

thm\_2Elim\_2EDIFF\_\_LMAX  
(TMRh3H9dum6HiYrNzfcKagEAHcKVVaVCwWdx)

October 26, 2020

**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Ebool\_2ET$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

**Definition 3** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c\_2Emin\_2E\_3D (2^{A-27a})) (\lambda V0t \in 2.V0t)) (\lambda V1t \in 2.V1t)))$

**Definition 4** We define  $c\_2Ebool\_2EF$  to be  $(ap (c\_2Ebool\_2E\_21 2) (\lambda V0t \in 2.V0t))$ .

Let  $c\_2Enum\_2EZERO\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2EZERO\_REP \in \omega \tag{1}$$

Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{2}$$

Let  $c\_2Enum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum^{\omega}) \tag{3}$$

**Definition 5** We define  $c\_2Enum\_2E0$  to be  $(ap\ c\_2Enum\_2EABS\_num\ c\_2Enum\_2EZERO\_REP)$ .

Let  $ty\_2Erealx\_2Ereal : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Erealx\_2Ereal \tag{4}$$

Let  $c\_2Ereal\_2Ereal\_of\_num : \iota$  be given. Assume the following.

$$c\_2Ereal\_2Ereal\_of\_num \in (ty\_2Erealx\_2Ereal^{ty\_2Enum\_2Enum}) \tag{5}$$

Let  $ty\_2Ehreal\_2Ehreal : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Ehreal\_2Ehreal \tag{6}$$

Let  $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \quad (7)$$

Let  $c\_2Erealax\_2Ereal\_REP\_CLASS : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Ereal\_REP\_CLASS \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})_{ty\_2Erealax}) \quad (8)$$

**Definition 6** We define  $c\_2Emin\_2E40$  to be  $\lambda A.\lambda P \in 2^A$ . **if**  $(\exists x \in A.p\ (ap\ P\ x))$  **then** (the  $(\lambda x.x \in A \wedge p\ x)$  of type  $\iota \Rightarrow \iota$ ).

**Definition 7** We define  $c\_2Erealax\_2Ereal\_REP$  to be  $\lambda V0a \in ty\_2Erealax\_2Ereal.(ap\ (c\_2Emin\_2E40\ (ty\_2Erealax\_2Ereal\_REP\_CLASS\ a)))$

Let  $c\_2Erealax\_2Etrealm\_add : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etrealm\_add \in (((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)_{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal)})_{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal)})_{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal)} \quad (9)$$

Let  $c\_2Erealax\_2Etrealm\_eq : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etrealm\_eq \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})_{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal)})_{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal)} \quad (10)$$

Let  $c\_2Erealax\_2Ereal\_ABS\_CLASS : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Ereal\_ABS\_CLASS \in (ty\_2Erealax\_2Ereal)^{(2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})} \quad (11)$$

**Definition 8** We define  $c\_2Erealax\_2Ereal\_ABS$  to be  $\lambda V0r \in (ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)$

**Definition 9** We define  $c\_2Erealax\_2Ereal\_add$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Erealax\_2Ereal$

Let  $c\_2Erealax\_2Etrealm\_neg : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etrealm\_neg \in ((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)_{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})_{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)} \quad (12)$$

**Definition 10** We define  $c\_2Erealax\_2Ereal\_neg$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.(ap\ c\_2Erealax\_2Ereal\_neg\ T1)$

**Definition 11** We define  $c\_2Ereal\_2Ereal\_sub$  to be  $\lambda V0x \in ty\_2Erealax\_2Ereal.\lambda V1y \in ty\_2Erealax\_2Ereal$

Let  $c\_2Erealax\_2Etrealm\_inv : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etrealm\_inv \in ((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)_{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})_{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)} \quad (13)$$

**Definition 12** We define  $c\_2Erealax\_2Einv$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.(ap\ c\_2Erealax\_2Ereal\_ABS\ T1)$



**Definition 23** We define  $c\_2Emetric\_2Emr1$  to be  $(ap (c\_2Emetric\_2Emetric\ ty\_2Erealx\_2Ereal) (ap (c\_2Epair\_2EABS\_prod : \iota \Rightarrow \iota \Rightarrow \iota)$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EABS\_prod\ A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{(2^{A\_27b})^{A\_27a}})$$
(20)

**Definition 24** We define  $c\_2Epair\_2E\_2C$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap (c\_2Enets\_2Etendsto : \iota \Rightarrow \iota)$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Enets\_2Etendsto\ A\_27a \in (((2^{A\_27a})^{A\_27a})^{(ty\_2Epair\_2Eprod\ (ty\_2Emetric\_2Emr1\ A\_27a\ A\_27b))})$$
(21)

Let  $c\_2Emetric\_2Edist : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Emetric\_2Edist\ A\_27a \in ((ty\_2Erealx\_2Ereal)^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)})$$
(22)

**Definition 25** We define  $c\_2Ebool\_2E\_3F$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ V0P\ (ap\ (c\_2Emin\_2E\_40$

Let  $ty\_2Etopology\_2Etopology : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Etopology\_2Etopology\ A0)$$
(23)

Let  $c\_2Etopology\_2Etopology : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Etopology\_2Etopology\ A\_27a \in ((ty\_2Etopology\_2Etopology\ A\_27a)^{(2^{(2^{A\_27a})})})$$
(24)

**Definition 26** We define  $c\_2Emetric\_2Emtop$  to be  $\lambda A\_27a : \iota.\lambda V0m \in (ty\_2Emetric\_2Emetric\ A\_27a).(ap$

Let  $c\_2Enets\_2Etends : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Enets\_2Etends\ A\_27a\ A\_27b \in (((2^{(ty\_2Epair\_2Eprod\ (ty\_2Etopology\_2Etopology\ A\_27a)\ (2^{A\_27b})^{A\_27b}))})^{A\_27a})^{(A\_27a)^{A\_27b}}$$
(25)

**Definition 27** We define  $c\_2Elim\_2Etends\_real\_real$  to be  $\lambda V0f \in (ty\_2Erealx\_2Ereal^{ty\_2Erealx\_2Ereal}).$

**Definition 28** We define  $c\_2Elim\_2Ediff1$  to be  $\lambda V0f \in (ty\_2Erealx\_2Ereal^{ty\_2Erealx\_2Ereal}).\lambda V1l \in ty\_2$

**Definition 29** We define  $c\_2Ebool\_2E\_5C\_2F$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21\ 2) (\lambda V2t \in$

Assume the following.

$$True$$
(26)

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \wedge True) \Leftrightarrow (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge (((p\ V0t) \wedge False) \Leftrightarrow False) \wedge (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t))))))$$
(27)

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow \\
& True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (( \\
& (p V0t) \Rightarrow False) \Leftrightarrow (\neg(p V0t))))))
\end{aligned} \tag{28}$$

Assume the following.

$$\begin{aligned}
& ((\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t)) \wedge (((\neg True) \Leftrightarrow False) \wedge \\
& ((\neg False) \Leftrightarrow True)))
\end{aligned} \tag{29}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty A.27a \Rightarrow (\forall V0x \in A.27a.((V0x = V0x) \Leftrightarrow \\
& True))
\end{aligned} \tag{30}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty A.27a \Rightarrow (\forall V0x \in A.27a.(\forall V1y \in \\
& A.27a.((V0x = V1y) \Leftrightarrow (V1y = V0x))))
\end{aligned} \tag{31}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow \\
& (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg( \\
& p V0t))))))
\end{aligned} \tag{32}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty A.27a \Rightarrow (\forall V0t1 \in A.27a.(\forall V1t2 \in \\
& A.27a.(((ap (ap (ap (c.2Ebool.2ECOND A.27a) c.2Ebool.2ET) V0t1) \\
& V1t2) = V0t1) \wedge ((ap (ap (ap (c.2Ebool.2ECOND A.27a) c.2Ebool.2EF \\
& V0t1) V1t2) = V1t2))))))
\end{aligned} \tag{33}$$

Assume the following.

$$\begin{aligned}
& (\forall V0f \in (ty.2Erealax.2Ereal^{ty.2Erealax.2Ereal}).(\forall V1x \in \\
& ty.2Erealax.2Ereal.(\forall V2l \in ty.2Erealax.2Ereal.(((p ( \\
& ap (ap (ap c.2Elim.2Ediff V0f) V2l) V1x)) \wedge (p (ap (ap c.2Erealax.2Ereal\_lt \\
& (ap c.2Ereal.2Ereal\_of\_num c.2Enum.2E0) V2l))) \Rightarrow (\exists V3d \in \\
& ty.2Erealax.2Ereal.((p (ap (ap c.2Erealax.2Ereal\_lt (ap c.2Ereal.2Ereal\_of\_num \\
& c.2Enum.2E0) V3d)) \wedge (\forall V4h \in ty.2Erealax.2Ereal.(((p ( \\
& ap (ap c.2Erealax.2Ereal\_lt (ap c.2Ereal.2Ereal\_of\_num c.2Enum.2E0) \\
& V4h)) \wedge (p (ap (ap c.2Erealax.2Ereal\_lt V4h) V3d))) \Rightarrow (p (ap (ap c.2Erealax.2Ereal\_lt \\
& (ap V0f V1x)) (ap V0f (ap (ap c.2Erealax.2Ereal\_add V1x) V4h))))))))))
\end{aligned} \tag{34}$$

Assume the following.

$$\begin{aligned}
& (\forall V0f \in (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}).(\forall V1x \in \\
& \quad ty\_2Erealax\_2Ereal.(\forall V2l \in ty\_2Erealax\_2Ereal.(((p ( \\
& \quad ap (ap (ap c\_2Elim\_2Ediff1 V0f) V2l) V1x)) \wedge (p (ap (ap c\_2Erealax\_2Ereal\_lt \\
& \quad V2l) (ap c\_2Ereal\_2Ereal\_of\_num c\_2Enum\_2E0)))) \Rightarrow (\exists V3d \in \\
& ty\_2Erealax\_2Ereal.((p (ap (ap c\_2Erealax\_2Ereal\_lt (ap c\_2Ereal\_2Ereal\_of\_num \\
& \quad c\_2Enum\_2E0)) V3d)) \wedge (\forall V4h \in ty\_2Erealax\_2Ereal.(((p ( \\
& \quad ap (ap c\_2Erealax\_2Ereal\_lt (ap c\_2Ereal\_2Ereal\_of\_num c\_2Enum\_2E0)) \\
& \quad V4h)) \wedge (p (ap (ap c\_2Erealax\_2Ereal\_lt V4h) V3d))) \Rightarrow (p (ap (ap c\_2Erealax\_2Ereal\_lt \\
& \quad (ap V0f V1x)) (ap V0f (ap (ap c\_2Ereal\_2Ereal\_sub V1x) V4h))))))))))))) \\
& \hspace{15em} (35)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal.(\forall V1y \in ty\_2Erealax\_2Ereal. \\
& ((V0x = V1y) \vee ((p (ap (ap c\_2Erealax\_2Ereal\_lt V0x) V1y)) \vee (p (ap \\
& \quad (ap c\_2Erealax\_2Ereal\_lt V1y) V0x)))))) \\
& \hspace{15em} (36)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal.(\forall V1y \in ty\_2Erealax\_2Ereal. \\
& ((\neg (p (ap (ap c\_2Erealax\_2Ereal\_lt V0x) V1y))) \Leftrightarrow (p (ap (ap c\_2Ereal\_2Ereal\_lte \\
& \quad V1y) V0x)))))) \\
& \hspace{15em} (37)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal.(\forall V1y \in ty\_2Erealax\_2Ereal. \\
& ((p (ap (ap c\_2Erealax\_2Ereal\_lt V0x) V1y)) \Rightarrow (p (ap (ap c\_2Ereal\_2Ereal\_lte \\
& \quad V0x) V1y)))))) \\
& \hspace{15em} (38)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal.(\forall V1y \in ty\_2Erealax\_2Ereal. \\
& (((p (ap (ap c\_2Erealax\_2Ereal\_lt (ap c\_2Ereal\_2Ereal\_of\_num \\
& \quad c\_2Enum\_2E0)) V0x)) \wedge (p (ap (ap c\_2Erealax\_2Ereal\_lt (ap c\_2Ereal\_2Ereal\_of\_num \\
& \quad c\_2Enum\_2E0)) V1y))) \Rightarrow (\exists V2z \in ty\_2Erealax\_2Ereal.((p ( \\
& \quad ap (ap c\_2Erealax\_2Ereal\_lt (ap c\_2Ereal\_2Ereal\_of\_num c\_2Enum\_2E0)) \\
& \quad V2z)) \wedge ((p (ap (ap c\_2Erealax\_2Ereal\_lt V2z) V0x)) \wedge (p (ap (ap c\_2Erealax\_2Ereal\_lt \\
& \quad V2z) V1y)))))))))) \\
& \hspace{15em} (39)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal.(\forall V1y \in ty\_2Erealax\_2Ereal. \\
& ((ap (ap c\_2Ereal\_2Ereal\_sub V0x) (ap (ap c\_2Ereal\_2Ereal\_sub \\
& \quad V0x) V1y)) = V1y))) \\
& \hspace{15em} (40)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal.(\forall V1y \in ty\_2Erealax\_2Ereal. \\
& ((ap (ap c\_2Ereal\_2Ereal\_sub V0x) (ap (ap c\_2Erealax\_2Ereal\_add \\
& \quad V0x) V1y)) = (ap c\_2Erealax\_2Ereal\_neg V1y)))) \\
& \hspace{15em} (41)
\end{aligned}$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.((ap\ c\_2Ereal\_2Eabs (ap\ c\_2Erealax\_2Ereal\_neg\ V0x)) = (ap\ c\_2Ereal\_2Eabs\ V0x))) \quad (42)$$

**Theorem 1**

$$\begin{aligned} & (\forall V0f \in (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}).(\forall V1x \in \\ & \quad ty\_2Erealax\_2Ereal.(\forall V2l \in ty\_2Erealax\_2Ereal.(((p\ ( \\ & \quad ap\ (ap\ (ap\ c\_2Elim\_2Ediff1\ V0f)\ V2l)\ V1x)) \wedge (\exists V3d \in ty\_2Erealax\_2Ereal. \\ & \quad ((p\ (ap\ (ap\ c\_2Erealax\_2Ereal\_lt\ (ap\ c\_2Ereal\_2Ereal\_of\_num \\ & \quad c\_2Enum\_2E0))\ V3d)) \wedge (\forall V4y \in ty\_2Erealax\_2Ereal.((p\ (ap \\ & \quad (ap\ c\_2Erealax\_2Ereal\_lt\ (ap\ c\_2Ereal\_2Eabs\ (ap\ (ap\ c\_2Ereal\_2Ereal\_sub \\ & \quad V1x)\ V4y)))\ V3d)) \Rightarrow (p\ (ap\ (ap\ c\_2Ereal\_2Ereal\_lte\ (ap\ V0f\ V4y)) \\ & \quad (ap\ V0f\ V1x)))))))))) \Rightarrow (V2l = (ap\ c\_2Ereal\_2Ereal\_of\_num\ c\_2Enum\_2E0)))))) \end{aligned}$$