

thm\_2Elim\_2ECONT\_INV  
(TMF6VUpuFV1i2Nb2ZCjHs1HqBDCB4jdoEcP)

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Let  $ty\_2Ehreal\_2Ehreal : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Ehreal\_2Ehreal \tag{1}$$

Let  $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \tag{2}$$

Let  $ty\_2Erealax\_2Ereal : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Erealax\_2Ereal \tag{3}$$

Let  $c\_2Erealax\_2Ereal\_REP\_CLASS : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Ereal\_REP\_CLASS \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{ty\_2Erealax\_2Ereal}) \tag{4}$$

**Definition 1** We define  $c\_2Emin\_2E40$  to be  $\lambda A.\lambda P \in 2^A.$ if  $(\exists x \in A.p\ (ap\ P\ x))$  then (the  $(\lambda x.x \in A \wedge p\ x)$  of type  $\iota \Rightarrow \iota$ ).

**Definition 2** We define  $c\_2Emin\_2E3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o\ (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 3** We define  $c\_2Ebool\_2E27$  to be  $(ap\ (ap\ (c\_2Emin\_2E3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x))$

**Definition 4** We define  $c\_2Ebool\_2E21$  to be  $\lambda A.27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap\ (ap\ (c\_2Emin\_2E3D\ (2^{A-27a}))\ (\lambda V1x \in 2.V1x))\ (\lambda V2x \in 2.V2x))$

**Definition 5** We define  $c\_2Erealax\_2Ereal\_REP$  to be  $\lambda V0a \in ty\_2Erealax\_2Ereal.(ap\ (c\_2Emin\_2E40\ (ty\_2Erealax\_2Ereal\ V0a)))$

Let  $c\_2Erealax\_2Etreall\_inv : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etreall\_inv \in ((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)}) \tag{5}$$

Let  $c\_2Erealax\_2Etreal\_eq : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etreal\_eq \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal)) \quad (6)$$

Let  $c\_2Erealax\_2Ereal\_ABS\_CLASS : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Ereal\_ABS\_CLASS \in (ty\_2Erealax\_2Ereal)^{(2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})} \quad (7)$$

**Definition 6** We define  $c\_2Erealax\_2Ereal\_ABS$  to be  $\lambda V0r \in (ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)$

**Definition 7** We define  $c\_2Erealax\_2Einv$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.(ap\ c\_2Erealax\_2Ereal\_ABS)$

Let  $c\_2Enum\_2EZERO\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2EZERO\_REP \in \omega \quad (8)$$

Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \quad (9)$$

Let  $c\_2Enum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum)^{\omega} \quad (10)$$

**Definition 8** We define  $c\_2Enum\_2E0$  to be  $(ap\ c\_2Enum\_2EABS\_num\ c\_2Enum\_2EZERO\_REP)$ .

Let  $c\_2Ereal\_2Ereal\_of\_num : \iota$  be given. Assume the following.

$$c\_2Ereal\_2Ereal\_of\_num \in (ty\_2Erealax\_2Ereal)^{ty\_2Enum\_2Enum} \quad (11)$$

**Definition 9** We define  $c\_2Ebool\_2EF$  to be  $(ap\ (c\_2Ebool\_2E21\ 2)\ (\lambda V0t \in 2.V0t))$ .

**Definition 10** We define  $c\_2Emin\_2E3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o\ (p\ P \Rightarrow p\ Q)$  of type  $\iota$ .

**Definition 11** We define  $c\_2Ebool\_2E7E$  to be  $(\lambda V0t \in 2.(ap\ (ap\ c\_2Emin\_2E3D\_3D\_3E\ V0t)\ c\_2Ebool\_2E21\ 2))$

**Definition 12** We define  $c\_2Ebool\_2E2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c\_2Ebool\_2E21\ 2)\ (\lambda V2t \in 2.V2t))))$

Let  $c\_2Erealax\_2Etreal\_neg : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etreal\_neg \in ((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)) \quad (12)$$

**Definition 13** We define  $c\_2Erealax\_2Ereal\_neg$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.(ap\ c\_2Erealax\_2Ereal\_neg)$

Let  $c\_2Erealax\_2Etreal\_add : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etreal\_add \in (((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)) \quad (13)$$

**Definition 14** We define  $c\_2Erealax\_2Ereal\_add$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Erealax\_2Ereal$

**Definition 15** We define  $c\_2Ereal\_2Ereal\_sub$  to be  $\lambda V0x \in ty\_2Erealax\_2Ereal.\lambda V1y \in ty\_2Erealax\_2Ereal$

Let  $c\_2Erealax\_2Etrealt\_lt : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etrealt\_lt \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal)}) \quad (14)$$

**Definition 16** We define  $c\_2Erealax\_2Ereal\_lt$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Erealax\_2Ereal$

**Definition 17** We define  $c\_2Ereal\_2Ereal\_lte$  to be  $\lambda V0x \in ty\_2Erealax\_2Ereal.\lambda V1y \in ty\_2Erealax\_2Ereal$

**Definition 18** We define  $c\_2Ebool\_2ECOND$  to be  $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.($

**Definition 19** We define  $c\_2Ereal\_2Eabs$  to be  $\lambda V0x \in ty\_2Erealax\_2Ereal.(ap\ (ap\ (ap\ (c\_2Ebool\_2ECOND$

Let  $c\_2Epair\_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2ESND \\ A\_27a\ A\_27b \in (A\_27b)^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)} \end{aligned} \quad (15)$$

Let  $c\_2Epair\_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EFST \\ A\_27a\ A\_27b \in (A\_27a)^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)} \end{aligned} \quad (16)$$

**Definition 20** We define  $c\_2Epair\_2EUNCURRY$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda V0f \in ((A\_27c)^{A\_27b}$

Let  $ty\_2Emetric\_2Emetric : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Emetric\_2Emetric\ A0) \quad (17)$$

Let  $c\_2Emetric\_2Emetric : \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Emetric\_2Emetric\ A\_27a \in ((ty\_2Emetric\_2Emetric \\ A\_27a)^{(ty\_2Erealax\_2Ereal)^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27a)}}) \end{aligned} \quad (18)$$

**Definition 21** We define  $c\_2Emetric\_2Emr1$  to be  $(ap\ (c\_2Emetric\_2Emetric\ ty\_2Erealax\_2Ereal)\ (ap\ (c$

Let  $c\_2Epair\_2EABS\_prod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EABS\_prod \\ A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{(2^{A\_27b})^{A\_27a}}) \end{aligned} \quad (19)$$

**Definition 22** We define  $c\_2Epair\_2E\_2C$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap\ (c$

Let  $c\_2Enets\_2Etendsto : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Enets\_2Etendsto\ A\_27a \in (((2^{A\_27a})^{A\_27a})^{ty\_2Epair\_2Eprod\ (ty\_2Emetric\_2E0)}) \quad (20)$$

Let  $c\_2Emetric\_2Edist : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Emetric\_2Edist\ A\_27a \in ((ty\_2Erealax\_2Ereal)^{ty\_2Epair\_2Eprod\ A\_27a\ A\_27a}) \quad (21)$$

**Definition 23** We define  $c\_2Ebool\_2E3F$  to be  $\lambda A\_27a : \iota. (\lambda V0P \in (2^{A\_27a}). (ap\ V0P\ (ap\ (c\_2Emin\_2E40$

Let  $ty\_2Etopology\_2Etopology : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Etopology\_2Etopology\ A0) \quad (22)$$

Let  $c\_2Etopology\_2Etopology : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Etopology\_2Etopology\ A\_27a \in ((ty\_2Etopology\_2Etopology\ A\_27a)^{(2^{(2^{A\_27a})})}) \quad (23)$$

**Definition 24** We define  $c\_2Emetric\_2Emtop$  to be  $\lambda A\_27a : \iota. \lambda V0m \in (ty\_2Emetric\_2Emetric\ A\_27a). (ap$

Let  $c\_2Enets\_2Etends : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Enets\_2Etends\ A\_27a\ A\_27b \in (((2^{(ty\_2Epair\_2Eprod\ (ty\_2Etopology\_2Etopology\ A\_27a)\ (2^{A\_27b})^{A\_27b}))})^{A\_27a})^{(A\_27a^{A\_27b})}) \quad (24)$$

**Definition 25** We define  $c\_2Elim\_2Etends\_real\_real$  to be  $\lambda V0f \in (ty\_2Erealax\_2Ereal)^{ty\_2Erealax\_2Ereal}.$

**Definition 26** We define  $c\_2Elim\_2Econtl$  to be  $\lambda V0f \in (ty\_2Erealax\_2Ereal)^{ty\_2Erealax\_2Ereal}. \lambda V1x \in ty$

Assume the following.

$$\begin{aligned} & (\forall V0f \in (ty\_2Erealax\_2Ereal)^{ty\_2Erealax\_2Ereal}). (\forall V1l \in \\ & ty\_2Erealax\_2Ereal. (\forall V2x \in ty\_2Erealax\_2Ereal. ((p\ ( \\ & ap\ (ap\ (ap\ c\_2Elim\_2Etends\_real\_real\ V0f)\ V1l)\ V2x)) \wedge (\neg(V1l = \\ & (ap\ c\_2Ereal\_2Ereal\_of\_num\ c\_2Enum\_2E0)))) \Rightarrow (p\ (ap\ (ap\ (ap\ c\_2Elim\_2Etends\_real\_real \\ & (\lambda V3x \in ty\_2Erealax\_2Ereal. (ap\ c\_2Erealax\_2Einv\ (ap\ V0f\ V3x)))) \\ & (ap\ c\_2Erealax\_2Einv\ V1l))\ V2x)))))) \end{aligned} \quad (25)$$

Assume the following.

$$\begin{aligned} & (\forall V0f \in (ty\_2Erealax\_2Ereal)^{ty\_2Erealax\_2Ereal}). (\forall V1x \in \\ & ty\_2Erealax\_2Ereal. ((p\ (ap\ (ap\ c\_2Elim\_2Econtl\ V0f)\ V1x)) \Leftrightarrow (p \\ & (ap\ (ap\ (ap\ c\_2Elim\_2Etends\_real\_real\ V0f)\ (ap\ V0f\ V1x))\ V1x)))) \end{aligned} \quad (26)$$

**Theorem 1**

$$\begin{aligned} & (\forall V0f \in (ty\_2Erealax\_2Ereal)^{ty\_2Erealax\_2Ereal}). (\forall V1x \in \\ & ty\_2Erealax\_2Ereal. ((p\ (ap\ (ap\ c\_2Elim\_2Econtl\ V0f)\ V1x)) \wedge ( \\ & \neg((ap\ V0f\ V1x) = (ap\ c\_2Ereal\_2Ereal\_of\_num\ c\_2Enum\_2E0)))) \Rightarrow \\ & (p\ (ap\ (ap\ c\_2Elim\_2Econtl\ (\lambda V2x \in ty\_2Erealax\_2Ereal. (ap\ c\_2Erealax\_2Einv \\ & (ap\ V0f\ V2x))))\ V1x)))) \end{aligned}$$