

thm_2Elim_2ELIM (TMLkRurhwaEUKiZyS- GuPP9HMahYcYJmnEpj)

October 26, 2020

Let $ty_2Ehreal_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Ehreal_2Ehreal \tag{1}$$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \tag{2}$$

Let $ty_2Erealax_2Ereal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Erealax_2Ereal \tag{3}$$

Let $c_2Erealax_2Ereal_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_REP_CLASS \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{ty_2Erealax_2Ereal}) \tag{4}$$

Definition 1 We define c_2Emin_2E40 to be $\lambda A.\lambda P \in 2^A$.if $(\exists x \in A.p\ (ap\ P\ x))$ **then** (the $(\lambda x.x \in A \wedge p\ x)$ of type $\iota \Rightarrow \iota$).

Definition 2 We define c_2Emin_2E3D to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o\ (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 3 We define c_2Ebool_2E2T to be $(ap\ (ap\ (c_2Emin_2E3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x))$

Definition 4 We define c_2Ebool_2E21 to be $\lambda A.27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap\ (ap\ (c_2Emin_2E3D\ (2^{A-27a})))$

Definition 5 We define $c_2Erealax_2Ereal_REP$ to be $\lambda V0a \in ty_2Erealax_2Ereal.(ap\ (c_2Emin_2E40\ (ty_2Erealax_2Ereal_REP_CLASS\ a)))$

Let $c_2Erealax_2Etrealm_neg : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealm_neg \in ((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)}) \tag{5}$$

Let $c_2Erealax_2Etreall_eq : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreall_eq \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal)}) \quad (6)$$

Let $c_2Erealax_2Ereal_ABS_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_ABS_CLASS \in (ty_2Erealax_2Ereal)^{(2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})} \quad (7)$$

Definition 6 We define $c_2Erealax_2Ereal_ABS$ to be $\lambda V0r \in (ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)$

Definition 7 We define $c_2Erealax_2Ereal_neg$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.(ap\ c_2Erealax_2Ereal_ABS)$

Let $c_2Erealax_2Etreall_add : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreall_add \in (((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)}) \quad (8)$$

Definition 8 We define $c_2Erealax_2Ereal_add$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal$

Definition 9 We define $c_2Ereal_2Ereal_sub$ to be $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1y \in ty_2Erealax_2Ereal$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \quad (9)$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \quad (10)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum)^{\omega} \quad (11)$$

Definition 10 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Let $c_2Ereal_2Ereal_of_num : \iota$ be given. Assume the following.

$$c_2Ereal_2Ereal_of_num \in (ty_2Erealax_2Ereal)^{ty_2Enum_2Enum} \quad (12)$$

Let $c_2Erealax_2Etreall_lt : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreall_lt \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal)}) \quad (13)$$

Definition 11 We define $c_2Erealax_2Ereal_lt$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal$

Definition 12 We define c_2Ebool_2EF to be $(ap\ (c_2Ebool_2E21\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 13 We define $c_2Emin_2E3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o\ (p\ P \Rightarrow p\ Q)$ of type ι .

Definition 14 We define $c_Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_Emin_2E_3D_3D_3E V0t) c_Ebool_2E_7E))$

Definition 15 We define $c_Ereal_2Ereal_lte$ to be $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1y \in ty_2Erealax_2Ereal.$

Definition 16 We define $c_Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_Ebool_2E_21) 2) (\lambda V2t \in 2.))$

Definition 17 We define c_Ebool_2ECOND to be $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a.))$

Definition 18 We define c_Ereal_2Eabs to be $\lambda V0x \in ty_2Erealax_2Ereal.(ap (ap (ap (c_Ebool_2ECOND$

Let $c_Epair_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_Epair_2ESND \\ A_27a\ A_27b \in (A_27b^{(ty_2Epair_2Eprod\ A_27a\ A_27b)}) \end{aligned} \quad (14)$$

Let $c_Epair_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_Epair_2EFST \\ A_27a\ A_27b \in (A_27a^{(ty_2Epair_2Eprod\ A_27a\ A_27b)}) \end{aligned} \quad (15)$$

Definition 19 We define $c_Epair_2EUNCURRY$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda V0f \in ((A_27c^{A_27a})$

Let $ty_2Emetric_2Emetric : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Emetric_2Emetric\ A0) \quad (16)$$

Let $c_2Emetric_2Emetric : \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow c_2Emetric_2Emetric\ A_27a \in ((ty_2Emetric_2Emetric \\ A_27a)^{(ty_2Erealax_2Ereal^{(ty_2Epair_2Eprod\ A_27a\ A_27a)})}) \end{aligned} \quad (17)$$

Definition 20 We define $c_2Emetric_2Emr1$ to be $(ap (c_2Emetric_2Emetric\ ty_2Erealax_2Ereal) (ap (c_2Emetric_2Emr1$

Let $c_Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_Epair_2EABS_prod \\ A_27a\ A_27b \in ((ty_2Epair_2Eprod\ A_27a\ A_27b)^{(2^{A_27b})^{A_27a}}) \end{aligned} \quad (18)$$

Definition 21 We define $c_Epair_2E_2C$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap (c_2Emetric_2Emr1$

Let $c_2Enets_2Etendsto : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Enets_2Etendsto\ A_27a \in (((2^{A_27a})^{A_27a})^{(ty_2Epair_2Eprod\ (ty_2Emetric_2Emr1\ A_27a\ A_27a))}) \quad (19)$$

Let $c_2Emetric_2Edist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Emetric_2Edist\ A_27a \in ((ty_2Erealax_2Ereal^{(ty_2Epair_2Eprod\ A_27a\ A_27a)})^{(ty_2Emetric_2Edist\ A_27a)}) \quad (20)$$

Definition 22 We define $c_Ebool_2E_3F$ to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A-27a}). (ap V0P (ap (c_2Emin_2E_40$
Let $ty_2Etopology_2Etopology : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty_2Etopology_2Etopology A0) \quad (21)$$

Let $c_2Etopology_2Etopology : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Etopology_2Etopology A_27a \in ((ty_2Etopology_2Etopology A_27a)^{(2^{(2^A-27a)})}) \quad (22)$$

Definition 23 We define $c_2Emetric_2Emtop$ to be $\lambda A_27a : \iota. \lambda V0m \in (ty_2Emetric_2Emetric A_27a). (ap$
Let $c_2Enets_2Eextends : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Enets_2Eextends A_27a A_27b \in (((2^{(ty_2Epair_2Eprod (ty_2Etopology_2Etopology A_27a) ((2^{A-27b})^{A-27b}))})_{A_27a})_{(A_27a)^{A-27b}})) \quad (23)$$

Definition 24 We define $c_2Elim_2Eextends_real_real$ to be $\lambda V0f \in (ty_2Erealax_2Ereal^{ty_2Erealax_2Ereal}).$

Definition 25 We define $c_2Epred_set_2EUNIV$ to be $\lambda A_27a : \iota. (\lambda V0x \in A_27a.c_2Ebool_2ET).$

Let $c_2Etopology_2Eneigh : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Etopology_2Eneigh A_27a \in ((2^{(ty_2Epair_2Eprod (2^{A-27a}) A_27a)})(ty_2Etopology_2Etopology A_27a)) \quad (24)$$

Definition 26 We define $c_2Etopology_2Elimpt$ to be $\lambda A_27a : \iota. \lambda V0top \in (ty_2Etopology_2Etopology A_27a$
Assume the following.

$$(\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. ((ap (ap (c_2Emetric_2Edist ty_2Erealax_2Ereal) c_2Emetric_2Emr1) (ap (ap (c_2Epair_2E_2C ty_2Erealax_2Ereal ty_2Erealax_2Ereal) V0x) V1y)) = (ap c_2Ereal_2Eabs (ap (ap c_2Ereal_2Ereal_sub V1y) V0x)))))) \quad (25)$$

Assume the following.

$$(\forall V0x \in ty_2Erealax_2Ereal. (p (ap (ap (ap (c_2Etopology_2Elimpt ty_2Erealax_2Ereal) (ap (c_2Emetric_2Emtop ty_2Erealax_2Ereal) c_2Emetric_2Emr1)) V0x) (c_2Epred_set_2EUNIV ty_2Erealax_2Ereal)))) \quad (26)$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\
& \forall V0m1 \in (ty_2Emetric_2Emetric\ A.27a).(\forall V1m2 \in (ty_2Emetric_2Emetric \\
& \quad A.27b).(\forall V2f \in (A.27b^{A.27a}).(\forall V3x0 \in A.27a.(\forall V4y0 \in \\
& \quad A.27b.((p\ (ap\ (ap\ (ap\ (c_2Etopology_2Elimpt\ A.27a)\ (ap\ (c_2Emetric_2Emtp \\
& \quad \quad A.27a)\ V0m1))\ V3x0)\ (c_2Epred_set_2EUNIV\ A.27a)))) \Rightarrow ((p\ (ap\ (ap \\
& \quad \quad (ap\ (c_2Enets_2Etends\ A.27b\ A.27a)\ V2f)\ V4y0)\ (ap\ (ap\ (c_2Epair_2E_2C \\
& \quad \quad (ty_2Etopology_2Etopology\ A.27b)\ ((2^{A.27a})^{A.27a}))\ (ap\ (c_2Emetric_2Emtp \\
& \quad \quad \quad A.27b)\ V1m2))\ (ap\ (c_2Enets_2Etendsto\ A.27a)\ (ap\ (ap\ (c_2Epair_2E_2C \\
& \quad \quad \quad (ty_2Emetric_2Emetric\ A.27a)\ A.27a)\ V0m1)\ V3x0)))))) \Leftrightarrow (\forall V5e \in \\
& \quad ty_2Erealax_2Ereal.((p\ (ap\ (ap\ c_2Erealax_2Ereal_lt\ (ap\ c_2Ereal_2Ereal_of_num \\
& \quad \quad c_2Enum_2E0))\ V5e)) \Rightarrow (\exists V6d \in ty_2Erealax_2Ereal.((p\ (ap \\
& \quad \quad (ap\ c_2Erealax_2Ereal_lt\ (ap\ c_2Ereal_2Ereal_of_num\ c_2Enum_2E0)) \\
& \quad \quad \quad V6d)) \wedge (\forall V7x \in A.27a.(((p\ (ap\ (ap\ c_2Erealax_2Ereal_lt \\
& \quad \quad (ap\ c_2Ereal_2Ereal_of_num\ c_2Enum_2E0))\ (ap\ (ap\ (c_2Emetric_2Edist \\
& \quad \quad \quad A.27a)\ V0m1)\ (ap\ (ap\ (c_2Epair_2E_2C\ A.27a\ A.27a)\ V7x)\ V3x0)))) \wedge \\
& \quad \quad \quad (p\ (ap\ (ap\ c_2Erealax_2Ereal_lt\ (ap\ (ap\ (c_2Emetric_2Edist\ A.27a) \\
& \quad \quad \quad V0m1)\ (ap\ (ap\ (c_2Epair_2E_2C\ A.27a\ A.27a)\ V7x)\ V3x0))))\ V6d))) \Rightarrow \\
& \quad \quad \quad (p\ (ap\ (ap\ c_2Erealax_2Ereal_lt\ (ap\ (ap\ (c_2Emetric_2Edist\ A.27b) \\
& \quad \quad \quad V1m2)\ (ap\ (ap\ (c_2Epair_2E_2C\ A.27b\ A.27b)\ (ap\ V2f\ V7x))\ V4y0))) \\
& \quad \quad \quad V5e))))))))))))) \\
& \hspace{15em} (27)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal.(\forall V1y \in ty_2Erealax_2Ereal. \\
& \quad ((ap\ c_2Ereal_2Eabs\ (ap\ (ap\ c_2Ereal_2Ereal_sub\ V0x)\ V1y)) = (\\
& \quad \quad ap\ c_2Ereal_2Eabs\ (ap\ (ap\ c_2Ereal_2Ereal_sub\ V1y)\ V0x)))) \\
& \hspace{15em} (28)
\end{aligned}$$

Theorem 1

$$\begin{aligned}
& (\forall V0f \in (ty_2Erealax_2Ereal^{ty_2Erealax_2Ereal}).(\forall V1y0 \in \\
& \quad ty_2Erealax_2Ereal.(\forall V2x0 \in ty_2Erealax_2Ereal.((p\ (\\
& \quad \quad ap\ (ap\ (ap\ c_2Elim_2Etends_real_real\ V0f)\ V1y0)\ V2x0)) \Leftrightarrow (\forall V3e \in \\
& \quad ty_2Erealax_2Ereal.((p\ (ap\ (ap\ c_2Erealax_2Ereal_lt\ (ap\ c_2Ereal_2Ereal_of_num \\
& \quad \quad c_2Enum_2E0))\ V3e)) \Rightarrow (\exists V4d \in ty_2Erealax_2Ereal.((p\ (ap \\
& \quad \quad (ap\ c_2Erealax_2Ereal_lt\ (ap\ c_2Ereal_2Ereal_of_num\ c_2Enum_2E0)) \\
& \quad \quad \quad (ap\ c_2Ereal_2Eabs \\
& \quad \quad \quad (ap\ c_2Ereal_2Ereal_sub\ V5x)\ V2x0)))) \wedge (p\ (ap\ (ap\ c_2Erealax_2Ereal_lt \\
& \quad \quad \quad (ap\ c_2Ereal_2Eabs\ (ap\ (ap\ c_2Ereal_2Ereal_sub\ V5x)\ V2x0)))\ V4d))) \Rightarrow \\
& \quad (p\ (ap\ (ap\ c_2Erealax_2Ereal_lt\ (ap\ c_2Ereal_2Eabs\ (ap\ (ap\ c_2Ereal_2Ereal_sub \\
& \quad \quad \quad (ap\ V0f\ V5x))\ V1y0)))\ V3e))))))))) \\
& \hspace{15em} (29)
\end{aligned}$$