

# thm\_2Elim\_2INTERVAL\_CLEMMMA (TMZnm- NFD52CvP7ewwzNkQNJtWjpS5TCm6jT)

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**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Ebool\_2ET$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

**Definition 3** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A.^{27a} : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c\_2Emin\_2E\_3D (2^{A-27a}))$

**Definition 4** We define  $c\_2Ebool\_2EF$  to be  $(ap (c\_2Ebool\_2E\_21 2) (\lambda V0t \in 2.V0t))$ .

**Definition 5** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p P \Rightarrow p Q)$  of type  $\iota$ .

**Definition 6** We define  $c\_2Ebool\_2E\_7E$  to be  $(\lambda V0t \in 2.(ap (ap c\_2Emin\_2E\_3D\_3D\_3E V0t) c\_2Ebool\_2EF$

Let  $ty\_2Ehreal\_2Ehreal : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Ehreal\_2Ehreal \tag{1}$$

Let  $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \tag{2}$$

Let  $ty\_2Erealax\_2Ereal : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Erealax\_2Ereal \tag{3}$$

Let  $c\_2Erealax\_2Ereal\_REP\_CLASS : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Ereal\_REP\_CLASS \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})\ ty\_2Erealax\_2Ereal) \tag{4}$$

**Definition 7** We define  $c\_2Emin\_2E\_40$  to be  $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x)) \mathbf{then} (the (\lambda x.x \in A \wedge p x))$  of type  $\iota \Rightarrow \iota$ .

**Definition 8** We define  $c\_2Erealax\_2Ereal\_REP$  to be  $\lambda V0a \in ty\_2Erealax\_2Ereal.(ap (c\_2Emin\_2E\_40 (ty\_2Erealax\_2Ereal\_REP\_CLASS$

Let  $c\_2Erealax\_2Etreal\_neg : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etreal\_neg \in ((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)\ (ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)) \quad (5)$$

Let  $c\_2Erealax\_2Etreal\_eq : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etreal\_eq \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})\ (ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal)) \quad (6)$$

Let  $c\_2Erealax\_2Ereal\_ABS\_CLASS : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Ereal\_ABS\_CLASS \in (ty\_2Erealax\_2Ereal)^{(2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})} \quad (7)$$

**Definition 9** We define  $c\_2Erealax\_2Ereal\_ABS$  to be  $\lambda V0r \in (ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)$

**Definition 10** We define  $c\_2Erealax\_2Ereal\_neg$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.(ap\ c\_2Erealax\_2Ereal$

Let  $c\_2Enum\_2EZERO\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2EZERO\_REP \in \omega\ \text{omega} \quad (8)$$

Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$\text{nonempty } ty\_2Enum\_2Enum \quad (9)$$

Let  $c\_2Enum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum)^{\omega\ \text{omega}} \quad (10)$$

**Definition 11** We define  $c\_2Enum\_2E0$  to be  $(ap\ c\_2Enum\_2EABS\_num\ c\_2Enum\_2EZERO\_REP)$ .

Let  $c\_2Ereal\_2Ereal\_of\_num : \iota$  be given. Assume the following.

$$c\_2Ereal\_2Ereal\_of\_num \in (ty\_2Erealax\_2Ereal)^{ty\_2Enum\_2Enum} \quad (11)$$

Let  $c\_2Erealax\_2Etreal\_lt : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etreal\_lt \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})\ (ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal)) \quad (12)$$

**Definition 12** We define  $c\_2Erealax\_2Ereal\_lt$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Erealax\_2Ereal$

**Definition 13** We define  $c\_2Ereal\_2Ereal\_lte$  to be  $\lambda V0x \in ty\_2Erealax\_2Ereal.\lambda V1y \in ty\_2Erealax\_2Ereal$

**Definition 14** We define  $c\_2Ebool\_2E2F5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c\_2Ebool\_2E21\ 2)\ (\lambda V2t \in 2$

**Definition 15** We define  $c\_2Ebool\_2ECOND$  to be  $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.($

**Definition 16** We define  $c\_2Ereal\_2Eabs$  to be  $\lambda V0x \in ty\_2Erealax\_2Ereal.(ap\ (ap\ (ap\ (c\_2Ebool\_2ECOND$



Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal. (\forall V1y \in ty\_2Erealax\_2Ereal. \\
& (\forall V2z \in ty\_2Erealax\_2Ereal. ((p (ap (ap c\_2Erealax\_2Ereal\_lt \\
& (ap (ap c\_2Erealax\_2Ereal\_add V0x) V1y)) (ap (ap c\_2Erealax\_2Ereal\_add \\
& V0x) V2z))) \Leftrightarrow (p (ap (ap c\_2Erealax\_2Ereal\_lt V1y) V2z))))))
\end{aligned} \tag{21}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal. (\forall V1y \in ty\_2Erealax\_2Ereal. \\
& (\forall V2z \in ty\_2Erealax\_2Ereal. ((p (ap (ap c\_2Erealax\_2Ereal\_lt \\
& V0x) V1y)) \wedge (p (ap (ap c\_2Ereal\_2Ereal\_lte V1y) V2z))) \Rightarrow (p (ap ( \\
& ap c\_2Erealax\_2Ereal\_lt V0x) V2z))))))
\end{aligned} \tag{22}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal. (\forall V1y \in ty\_2Erealax\_2Ereal. \\
& (\forall V2z \in ty\_2Erealax\_2Ereal. ((p (ap (ap c\_2Ereal\_2Ereal\_lte \\
& V0x) V1y)) \wedge (p (ap (ap c\_2Erealax\_2Ereal\_lt V1y) V2z))) \Rightarrow (p (ap \\
& (ap c\_2Erealax\_2Ereal\_lt V0x) V2z))))))
\end{aligned} \tag{23}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal. (\forall V1y \in ty\_2Erealax\_2Ereal. \\
& ((p (ap (ap c\_2Erealax\_2Ereal\_lt (ap c\_2Ereal\_2Ereal\_of\_num \\
& c\_2Enum\_2E0)) (ap (ap c\_2Ereal\_2Ereal\_sub V0x) V1y))) \Leftrightarrow (p (ap \\
& (ap c\_2Erealax\_2Ereal\_lt V1y) V0x))))))
\end{aligned} \tag{24}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal. (\forall V1y \in ty\_2Erealax\_2Ereal. \\
& (((p (ap (ap c\_2Erealax\_2Ereal\_lt (ap c\_2Ereal\_2Ereal\_of\_num \\
& c\_2Enum\_2E0)) V0x)) \wedge (p (ap (ap c\_2Erealax\_2Ereal\_lt (ap c\_2Ereal\_2Ereal\_of\_num \\
& c\_2Enum\_2E0)) V1y))) \Rightarrow (\exists V2z \in ty\_2Erealax\_2Ereal. ((p ( \\
& ap (ap c\_2Erealax\_2Ereal\_lt (ap c\_2Ereal\_2Ereal\_of\_num c\_2Enum\_2E0)) \\
& V2z)) \wedge ((p (ap (ap c\_2Erealax\_2Ereal\_lt V2z) V0x)) \wedge (p (ap (ap c\_2Erealax\_2Ereal\_lt \\
& V2z) V1y))))))))))
\end{aligned} \tag{25}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal. (\forall V1y \in ty\_2Erealax\_2Ereal. \\
& (\forall V2z \in ty\_2Erealax\_2Ereal. ((p (ap (ap c\_2Erealax\_2Ereal\_lt \\
& V0x) (ap (ap c\_2Ereal\_2Ereal\_sub V1y) V2z))) \Leftrightarrow (p (ap (ap c\_2Erealax\_2Ereal\_lt \\
& (ap (ap c\_2Erealax\_2Ereal\_add V0x) V2z)) V1y))))))
\end{aligned} \tag{26}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal. (\forall V1y \in ty\_2Erealax\_2Ereal. \\
& (\forall V2z \in ty\_2Erealax\_2Ereal. ((p (ap (ap c\_2Ereal\_2Ereal\_lte \\
& (ap (ap c\_2Ereal\_2Ereal\_sub V0x) V1y)) V2z)) \Leftrightarrow (p (ap (ap c\_2Ereal\_2Ereal\_lte \\
& V0x) (ap (ap c\_2Erealax\_2Ereal\_add V2z) V1y))))))
\end{aligned} \tag{27}$$

**Theorem 1**

$$\begin{aligned} & (\forall V0a \in ty\_2Erealax\_2Ereal. (\forall V1b \in ty\_2Erealax\_2Ereal. \\ & (\forall V2x \in ty\_2Erealax\_2Ereal. (((p (ap (ap c\_2Erealax\_2Ereal\_lt \\ & V0a) V2x)) \wedge (p (ap (ap c\_2Erealax\_2Ereal\_lt V2x) V1b))) \Rightarrow (\exists V3d \in \\ ty\_2Erealax\_2Ereal. ((p (ap (ap c\_2Erealax\_2Ereal\_lt (ap c\_2Ereal\_2Ereal\_of\_num \\ & c\_2Enum\_2E0)) V3d)) \wedge (\forall V4y \in ty\_2Erealax\_2Ereal. ((p (ap \\ & (ap c\_2Ereal\_2Ereal\_lte (ap c\_2Ereal\_2Eabs (ap (ap c\_2Ereal\_2Ereal\_sub \\ & V4y) V2x))) V3d)) \Rightarrow ((p (ap (ap c\_2Erealax\_2Ereal\_lt V0a) V4y)) \wedge \\ & (p (ap (ap c\_2Erealax\_2Ereal\_lt V4y) V1b)))))))))) \end{aligned}$$