall V3x \in ty\_2Erealax\_2Ereal.((p\ (ap\ V0P\ V3x))\Rightarrow(p\ (ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_lte}\ V3x)\ V2z))))))\Rightarrow(\forall V4y \in ty\_2Erealax\_2Ereal.((\exists V5x \in ty\_2Erealax\_2Ereal.((p\ (ap\ V0P\ V5x))\wedge(p\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_lt}\ V4y)\ V5x))))\Leftrightarrow(p\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_lt}\ V4y)\ (ap\ \mathsf{c\_2Ereal\_2Esup}\ V0P)))))))\tag{38}$$

Theorem 1

$$(\forall V0f \in (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}).(\forall V1a \in ty\_2Erealax\_2Ereal.(\forall V2b \in ty\_2Erealax\_2Ereal.(((p\ (ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_lte}\ V1a)\ V2b))\wedge(\forall V3x \in ty\_2Erealax\_2Ereal.(((p\ (ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_lte}\ V1a)\ V3x))\wedge(p\ (ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_lte}\ V3x)\ V2b)))\Rightarrow(p\ (ap\ (ap\ \mathsf{c\_2Elim\_2Econtl}\ V0f)\ V3x)))))\Rightarrow(\exists V4M \in ty\_2Erealax\_2Ereal.((\forall V5x \in ty\_2Erealax\_2Ereal.(((p\ (ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_lte}\ V1a)\ V5x))\wedge(p\ (ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_lte}\ V5x)\ V2b)))\Rightarrow(p\ (ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_lte}\ (ap\ V0f\ V5x))\ V4M))))\wedge(\forall V6N \in ty\_2Erealax\_2Ereal.((p\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_lt}\ V6N)\ V4M))\Rightarrow(\exists V7x \in ty\_2Erealax\_2Ereal.((p\ (ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_lte}\ V1a)\ V7x))\wedge((p\ (ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_lte}\ V7x)\ V2b))\wedge(p\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_lt}\ V6N)\ (ap\ V0f\ V7x)))))))))))))$$