

# thm\_2Elim\_2ELIM\_\_TRANSFORM (TMTBrZyN2WNhiRd9UAefEfrdssgVCj82eU5)

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**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Ebool\_2E\_7E$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

**Definition 3** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A.\lambda P \in 2^A.(ap (ap (c\_2Emin\_2E\_3D (2^{A-27a})) (\lambda V0P \in (2^{A-27a}).(ap (ap (c\_2Emin\_2E\_3D (2^{A-27a})) (\lambda V0t \in 2.V0t)) (\lambda V1t \in 2.V1t))$

**Definition 4** We define  $c\_2Ebool\_2E\_7E$  to be  $(ap (c\_2Ebool\_2E\_21 2) (\lambda V0t \in 2.V0t))$ .

**Definition 5** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p P \Rightarrow p Q)$  of type  $\iota$ .

**Definition 6** We define  $c\_2Ebool\_2E\_7E$  to be  $(\lambda V0t \in 2.(ap (ap c\_2Emin\_2E\_3D\_3D\_3E V0t) c\_2Ebool\_2E\_7E))$

Let  $ty\_2Ehreal\_2Ehreal : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Ehreal\_2Ehreal \tag{1}$$

Let  $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \tag{2}$$

Let  $ty\_2Erealax\_2Ereal : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Erealax\_2Ereal \tag{3}$$

Let  $c\_2Erealax\_2Ereal\_REP\_CLASS : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Ereal\_REP\_CLASS \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})\ ty\_2Erealax\_2Ereal) \tag{4}$$

**Definition 7** We define  $c\_2Emin\_2E\_40$  to be  $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x)) \mathbf{then} (the (\lambda x.x \in A \wedge p (ap P x)))$  of type  $\iota \Rightarrow \iota$ .

**Definition 8** We define  $c\_2Erealax\_2Ereal\_REP$  to be  $\lambda V0a \in ty\_2Erealax\_2Ereal.(ap (c\_2Emin\_2E\_40 (ty\_2Erealax\_2Ereal\_REP\_CLASS V0a)))$

Let  $c\_2Erealax\_2Etreal\_neg : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etreal\_neg \in ((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)) \quad (5)$$

Let  $c\_2Erealax\_2Etreal\_eq : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etreal\_eq \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal)) \quad (6)$$

Let  $c\_2Erealax\_2Ereal\_ABS\_CLASS : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Ereal\_ABS\_CLASS \in (ty\_2Erealax\_2Ereal)^{(2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})} \quad (7)$$

**Definition 9** We define  $c\_2Erealax\_2Ereal\_ABS$  to be  $\lambda V0r \in (ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)$

**Definition 10** We define  $c\_2Erealax\_2Ereal\_neg$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.(ap\ c\_2Erealax\_2Ereal)$

Let  $c\_2Erealax\_2Etreal\_add : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etreal\_add \in (((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal))^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)}) \quad (8)$$

**Definition 11** We define  $c\_2Erealax\_2Ereal\_add$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Erealax\_2Ereal$

**Definition 12** We define  $c\_2Ereal\_2Ereal\_sub$  to be  $\lambda V0x \in ty\_2Erealax\_2Ereal.\lambda V1y \in ty\_2Erealax\_2Ereal$

Let  $c\_2Enum\_2EZERO\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2EZERO\_REP \in \omega \quad (9)$$

Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \quad (10)$$

Let  $c\_2Enum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum)^{\omega} \quad (11)$$

**Definition 13** We define  $c\_2Enum\_2E0$  to be  $(ap\ c\_2Enum\_2EABS\_num\ c\_2Enum\_2EZERO\_REP)$ .

Let  $c\_2Ereal\_2Ereal\_of\_num : \iota$  be given. Assume the following.

$$c\_2Ereal\_2Ereal\_of\_num \in (ty\_2Erealax\_2Ereal)^{ty\_2Enum\_2Enum} \quad (12)$$

Let  $c\_2Erealax\_2Etreal\_lt : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etreal\_lt \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal)) \quad (13)$$



**Definition 22** We define  $c\_Ebool\_2E\_3F$  to be  $\lambda A\_27a : \iota. (\lambda V0P \in (2^{A-27a}). (ap\ V0P\ (ap\ (c\_2Emin\_2E\_40\ (ty\_2Etopology\_2Etopology\ \iota \Rightarrow \iota))))$

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Etopology\_2Etopology\ A0) \quad (21)$$

Let  $c\_2Etopology\_2Etopology : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Etopology\_2Etopology\ A\_27a \in ((ty\_2Etopology\_2Etopology\ A\_27a)^{(2^{(2^A-27a)})}) \quad (22)$$

**Definition 23** We define  $c\_2Emetric\_2Emtop$  to be  $\lambda A\_27a : \iota. \lambda V0m \in (ty\_2Emetric\_2Emetric\ A\_27a). (ap\ (c\_2Enets\_2Eends\ \iota \Rightarrow \iota \Rightarrow \iota))$

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Enets\_2Eends\ A\_27a\ A\_27b \in (((2^{(ty\_2Epair\_2Eprod\ (ty\_2Etopology\_2Etopology\ A\_27a)\ ((2^{A-27b})^{A-27b}))})^{A\_27a})^{(A\_27a^{A-27b})}) \quad (23)$$

**Definition 24** We define  $c\_2Elim\_2Eends\_real\_real$  to be  $\lambda V0f \in (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal})$

**Definition 25** We define  $c\_2Earithmetic\_2EZERO$  to be  $c\_2Enum\_2E0$ .

Let  $c\_2Enum\_2EREP\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EREP\_num \in (\omega^{ty\_2Enum\_2Enum}) \quad (24)$$

Let  $c\_2Enum\_2ESUC\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2ESUC\_REP \in (\omega^{\omega}) \quad (25)$$

**Definition 26** We define  $c\_2Enum\_2ESUC$  to be  $\lambda V0m \in ty\_2Enum\_2Enum. (ap\ c\_2Enum\_2EABS\_num)$

Let  $c\_2Earithmetic\_2E\_2B : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (26)$$

**Definition 27** We define  $c\_2Earithmetic\_2EBIT2$  to be  $\lambda V0n \in ty\_2Enum\_2Enum. (ap\ (ap\ c\_2Earithmetic\_2E\_2B))$

**Definition 28** We define  $c\_2Earithmetic\_2ENUMERAL$  to be  $\lambda V0x \in ty\_2Enum\_2Enum. V0x$ .

Let  $c\_2Erealax\_2Etrealm\_inv : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etrealm\_inv \in ((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)}) \quad (27)$$

**Definition 29** We define  $c\_2Erealax\_2Einv$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal. (ap\ c\_2Erealax\_2Ereal\_ABS)$

Let  $c\_2Erealax\_2Etrealmul : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etrealmul \in (((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal)) (28)$$

**Definition 30** We define  $c\_2Erealax\_2Ereal\_mul$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Erealax\_2Ereal.$

**Definition 31** We define  $c\_2Ereal\_2E\_2F$  to be  $\lambda V0x \in ty\_2Erealax\_2Ereal.\lambda V1y \in ty\_2Erealax\_2Ereal.$

Assume the following.

$$True \quad (29)$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \wedge True) \Leftrightarrow (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge (((p\ V0t) \wedge False) \Leftrightarrow False) \wedge (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)))))) \quad (30)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow False) \Leftrightarrow \neg(p\ V0t)))))) \quad (31)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow \neg(p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow \neg(p\ V0t)))))) \quad (32)$$

Assume the following.

$$(\forall V0f \in (ty\_2Erealax\_2Ereal\ ty\_2Erealax\_2Ereal).(\forall V1y0 \in ty\_2Erealax\_2Ereal.(\forall V2x0 \in ty\_2Erealax\_2Ereal.((p\ (ap\ (ap\ (ap\ c\_2Elim\_2Eends\_real\_real\ V0f)\ V1y0)\ V2x0)) \Leftrightarrow (\forall V3e \in ty\_2Erealax\_2Ereal.((p\ (ap\ (ap\ c\_2Erealax\_2Ereal\_lt\ (ap\ c\_2Ereal\_2Ereal\_of\_num\ c\_2Enum\_2E0))\ V3e)) \Rightarrow (\exists V4d \in ty\_2Erealax\_2Ereal.((p\ (ap\ (ap\ c\_2Erealax\_2Ereal\_lt\ (ap\ c\_2Ereal\_2Ereal\_of\_num\ c\_2Enum\_2E0))\ V4d)) \wedge (\forall V5x \in ty\_2Erealax\_2Ereal.((p\ (ap\ (ap\ c\_2Erealax\_2Ereal\_lt\ (ap\ c\_2Ereal\_2Ereal\_of\_num\ c\_2Enum\_2E0))\ (ap\ c\_2Ereal\_2Eabs\ (ap\ (ap\ c\_2Ereal\_2Ereal\_sub\ V5x)\ V2x0)))))) \wedge (p\ (ap\ (ap\ c\_2Erealax\_2Ereal\_lt\ (ap\ c\_2Ereal\_2Eabs\ (ap\ (ap\ c\_2Ereal\_2Ereal\_sub\ V5x)\ V2x0)))\ V4d))) \Rightarrow (p\ (ap\ (ap\ c\_2Erealax\_2Ereal\_lt\ (ap\ c\_2Ereal\_2Eabs\ (ap\ (ap\ c\_2Ereal\_2Ereal\_sub\ (ap\ V0f\ V5x))\ V1y0)))\ V3e)))))))))) \quad (33)$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.(\forall V1y \in ty\_2Erealax\_2Ereal.(\forall V2z \in ty\_2Erealax\_2Ereal.(((p\ (ap\ (ap\ c\_2Erealax\_2Ereal\_lt\ V0x)\ V1y)) \wedge (p\ (ap\ (ap\ c\_2Erealax\_2Ereal\_lt\ V1y)\ V2z))) \Rightarrow (p\ (ap\ (ap\ c\_2Erealax\_2Ereal\_lt\ V0x)\ V2z)))))) \quad (34)$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.(p (ap (ap c\_2Ereal\_2Ereal\_lte V0x) V0x))) \quad (35)$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.(\forall V1y \in ty\_2Erealax\_2Ereal. (\forall V2z \in ty\_2Erealax\_2Ereal.(((p (ap (ap c\_2Erealax\_2Ereal\_lte V0x) V1y)) \wedge (p (ap (ap c\_2Ereal\_2Ereal\_lte V1y) V2z)))) \Rightarrow (p (ap (ap c\_2Erealax\_2Ereal\_lte V0x) V2z)))))) \quad (36)$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.(\forall V1y \in ty\_2Erealax\_2Ereal. (\forall V2z \in ty\_2Erealax\_2Ereal.(((p (ap (ap c\_2Erealax\_2Ereal\_lte V0x) V1y)) \wedge (p (ap (ap c\_2Erealax\_2Ereal\_lte V1y) V2z)))) \Rightarrow (p (ap (ap c\_2Erealax\_2Ereal\_lte V0x) V2z)))))) \quad (37)$$

Assume the following.

$$(\forall V0w \in ty\_2Erealax\_2Ereal.(\forall V1x \in ty\_2Erealax\_2Ereal. (\forall V2y \in ty\_2Erealax\_2Ereal.(\forall V3z \in ty\_2Erealax\_2Ereal. (((p (ap (ap c\_2Erealax\_2Ereal\_lte V0w) V1x)) \wedge (p (ap (ap c\_2Erealax\_2Ereal\_lte V2y) V3z)))) \Rightarrow (p (ap (ap c\_2Erealax\_2Ereal\_lte (ap (ap c\_2Erealax\_2Ereal\_add V0w) V2y)) (ap (ap c\_2Erealax\_2Ereal\_add V1x) V3z))))))))) \quad (38)$$

Assume the following.

$$(\forall V0d \in ty\_2Erealax\_2Ereal.(((p (ap (ap c\_2Erealax\_2Ereal\_lte (ap c\_2Ereal\_2Ereal\_of\_num c\_2Enum\_2E0) (ap (ap c\_2Ereal\_2E\_2F V0d) (ap c\_2Ereal\_2Ereal\_of\_num (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT2 c\_2Earithmetic\_2EZERO)))))) \Leftrightarrow (p (ap (ap c\_2Erealax\_2Ereal\_lte (ap c\_2Ereal\_2Ereal\_of\_num c\_2Enum\_2E0) V0d)))))) \quad (39)$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.(((ap (ap c\_2Erealax\_2Ereal\_add (ap (ap c\_2Ereal\_2E\_2F V0x) (ap c\_2Ereal\_2Ereal\_of\_num (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT2 c\_2Earithmetic\_2EZERO)))))) (ap (ap c\_2Ereal\_2E\_2F V0x) (ap c\_2Ereal\_2Ereal\_of\_num (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT2 c\_2Earithmetic\_2EZERO)))))) = V0x)) \quad (40)$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal. (\forall V1y \in ty\_2Erealax\_2Ereal. \\
& ((p (ap (ap c\_2Erealax\_2Ereal\_lt (ap c\_2Ereal\_2Ereal\_of\_num \\
& c\_2Enum\_2E0)) V0x)) \wedge (p (ap (ap c\_2Erealax\_2Ereal\_lt (ap c\_2Ereal\_2Ereal\_of\_num \\
& c\_2Enum\_2E0)) V1y))) \Rightarrow (\exists V2z \in ty\_2Erealax\_2Ereal. ((p ( \\
& ap (ap c\_2Erealax\_2Ereal\_lt (ap c\_2Ereal\_2Ereal\_of\_num c\_2Enum\_2E0)) \\
& V2z)) \wedge ((p (ap (ap c\_2Erealax\_2Ereal\_lt V2z) V0x)) \wedge (p (ap (ap c\_2Erealax\_2Ereal\_lt \\
& V2z) V1y)))))))))
\end{aligned} \tag{41}$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal. ((ap (ap c\_2Ereal\_2Ereal\_sub V0x) (ap c\_2Ereal\_2Ereal\_of\_num c\_2Enum\_2E0)) = V0x)) \tag{42}$$

Assume the following.

$$\begin{aligned}
& (\forall V0a \in ty\_2Erealax\_2Ereal. (\forall V1b \in ty\_2Erealax\_2Ereal. \\
& (\forall V2c \in ty\_2Erealax\_2Ereal. ((ap (ap c\_2Erealax\_2Ereal\_add \\
& (ap (ap c\_2Ereal\_2Ereal\_sub V0a) V1b)) (ap (ap c\_2Ereal\_2Ereal\_sub \\
& V1b) V2c)) = (ap (ap c\_2Ereal\_2Ereal\_sub V0a) V2c))))))
\end{aligned} \tag{43}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal. (\forall V1y \in ty\_2Erealax\_2Ereal. \\
& (p (ap (ap c\_2Ereal\_2Ereal\_lte (ap c\_2Ereal\_2Eabs (ap (ap c\_2Erealax\_2Ereal\_add \\
& V0x) V1y))) (ap (ap c\_2Erealax\_2Ereal\_add (ap c\_2Ereal\_2Eabs \\
& V0x)) (ap c\_2Ereal\_2Eabs V1y))))))
\end{aligned} \tag{44}$$

### Theorem 1

$$\begin{aligned}
& (\forall V0f \in (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}). (\forall V1g \in \\
& (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}). (\forall V2x0 \in ty\_2Erealax\_2Ereal. \\
& (\forall V3l \in ty\_2Erealax\_2Ereal. (((p (ap (ap (ap c\_2Elim\_2Etends\_real\_real \\
& (\lambda V4x \in ty\_2Erealax\_2Ereal. (ap (ap c\_2Ereal\_2Ereal\_sub ( \\
& ap V0f) V4x)) (ap V1g) V4x)))) (ap c\_2Ereal\_2Ereal\_of\_num c\_2Enum\_2E0)) \\
& V2x0)) \wedge (p (ap (ap (ap c\_2Elim\_2Etends\_real\_real V1g) V3l) V2x0))) \Rightarrow \\
& (p (ap (ap (ap c\_2Elim\_2Etends\_real\_real V0f) V3l) V2x0))))))
\end{aligned}$$