

thm_2Elim_2EDIFF_ISCONST_ALL (TM- SjVTHV2d6Rur6ca8qB8VF87DPGHw7DqKU)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a}))$

Definition 4 We define c_2Ebool_2EF to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 5 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 6 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2EF$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \tag{1}$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \tag{2}$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \tag{3}$$

Definition 7 We define c_2Enum_2E0 to be $(ap c_2Enum_2EABS_num c_2Enum_2EZERO_REP)$.

Let $ty_2Erealx_2Ereal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Erealx_2Ereal \tag{4}$$

Let $c_2Ereal_2Ereal_of_num : \iota$ be given. Assume the following.

$$c_2Ereal_2Ereal_of_num \in (ty_2Erealx_2Ereal^{ty_2Enum_2Enum}) \tag{5}$$

Let $ty_2Ehreal_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Ehreal_2Ehreal \quad (6)$$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \quad (7)$$

Let $c_2Erealax_2Ereal_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_REP_CLASS \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{ty_2Erealax}) \quad (8)$$

Definition 8 We define c_2Emin_2E40 to be $\lambda A.\lambda P \in 2^A$.if $(\exists x \in A.p\ (ap\ P\ x))$ then (the $(\lambda x.x \in A \wedge p$ of type $\iota \Rightarrow \iota$).

Definition 9 We define $c_2Erealax_2Ereal_REP$ to be $\lambda V0a \in ty_2Erealax_2Ereal$. $(ap\ (c_2Emin_2E40\ (ty_2Erealax_2Ereal_REP_CLASS\ a)))$

Let $c_2Erealax_2Etrealm_add : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealm_add \in (((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)^{ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal})^{ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal}) \quad (9)$$

Let $c_2Erealax_2Etrealm_eq : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealm_eq \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal}) \quad (10)$$

Let $c_2Erealax_2Ereal_ABS_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_ABS_CLASS \in (ty_2Erealax_2Ereal)^{(2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})} \quad (11)$$

Definition 10 We define $c_2Erealax_2Ereal_ABS$ to be $\lambda V0r \in (ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)$

Definition 11 We define $c_2Erealax_2Ereal_add$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal$. $\lambda V1T2 \in ty_2Erealax_2Ereal$

Let $c_2Erealax_2Etrealm_neg : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealm_neg \in ((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)^{ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal}) \quad (12)$$

Definition 12 We define $c_2Erealax_2Ereal_neg$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal$. $(ap\ c_2Erealax_2Ereal_neg)$

Definition 13 We define $c_2Ereal_2Ereal_sub$ to be $\lambda V0x \in ty_2Erealax_2Ereal$. $\lambda V1y \in ty_2Erealax_2Ereal$

Let $c_2Erealax_2Etrealm_inv : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealm_inv \in ((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)^{ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal}) \quad (13)$$

Definition 14 We define $c_Erealax_Einv$ to be $\lambda V0T1 \in ty_Erealax_Ereal.(ap\ c_Erealax_Ereal_ABS)$
Let $c_Erealax_Etrealmul : \iota$ be given. Assume the following.

$$\begin{gathered} c_Erealax_Etrealmul \in (((ty_Epair_Eprod\ ty_Ehreal_Ehreal \\ ty_Ehreal_Ehreal)(ty_Epair_Eprod\ ty_Ehreal_Ehreal\ ty_Ehreal_Ehreal\ ty_Ehreal_Ehreal))(ty_Epair_Eprod\ ty_Ehreal_Ehreal)) \\ (14) \end{gathered}$$

Definition 15 We define $c_Erealax_Ereal_mul$ to be $\lambda V0T1 \in ty_Erealax_Ereal.\lambda V1T2 \in ty_Erealax_Ereal.$

Definition 16 We define $c_Ereal_E_E2F$ to be $\lambda V0x \in ty_Erealax_Ereal.\lambda V1y \in ty_Erealax_Ereal.$

Let $c_Erealax_Etreallt : \iota$ be given. Assume the following.

$$\begin{gathered} c_Erealax_Etreallt \in ((2^{(ty_Epair_Eprod\ ty_Ehreal_Ehreal\ ty_Ehreal_Ehreal\ ty_Ehreal_Ehreal)})(ty_Epair_Eprod\ ty_Ehreal_Ehreal)) \\ (15) \end{gathered}$$

Definition 17 We define $c_Erealax_Ereal_lt$ to be $\lambda V0T1 \in ty_Erealax_Ereal.\lambda V1T2 \in ty_Erealax_Ereal.$

Definition 18 We define $c_Ereal_Ereal_lte$ to be $\lambda V0x \in ty_Erealax_Ereal.\lambda V1y \in ty_Erealax_Ereal.$

Definition 19 We define $c_Ebool_E_E2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_Ebool_E_E21\ 2)\ (\lambda V2t \in 2.))$

Definition 20 We define c_Ebool_ECOND to be $\lambda A.27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A.27a.(\lambda V2t2 \in A.27a.))$

Definition 21 We define c_Ereal_Eabs to be $\lambda V0x \in ty_Erealax_Ereal.(ap\ (ap\ (ap\ (c_Ebool_ECOND)$

Let $c_Epair_EESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{gathered} \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow c_Epair_EESND \\ A.27a\ A.27b \in (A.27b^{(ty_Epair_Eprod\ A.27a\ A.27b)}) \\ (16) \end{gathered}$$

Let $c_Epair_EFAST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{gathered} \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow c_Epair_EFAST \\ A.27a\ A.27b \in (A.27a^{(ty_Epair_Eprod\ A.27a\ A.27b)}) \\ (17) \end{gathered}$$

Definition 22 We define $c_Epair_EUNCURRY$ to be $\lambda A.27a : \iota.\lambda A.27b : \iota.\lambda A.27c : \iota.\lambda V0f \in ((A.27c^{A.27a})$

Let $ty_Emetric_Emetric : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_Emetric_Emetric\ A0) \quad (18)$$

Let $c_Emetric_Emetric : \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{gathered} \forall A.27a.nonempty\ A.27a \Rightarrow c_Emetric_Emetric\ A.27a \in ((ty_Emetric_Emetric \\ A.27a)^{(ty_Erealax_Ereal^{(ty_Epair_Eprod\ A.27a\ A.27a)}})) \\ (19) \end{gathered}$$

Definition 23 We define $c_Emetric_Emr1$ to be $(ap\ (c_Emetric_Emetric\ ty_Erealax_Ereal))\ (ap\ (c_Emetric_Emetric))$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EABS_prod \\ A_27a\ A_27b \in ((ty_2Epair_2Eprod\ A_27a\ A_27b)^{(2^{A_27b})^{A_27a}}) \end{aligned} \quad (20)$$

Definition 24 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap\ (c_2E$

Let $c_2Enets_2Eextendsto : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Enets_2Eextendsto\ A_27a \in (((2^{A_27a})^{A_27a})^{(ty_2Epair_2Eprod\ (ty_2Emetric_2E$$

Let $c_2Emetric_2Edist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Emetric_2Edist\ A_27a \in ((ty_2Erealax_2Ereal^{(ty_2Epair_2Eprod\ A_27a\ A_27b)}$$

Definition 25 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ V0P\ (ap\ (c_2Emin_2E_40$

Let $ty_2Etopology_2Etopology : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Etopology_2Etopology\ A0) \quad (23)$$

Let $c_2Etopology_2Etopology : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Etopology_2Etopology\ A_27a \in ((ty_2Etopology_2Etopology\ A_27a)^{(2^{(2^{A_27a})})}) \quad (24)$$

Definition 26 We define $c_2Emetric_2Eemptop$ to be $\lambda A_27a : \iota.\lambda V0m \in (ty_2Emetric_2Emetric\ A_27a).(ap$

Let $c_2Enets_2Eextends : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Enets_2Eextends \\ A_27a\ A_27b \in (((2^{(ty_2Epair_2Eprod\ (ty_2Etopology_2Etopology\ A_27a)\ (2^{A_27b})^{A_27b}}))^{A_27a})^{(A_27a)^{A_27b}}) \end{aligned} \quad (25)$$

Definition 27 We define $c_2Elim_2Eextends_real_real$ to be $\lambda V0f \in (ty_2Erealax_2Ereal^{ty_2Erealax_2Ereal}).$

Definition 28 We define $c_2Elim_2Ediff1$ to be $\lambda V0f \in (ty_2Erealax_2Ereal^{ty_2Erealax_2Ereal}).\lambda V1l \in ty_2$

Definition 29 We define $c_2Elim_2Econt1$ to be $\lambda V0f \in (ty_2Erealax_2Ereal^{ty_2Erealax_2Ereal}).\lambda V1x \in ty_2$

Definition 30 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in 2.$

Assume the following.

$$True \quad (26)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A_27a.(p\ V0t) \Leftrightarrow (p\ V0t))) \quad (27)$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \wedge True) \Leftrightarrow (p \ V0t)) \wedge (((False \wedge (p \ V0t)) \Leftrightarrow False) \wedge (((p \ V0t) \wedge False) \Leftrightarrow False) \wedge (((p \ V0t) \wedge (p \ V0t)) \Leftrightarrow (p \ V0t)))))) \quad (28)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p \ V0t)) \Leftrightarrow True) \wedge (((p \ V0t) \Rightarrow (p \ V0t)) \Leftrightarrow True) \wedge (((p \ V0t) \Rightarrow False) \Leftrightarrow \neg(p \ V0t)))))) \quad (29)$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in A_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (30)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Leftrightarrow True) \Leftrightarrow (p \ V0t)) \wedge (((False \Leftrightarrow (p \ V0t)) \Leftrightarrow \neg(p \ V0t)) \wedge (((p \ V0t) \Leftrightarrow False) \Leftrightarrow \neg(p \ V0t)))))) \quad (31)$$

Assume the following.

$$(\forall V0f \in (ty_2Erealax_2Ereal^{ty_2Erealax_2Ereal}). (\forall V1l \in ty_2Erealax_2Ereal. (\forall V2x \in ty_2Erealax_2Ereal. ((p \ (ap \ (ap \ c_2Elim_2Ediff \ V0f) \ V1l) \ V2x)) \Rightarrow (p \ (ap \ (ap \ c_2Elim_2Econtl \ V0f) \ V2x)))))) \quad (32)$$

Assume the following.

$$(\forall V0f \in (ty_2Erealax_2Ereal^{ty_2Erealax_2Ereal}). (\forall V1a \in ty_2Erealax_2Ereal. (\forall V2b \in ty_2Erealax_2Ereal. (((p \ (ap \ (ap \ c_2Erealax_2Ereal_lt \ V1a) \ V2b)) \wedge ((\forall V3x \in ty_2Erealax_2Ereal. (((p \ (ap \ (ap \ c_2Ereal_2Ereal_lte \ V1a) \ V3x)) \wedge (p \ (ap \ (ap \ c_2Ereal_2Ereal_lte \ V3x) \ V2b))) \Rightarrow (p \ (ap \ (ap \ c_2Elim_2Econtl \ V0f) \ V3x)))))) \wedge (\forall V4x \in ty_2Erealax_2Ereal. (((p \ (ap \ (ap \ c_2Erealax_2Ereal_lt \ V1a) \ V4x)) \wedge (p \ (ap \ (ap \ c_2Erealax_2Ereal_lt \ V4x) \ V2b))) \Rightarrow (p \ (ap \ (ap \ c_2Elim_2Ediff \ V0f) \ (ap \ c_2Ereal_2Ereal_of_num \ c_2Enum_2E0) \ V4x)))))) \Rightarrow (ap \ V0f \ V2b) = (ap \ V0f \ V1a)))))) \quad (33)$$

Assume the following.

$$(\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. ((V0x = V1y) \vee ((p \ (ap \ (ap \ c_2Erealax_2Ereal_lt \ V0x) \ V1y)) \vee (p \ (ap \ (ap \ c_2Erealax_2Ereal_lt \ V1y) \ V0x)))))) \quad (34)$$

Theorem 1

$$(\forall V0f \in (ty_2Erealax_2Ereal^{ty_2Erealax_2Ereal}). ((\forall V1x \in ty_2Erealax_2Ereal. (p \ (ap \ (ap \ (ap \ c_2Elim_2Ediff \ V0f) \ (ap \ c_2Ereal_2Ereal_of_num \ c_2Enum_2E0) \ V1x))) \Rightarrow (\forall V2x \in ty_2Erealax_2Ereal. (\forall V3y \in ty_2Erealax_2Ereal. ((ap \ V0f \ V2x) = (ap \ V0f \ V3y))))))$$