

# thm\_2Elim\_2EDIFF\_INVERSE (TMJreki9eFd4b9By6ohWARrScNgjDxw1Nq9)

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**Definition 1** We define `c_2Emin_2E_3D` to be  $\lambda A. \lambda x \in A. \lambda y \in A. inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define `c_2Ebool_2E_2T` to be  $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V 0x \in 2.V 0x)) (\lambda V 1x \in 2.V 1x))$

**Definition 3** We define `c_2Ebool_2E_21` to be  $\lambda A. 27a : \iota. (\lambda V 0P \in (2^{A-27a}). (ap (ap (c_2Emin_2E_3D (2^{A-27a})))$

**Definition 4** We define `c_2Ebool_2E_2F` to be  $(ap (c_2Ebool_2E_21 2) (\lambda V 0t \in 2.V 0t))$ .

**Definition 5** We define `c_2Emin_2E_3D_3D_3E` to be  $\lambda P \in 2. \lambda Q \in 2. inj\_o (p P \Rightarrow p Q)$  of type  $\iota$ .

**Definition 6** We define `c_2Ebool_2E_2F_5C` to be  $(\lambda V 0t1 \in 2. (\lambda V 1t2 \in 2. (ap (c_2Ebool_2E_21 2) (\lambda V 2t \in 2.V 2t))$

**Definition 7** We define `c_2Emin_2E_40` to be  $\lambda A. \lambda P \in 2^A. \mathbf{if} (\exists x \in A. p (ap P x)) \mathbf{then} (the (\lambda x. x \in A \wedge p x))$  of type  $\iota \Rightarrow \iota$ .

**Definition 8** We define `c_2Ebool_2ECOND` to be  $\lambda A. 27a : \iota. (\lambda V 0t \in 2. (\lambda V 1t1 \in A. 27a. (\lambda V 2t2 \in A. 27a. (ap$

Let `ty_2Ehreal_2Ehreal` :  $\iota$  be given. Assume the following.

$$nonempty\ ty\_2Ehreal\_2Ehreal \tag{1}$$

Let `ty_2Epair_2Eprod` :  $\iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0. nonempty\ A0 \Rightarrow \forall A1. nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \tag{2}$$

Let `ty_2Erealax_2Ereal` :  $\iota$  be given. Assume the following.

$$nonempty\ ty\_2Erealax\_2Ereal \tag{3}$$

Let `c_2Erealax_2Ereal_REP_CLASS` :  $\iota$  be given. Assume the following.

$$c\_2Erealax\_2Ereal\_REP\_CLASS \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})\ ty\_2Erealax) \tag{4}$$

**Definition 9** We define  $c\_Erealax\_Ereal\_REP$  to be  $\lambda V0a \in ty\_Erealax\_Ereal.(ap (c\_Emin\_E40 (ty\_Erealax\_Ereal\_neg : \iota$  be given. Assume the following.

$$c\_Erealax\_Ereal\_neg \in ((ty\_Epair\_Eprod ty\_Ehreal\_Ehreal ty\_Ehreal\_Ehreal)(ty\_Epair\_Eprod ty\_Ehreal\_Ehreal ty\_Ehreal\_Ehreal)) \quad (5)$$

Let  $c\_Erealax\_Ereal\_eq : \iota$  be given. Assume the following.

$$c\_Erealax\_Ereal\_eq \in ((2^{(ty\_Epair\_Eprod ty\_Ehreal\_Ehreal ty\_Ehreal\_Ehreal)})(ty\_Epair\_Eprod ty\_Ehreal\_Ehreal ty\_Ehreal\_Ehreal)) \quad (6)$$

Let  $c\_Erealax\_Ereal\_ABS\_CLASS : \iota$  be given. Assume the following.

$$c\_Erealax\_Ereal\_ABS\_CLASS \in (ty\_Erealax\_Ereal)^{(2^{(ty\_Epair\_Eprod ty\_Ehreal\_Ehreal ty\_Ehreal\_Ehreal)})} \quad (7)$$

**Definition 10** We define  $c\_Erealax\_Ereal\_ABS$  to be  $\lambda V0r \in (ty\_Epair\_Eprod ty\_Ehreal\_Ehreal ty\_Ehreal\_Ehreal)$

**Definition 11** We define  $c\_Erealax\_Ereal\_neg$  to be  $\lambda V0T1 \in ty\_Erealax\_Ereal.(ap c\_Erealax\_Ereal\_neg$

Let  $c\_Erealax\_Ereal\_add : \iota$  be given. Assume the following.

$$c\_Erealax\_Ereal\_add \in (((ty\_Epair\_Eprod ty\_Ehreal\_Ehreal ty\_Ehreal\_Ehreal)(ty\_Epair\_Eprod ty\_Ehreal\_Ehreal ty\_Ehreal\_Ehreal))^{(ty\_Epair\_Eprod ty\_Ehreal\_Ehreal ty\_Ehreal\_Ehreal)}) \quad (8)$$

**Definition 12** We define  $c\_Erealax\_Ereal\_add$  to be  $\lambda V0T1 \in ty\_Erealax\_Ereal.\lambda V1T2 \in ty\_Erealax\_Ereal$

**Definition 13** We define  $c\_Ereal\_Ereal\_sub$  to be  $\lambda V0x \in ty\_Erealax\_Ereal.\lambda V1y \in ty\_Erealax\_Ereal$

Let  $c\_Eenum\_EZERO\_REP : \iota$  be given. Assume the following.

$$c\_Eenum\_EZERO\_REP \in \omega \quad (9)$$

Let  $ty\_Eenum\_Eenum : \iota$  be given. Assume the following.

$$nonempty\ ty\_Eenum\_Eenum \quad (10)$$

Let  $c\_Eenum\_EABS\_num : \iota$  be given. Assume the following.

$$c\_Eenum\_EABS\_num \in (ty\_Eenum\_Eenum)^{\omega} \quad (11)$$

**Definition 14** We define  $c\_Eenum\_E0$  to be  $(ap c\_Eenum\_EABS\_num c\_Eenum\_EZERO\_REP)$ .

Let  $c\_Ereal\_Ereal\_of\_num : \iota$  be given. Assume the following.

$$c\_Ereal\_Ereal\_of\_num \in (ty\_Erealax\_Ereal)^{ty\_Eenum\_Eenum} \quad (12)$$

Let  $c\_Erealax\_Ereal\_lt : \iota$  be given. Assume the following.

$$c\_Erealax\_Ereal\_lt \in ((2^{(ty\_Epair\_Eprod ty\_Ehreal\_Ehreal ty\_Ehreal\_Ehreal)})(ty\_Epair\_Eprod ty\_Ehreal\_Ehreal ty\_Ehreal\_Ehreal)) \quad (13)$$

**Definition 15** We define  $c\_2Erealax\_2Ereal\_lt$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Erealax\_2Ereal$ .

**Definition 16** We define  $c\_2Ebool\_2E\_7E$  to be  $(\lambda V0t \in 2.(ap (ap c\_2Emin\_2E\_3D\_3D\_3E V0t) c\_2Ebool\_2E\_7E))$ .

**Definition 17** We define  $c\_2Ereal\_2Ereal\_lte$  to be  $\lambda V0x \in ty\_2Erealax\_2Ereal.\lambda V1y \in ty\_2Erealax\_2Ereal$ .

**Definition 18** We define  $c\_2Ereal\_2Eabs$  to be  $\lambda V0x \in ty\_2Erealax\_2Ereal.(ap (ap (ap (c\_2Ebool\_2ECONJ))$

Let  $c\_2Epair\_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow c\_2Epair\_2ESND \\ A\_27a A\_27b \in (A\_27b)^{(ty\_2Epair\_2Eprod A\_27a A\_27b)} \end{aligned} \quad (14)$$

Let  $c\_2Epair\_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow c\_2Epair\_2EFST \\ A\_27a A\_27b \in (A\_27a)^{(ty\_2Epair\_2Eprod A\_27a A\_27b)} \end{aligned} \quad (15)$$

**Definition 19** We define  $c\_2Epair\_2EUNCURRY$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda V0f \in ((A\_27c)^{A\_27a})$ .

Let  $ty\_2Emetric\_2Emetric : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty\_2Emetric\_2Emetric A0) \quad (16)$$

Let  $c\_2Emetric\_2Emetric : \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty A\_27a \Rightarrow c\_2Emetric\_2Emetric A\_27a \in ((ty\_2Emetric\_2Emetric \\ A\_27a)^{(ty\_2Erealax\_2Ereal)^{(ty\_2Epair\_2Eprod A\_27a A\_27a)}}) \end{aligned} \quad (17)$$

**Definition 20** We define  $c\_2Emetric\_2Emr1$  to be  $(ap (c\_2Emetric\_2Emetric ty\_2Erealax\_2Ereal) (ap (c\_2Ebool\_2ECONJ))$

Let  $c\_2Epair\_2EABS\_prod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow c\_2Epair\_2EABS\_prod \\ A\_27a A\_27b \in ((ty\_2Epair\_2Eprod A\_27a A\_27b)^{(2^{A\_27b})^{A\_27a}}) \end{aligned} \quad (18)$$

**Definition 21** We define  $c\_2Epair\_2E\_2C$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap (c\_2Ebool\_2ECONJ))$

Let  $c\_2Enets\_2Eextendsto : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_2Enets\_2Eextendsto A\_27a \in (((2^{A\_27a})^{A\_27a})^{(ty\_2Epair\_2Eprod (ty\_2Emetric\_2Emetric A\_27a) A\_27a)}) \quad (19)$$

Let  $c\_2Emetric\_2Edist : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_2Emetric\_2Edist A\_27a \in ((ty\_2Erealax\_2Ereal)^{(ty\_2Epair\_2Eprod A\_27a A\_27a)}) \quad (20)$$

**Definition 22** We define  $c\_2Ebool\_2E\_3F$  to be  $\lambda A\_27a : \iota.(2^{A\_27a}).(ap V0P (ap (c\_2Emin\_2E\_4D))$

Let  $ty\_2Etopology\_2Etopology : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Etopology\_2Etopology\ A0) \quad (21)$$

Let  $c\_2Etopology\_2Etopology : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Etopology\_2Etopology\ A\_27a \in ((ty\_2Etopology\_2Etopology\ A\_27a)^{(2^{(2^A-27a)})}) \quad (22)$$

**Definition 23** We define  $c\_2Emetric\_2Emtop$  to be  $\lambda A\_27a : \iota.\lambda V0m \in (ty\_2Emetric\_2Emetric\ A\_27a).(ap$

Let  $c\_2Enets\_2Etends : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Enets\_2Etends\ A\_27a\ A\_27b \in (((2^{(ty\_2Epair\_2Eprod\ (ty\_2Etopology\_2Etopology\ A\_27a)\ (2^{A-27b})^{A-27b}))})_{A\_27a})_{(A\_27a^{A-27b})}) \quad (23)$$

**Definition 24** We define  $c\_2Elim\_2Etends\_real\_real$  to be  $\lambda V0f \in (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}).$

Let  $c\_2Erealax\_2Etrealm\_inv : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etrealm\_inv \in ((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)_{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)}) \quad (24)$$

**Definition 25** We define  $c\_2Erealax\_2Einv$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.(ap\ c\_2Erealax\_2Ereal\_ABS$

Let  $c\_2Erealax\_2Etrealm\_mul : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etrealm\_mul \in (((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)_{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})_{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)}) \quad (25)$$

**Definition 26** We define  $c\_2Erealax\_2Ereal\_mul$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Erealax\_2Ereal.$

**Definition 27** We define  $c\_2Ereal\_2E\_2F$  to be  $\lambda V0x \in ty\_2Erealax\_2Ereal.\lambda V1y \in ty\_2Erealax\_2Ereal.$

**Definition 28** We define  $c\_2Elim\_2Ediff$  to be  $\lambda V0f \in (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}).\lambda V1l \in ty\_2Erealax\_2Ereal.$

**Definition 29** We define  $c\_2Elim\_2Econt$  to be  $\lambda V0f \in (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}).\lambda V1x \in ty\_2Erealax\_2Ereal.$

**Definition 30** We define  $c\_2Earithmetic\_2EZERO$  to be  $c\_2Enum\_2E0$ .

Let  $c\_2Enum\_2EREP\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EREP\_num \in (\omega^{ty\_2Enum\_2Enum}) \quad (26)$$

Let  $c\_2Enum\_2ESUC\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2ESUC\_REP \in (\omega^{\omega^{omega}}) \quad (27)$$

**Definition 31** We define  $c\_2Enum\_2ESUC$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_num$   
Let  $c\_2Earithmetic\_2E\_2B : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (28)$$

**Definition 32** We define  $c\_2Earithmetic\_2EBIT1$  to be  $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic$

**Definition 33** We define  $c\_2Earithmetic\_2ENUMERAL$  to be  $\lambda V0x \in ty\_2Enum\_2Enum.V0x$ .

**Definition 34** We define  $c\_2Ebool\_2E\_5C\_2F$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c\_2Ebool\_2E\_21\ 2)\ (\lambda V2t \in$   
Assume the following.

$$True \quad (29)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \quad (30)$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p\ V0t))) \quad (31)$$

Assume the following.

$$(\forall V0t \in 2.((p\ V0t) \vee (\neg(p\ V0t)))) \quad (32)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A\_27a.(p\ V0t) \Leftrightarrow (p\ V0t)))) \quad (33)$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \wedge True) \Leftrightarrow (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge (((p\ V0t) \wedge False) \Leftrightarrow False) \wedge (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)))))) \quad (34)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((p\ V0t) \Rightarrow False) \Leftrightarrow (\neg(p\ V0t)))))) \quad (35)$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t)) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True)))) \quad (36)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a. ((V0x = V0x) \Leftrightarrow True)) \quad (37)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a. (\forall V1y \in A\_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (38)$$

Assume the following.

$$(\forall V0t \in 2. (((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow \neg(p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow \neg(p\ V0t)))))) \quad (39)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t1 \in A\_27a. (\forall V1t2 \in A\_27a. (((ap\ (ap\ (ap\ (c\_2Ebool\_2ECOND\ A\_27a)\ c\_2Ebool\_2ET)\ V0t1)\ V1t2) = V0t1) \wedge ((ap\ (ap\ (ap\ (c\_2Ebool\_2ECOND\ A\_27a)\ c\_2Ebool\_2EF)\ V0t1)\ V1t2) = V1t2)))))) \quad (40)$$

Assume the following.

$$(\forall V0f \in (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}). (\forall V1y0 \in ty\_2Erealax\_2Ereal. (\forall V2x0 \in ty\_2Erealax\_2Ereal. ((p\ (ap\ (ap\ (ap\ c\_2Elim\_2Etends\_real\_real\ V0f)\ V1y0)\ V2x0)) \Leftrightarrow (\forall V3e \in ty\_2Erealax\_2Ereal. ((p\ (ap\ (ap\ c\_2Erealax\_2Ereal\_lt\ (ap\ c\_2Ereal\_2Ereal\_of\_num\ c\_2Enum\_2E0))\ V3e)) \Rightarrow (\exists V4d \in ty\_2Erealax\_2Ereal. ((p\ (ap\ (ap\ c\_2Erealax\_2Ereal\_lt\ (ap\ c\_2Ereal\_2Ereal\_of\_num\ c\_2Enum\_2E0))\ V4d)) \wedge (\forall V5x \in ty\_2Erealax\_2Ereal. (((p\ (ap\ (ap\ c\_2Erealax\_2Ereal\_lt\ (ap\ c\_2Ereal\_2Ereal\_of\_num\ c\_2Enum\_2E0))\ (ap\ c\_2Ereal\_2Eabs\ (ap\ (ap\ c\_2Ereal\_2Ereal\_sub\ V5x)\ V2x0)))) \wedge (p\ (ap\ (ap\ c\_2Erealax\_2Ereal\_lt\ (ap\ c\_2Ereal\_2Eabs\ (ap\ (ap\ c\_2Ereal\_2Ereal\_sub\ (ap\ (ap\ c\_2Erealax\_2Ereal\_lt\ (ap\ c\_2Ereal\_2Eabs\ (ap\ (ap\ c\_2Ereal\_2Ereal\_sub\ (ap\ V0f\ V5x))\ V1y0))\ V3e)))))))))))))))))) \quad (41)$$

Assume the following.

$$(\forall V0f \in (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}). (\forall V1l \in ty\_2Erealax\_2Ereal. (\forall V2x \in ty\_2Erealax\_2Ereal. (((p\ (ap\ (ap\ (ap\ c\_2Elim\_2Etends\_real\_real\ V0f)\ V1l)\ V2x)) \wedge (\neg(V1l = (ap\ c\_2Ereal\_2Ereal\_of\_num\ c\_2Enum\_2E0)))))) \Rightarrow (p\ (ap\ (ap\ (ap\ c\_2Elim\_2Etends\_real\_real\ (\lambda V3x \in ty\_2Erealax\_2Ereal. (ap\ c\_2Erealax\_2Einv\ (ap\ V0f\ V3x)))\ (ap\ c\_2Erealax\_2Einv\ V1l))\ V2x)))))) \quad (42)$$

Assume the following.

$$\begin{aligned}
& (\forall V0f \in (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}).(\forall V1g \in \\
& (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}).(\forall V2x0 \in ty\_2Erealax\_2Ereal. \\
& (\forall V3l \in ty\_2Erealax\_2Ereal.(((p (ap (ap (ap c\_2Elim\_2Etends\_real\_real \\
& (\lambda V4x \in ty\_2Erealax\_2Ereal.(ap (ap c\_2Ereal\_2Ereal\_sub ( \\
& ap V0f V4x)) (ap V1g V4x)))) (ap c\_2Ereal\_2Ereal\_of\_num c\_2Enum\_2E0)) \\
& V2x0)) \wedge (p (ap (ap (ap c\_2Elim\_2Etends\_real\_real V1g) V3l) V2x0)))) \Rightarrow \\
& (p (ap (ap (ap c\_2Elim\_2Etends\_real\_real V0f) V3l) V2x0))))))
\end{aligned} \tag{43}$$

Assume the following.

$$\begin{aligned}
& (\forall V0f \in (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}).(\forall V1l \in \\
& ty\_2Erealax\_2Ereal.(\forall V2x \in ty\_2Erealax\_2Ereal.((p (ap \\
& (ap (ap c\_2Elim\_2Ediff1 V0f) V1l) V2x)) \Rightarrow (p (ap (ap c\_2Elim\_2Econt1 \\
& V0f) V2x))))))
\end{aligned} \tag{44}$$

Assume the following.

$$\begin{aligned}
& (\forall V0f \in (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}).(\forall V1x \in \\
& ty\_2Erealax\_2Ereal.((p (ap (ap c\_2Elim\_2Econt1 V0f) V1x)) \Leftrightarrow (p \\
& (ap (ap (ap c\_2Elim\_2Etends\_real\_real V0f) (ap V0f V1x)) V1x))))))
\end{aligned} \tag{45}$$

Assume the following.

$$\begin{aligned}
& (\forall V0f \in (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}).(\forall V1l \in \\
& ty\_2Erealax\_2Ereal.(\forall V2x \in ty\_2Erealax\_2Ereal.((p (ap \\
& (ap (ap c\_2Elim\_2Ediff1 V0f) V1l) V2x)) \Leftrightarrow (\exists V3g \in (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}). \\
& ((\forall V4z \in ty\_2Erealax\_2Ereal.((ap (ap c\_2Ereal\_2Ereal\_sub \\
& (ap V0f V4z)) (ap V0f V2x)) = (ap (ap c\_2Erealax\_2Ereal\_mul (ap V3g \\
& V4z)) (ap (ap c\_2Ereal\_2Ereal\_sub V4z) V2x)))) \wedge ((p (ap (ap c\_2Elim\_2Econt1 \\
& V3g) V2x)) \wedge ((ap V3g V2x) = V1l))))))
\end{aligned} \tag{46}$$

Assume the following.

$$\begin{aligned}
& (\forall V0f \in (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}).(\forall V1g \in \\
& (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}).(\forall V2x \in ty\_2Erealax\_2Ereal. \\
& (((p (ap (ap c\_2Elim\_2Econt1 V0f) V2x)) \wedge (p (ap (ap c\_2Elim\_2Econt1 \\
& V1g) (ap V0f V2x)))) \Rightarrow (p (ap (ap c\_2Elim\_2Econt1 (\lambda V3x \in ty\_2Erealax\_2Ereal. \\
& (ap V1g (ap V0f V3x))) V2x))))))
\end{aligned} \tag{47}$$

Assume the following.

$$\begin{aligned}
& (\forall V0f \in (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}).(\forall V1g \in \\
& (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}).(\forall V2x \in ty\_2Erealax\_2Ereal. \\
& (\forall V3d \in ty\_2Erealax\_2Ereal.(((p (ap (ap c\_2Erealax\_2Ereal\_lt \\
& (ap c\_2Ereal\_2Ereal\_of\_num c\_2Enum\_2E0)) V3d)) \wedge ((\forall V4z \in \\
& ty\_2Erealax\_2Ereal.((p (ap (ap c\_2Ereal\_2Ereal\_lte (ap c\_2Ereal\_2Eabs \\
& (ap (ap c\_2Ereal\_2Ereal\_sub V4z) V2x))) V3d)) \Rightarrow ((ap V1g (ap V0f \\
& V4z)) = V4z))) \wedge (\forall V5z \in ty\_2Erealax\_2Ereal.((p (ap (ap c\_2Ereal\_2Ereal\_lte \\
& (ap c\_2Ereal\_2Eabs (ap (ap c\_2Ereal\_2Ereal\_sub V5z) V2x))) V3d)) \Rightarrow \\
& (p (ap (ap c\_2Elim\_2Econtl V0f) V5z)))))) \Rightarrow (\exists V6e \in ty\_2Erealax\_2Ereal. \\
& ((p (ap (ap c\_2Erealax\_2Ereal\_lt (ap c\_2Ereal\_2Ereal\_of\_num \\
& c\_2Enum\_2E0)) V6e)) \wedge (\forall V7y \in ty\_2Erealax\_2Ereal.((p (ap \\
& (ap c\_2Ereal\_2Ereal\_lte (ap c\_2Ereal\_2Eabs (ap (ap c\_2Ereal\_2Ereal\_sub \\
& V7y) (ap V0f V2x))) V6e)) \Rightarrow (\exists V8z \in ty\_2Erealax\_2Ereal.( \\
& (p (ap (ap c\_2Ereal\_2Ereal\_lte (ap c\_2Ereal\_2Eabs (ap (ap c\_2Ereal\_2Ereal\_sub \\
& V8z) V2x))) V3d)) \wedge ((ap V0f V8z) = V7y)))))))))))))
\end{aligned} \tag{48}$$

Assume the following.

$$\begin{aligned}
& (\forall V0f \in (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}).(\forall V1g \in \\
& (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}).(\forall V2x \in ty\_2Erealax\_2Ereal. \\
& (\forall V3d \in ty\_2Erealax\_2Ereal.(((p (ap (ap c\_2Erealax\_2Ereal\_lt \\
& (ap c\_2Ereal\_2Ereal\_of\_num c\_2Enum\_2E0)) V3d)) \wedge ((\forall V4z \in \\
& ty\_2Erealax\_2Ereal.((p (ap (ap c\_2Ereal\_2Ereal\_lte (ap c\_2Ereal\_2Eabs \\
& (ap (ap c\_2Ereal\_2Ereal\_sub V4z) V2x))) V3d)) \Rightarrow ((ap V1g (ap V0f \\
& V4z)) = V4z))) \wedge (\forall V5z \in ty\_2Erealax\_2Ereal.((p (ap (ap c\_2Ereal\_2Ereal\_lte \\
& (ap c\_2Ereal\_2Eabs (ap (ap c\_2Ereal\_2Ereal\_sub V5z) V2x))) V3d)) \Rightarrow \\
& (p (ap (ap c\_2Elim\_2Econtl V0f) V5z)))))) \Rightarrow (p (ap (ap c\_2Elim\_2Econtl \\
& V1g) (ap V0f V2x)))))))))
\end{aligned} \tag{49}$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.(\neg(p (ap (ap c\_2Erealax\_2Ereal\_lt V0x) V0x)))) \tag{50}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal.(\forall V1y \in ty\_2Erealax\_2Ereal. \\
& (\forall V2z \in ty\_2Erealax\_2Ereal.(((p (ap (ap c\_2Erealax\_2Ereal\_lt \\
& V0x) V1y)) \wedge (p (ap (ap c\_2Erealax\_2Ereal\_lt V1y) V2z))) \Rightarrow (p (ap \\
& (ap c\_2Erealax\_2Ereal\_lt V0x) V2z))))))
\end{aligned} \tag{51}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal.(\forall V1y \in ty\_2Erealax\_2Ereal. \\
& ((ap (ap c\_2Erealax\_2Ereal\_mul V0x) V1y) = (ap (ap c\_2Erealax\_2Ereal\_mul \\
& V1y) V0x))))
\end{aligned} \tag{52}$$



Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal. (\forall V1y \in ty\_2Erealax\_2Ereal. \\
& (\forall V2z \in ty\_2Erealax\_2Ereal. ((ap (ap c\_2Erealax\_2Ereal\_mul \\
V0x) (ap (ap c\_2Erealax\_2Ereal\_mul V1y) V2z)) = (ap (ap c\_2Erealax\_2Ereal\_mul \\
& (ap (ap c\_2Erealax\_2Ereal\_mul V0x) V1y)) V2z))))))
\end{aligned} \tag{53}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal. ((ap (ap c\_2Erealax\_2Ereal\_mul \\
& (ap c\_2Ereal\_2Ereal\_of\_num (ap c\_2Earithmetic\_2ENUMERAL ( \\
ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO)))) V0x) = V0x))
\end{aligned} \tag{54}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal. ((\neg(V0x = (ap c\_2Ereal\_2Ereal\_of\_num \\
& c\_2Enum\_2E0))) \Rightarrow ((ap (ap c\_2Erealax\_2Ereal\_mul (ap c\_2Erealax\_2Einv \\
V0x)) V0x) = (ap c\_2Ereal\_2Ereal\_of\_num (ap c\_2Earithmetic\_2ENUMERAL \\
& (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO))))))
\end{aligned} \tag{55}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal. ((ap (ap c\_2Erealax\_2Ereal\_mul \\
V0x) (ap c\_2Ereal\_2Ereal\_of\_num c\_2Enum\_2E0)) = (ap c\_2Ereal\_2Ereal\_of\_num \\
& c\_2Enum\_2E0)))
\end{aligned} \tag{56}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal. (\forall V1y \in ty\_2Erealax\_2Ereal. \\
& ((p (ap (ap c\_2Erealax\_2Ereal\_lt V0x) V1y)) \Rightarrow (p (ap (ap c\_2Ereal\_2Ereal\_lte \\
& V0x) V1y))))))
\end{aligned} \tag{57}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal. ((ap (ap c\_2Ereal\_2Ereal\_sub \\
V0x) V0x) = (ap c\_2Ereal\_2Ereal\_of\_num c\_2Enum\_2E0)))
\end{aligned} \tag{58}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal. (\forall V1y \in ty\_2Erealax\_2Ereal. \\
& (((ap (ap c\_2Ereal\_2Ereal\_sub V0x) V1y) = (ap c\_2Ereal\_2Ereal\_of\_num \\
& c\_2Enum\_2E0))) \Leftrightarrow (V0x = V1y))))
\end{aligned} \tag{59}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal. (\forall V1y \in ty\_2Erealax\_2Ereal. \\
& ((\neg(V1y = (ap c\_2Ereal\_2Ereal\_of\_num c\_2Enum\_2E0))) \Rightarrow ((ap ( \\
ap c\_2Erealax\_2Ereal\_mul (ap (ap c\_2Ereal\_2E\_2F V0x) V1y)) V1y) = \\
& V0x))))
\end{aligned} \tag{60}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal. (\forall V1y \in ty\_2Erealax\_2Ereal. \\
& \quad (((p (ap (ap c\_2Erealax\_2Ereal\_lt (ap c\_2Ereal\_2Ereal\_of\_num \\
& \quad c\_2Enum\_2E0)) V0x)) \wedge (p (ap (ap c\_2Erealax\_2Ereal\_lt (ap c\_2Ereal\_2Ereal\_of\_num \\
& \quad c\_2Enum\_2E0)) V1y))) \Rightarrow (\exists V2z \in ty\_2Erealax\_2Ereal. ((p ( \\
& \quad ap (ap c\_2Erealax\_2Ereal\_lt (ap c\_2Ereal\_2Ereal\_of\_num c\_2Enum\_2E0)) \\
& \quad V2z)) \wedge ((p (ap (ap c\_2Erealax\_2Ereal\_lt V2z) V0x)) \wedge (p (ap (ap c\_2Erealax\_2Ereal\_lt \\
& \quad V2z) V1y))))))))))
\end{aligned} \tag{61}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal. ((ap (ap c\_2Ereal\_2Ereal\_sub \\
& (ap c\_2Ereal\_2Ereal\_of\_num c\_2Enum\_2E0)) V0x) = (ap c\_2Erealax\_2Ereal\_neg \\
& V0x)))
\end{aligned} \tag{62}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal. ((ap (ap c\_2Ereal\_2Ereal\_sub \\
& V0x) (ap c\_2Ereal\_2Ereal\_of\_num c\_2Enum\_2E0)) = V0x))
\end{aligned} \tag{63}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal. (\forall V1y \in ty\_2Erealax\_2Ereal. \\
& \quad (((\neg (V0x = (ap c\_2Ereal\_2Ereal\_of\_num c\_2Enum\_2E0))) \wedge (\neg (V1y = \\
& \quad (ap c\_2Ereal\_2Ereal\_of\_num c\_2Enum\_2E0)))) \Rightarrow ((ap c\_2Erealax\_2Ereal\_inv \\
& \quad (ap (ap c\_2Erealax\_2Ereal\_mul V0x) V1y)) = (ap (ap c\_2Erealax\_2Ereal\_mul \\
& \quad (ap c\_2Erealax\_2Ereal\_inv V0x)) (ap c\_2Erealax\_2Ereal\_inv V1y))))))
\end{aligned} \tag{64}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal. (((ap c\_2Ereal\_2Eabs V0x) = \\
& (ap c\_2Ereal\_2Ereal\_of\_num c\_2Enum\_2E0)) \Leftrightarrow (V0x = (ap c\_2Ereal\_2Ereal\_of\_num \\
& c\_2Enum\_2E0))))
\end{aligned} \tag{65}$$

Assume the following.

$$\begin{aligned}
& ((ap c\_2Ereal\_2Eabs (ap c\_2Ereal\_2Ereal\_of\_num c\_2Enum\_2E0)) = \\
& (ap c\_2Ereal\_2Ereal\_of\_num c\_2Enum\_2E0))
\end{aligned} \tag{66}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal. ((ap c\_2Ereal\_2Eabs (ap c\_2Erealax\_2Ereal\_neg \\
& V0x)) = (ap c\_2Ereal\_2Eabs V0x)))
\end{aligned} \tag{67}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal. ((\neg (V0x = (ap c\_2Ereal\_2Ereal\_of\_num \\
& c\_2Enum\_2E0))) \Leftrightarrow (p (ap (ap c\_2Erealax\_2Ereal\_lt (ap c\_2Ereal\_2Ereal\_of\_num \\
& c\_2Enum\_2E0)) (ap c\_2Ereal\_2Eabs V0x))))))
\end{aligned} \tag{68}$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \quad (69)$$

Assume the following.

$$(\forall V0A \in 2.((p V0A) \Rightarrow ((\neg(p V0A)) \Rightarrow False))) \quad (70)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p V0A) \vee (p V1B))) \Rightarrow False) \Leftrightarrow ((p V0A) \Rightarrow False) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \quad (71)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg(\neg(p V0A)) \vee (p V1B))) \Rightarrow False) \Leftrightarrow ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \quad (72)$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p V0A)) \Rightarrow False) \Rightarrow (((p V0A) \Rightarrow False) \Rightarrow False))) \quad (73)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \Leftrightarrow (p V2r)) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee ((\neg(p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee ((\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee ((\neg(p V1q)) \vee (\neg(p V0p)))))))))) \quad (74)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (p V0p)))) \quad (75)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (\neg(p V1q)))) \quad (76)$$

**Theorem 1**

$$(\forall V0f \in (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}).(\forall V1g \in (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}).(\forall V2l \in ty\_2Erealax\_2Ereal.(\forall V3x \in ty\_2Erealax\_2Ereal.(\forall V4d \in ty\_2Erealax\_2Ereal.(((p (ap (ap (ap c\_2Erealax\_2Ereal\_lt (ap c\_2Ereal\_2Ereal\_of\_num c\_2Enum\_2E0)) V4d)) \wedge ((\forall V5z \in ty\_2Erealax\_2Ereal.((p (ap (ap c\_2Ereal\_2Ereal\_lte (ap c\_2Ereal\_2Eabs (ap (ap c\_2Ereal\_2Ereal\_sub V5z) V3x))) V4d)) \Rightarrow ((ap V1g (ap V0f V5z)) = V5z))) \wedge ((\forall V6z \in ty\_2Erealax\_2Ereal.((p (ap (ap c\_2Ereal\_2Ereal\_lte (ap c\_2Ereal\_2Eabs (ap (ap c\_2Ereal\_2Ereal\_sub V6z) V3x))) V4d)) \Rightarrow (p (ap (ap c\_2Elim\_2Econtl V0f) V6z)))) \wedge ((p (ap (ap (ap c\_2Elim\_2Ediff1 V0f) V2l) V3x)) \wedge (\neg (V2l = (ap c\_2Ereal\_2Ereal\_of\_num c\_2Enum\_2E0)))))) \Rightarrow (p (ap (ap c\_2Elim\_2Ediff1 V1g) (ap c\_2Erealax\_2Einv V2l)) (ap V0f V3x))))))))))$$