thm_2Elim_2EMVT (TMN4LPYHVgY3vtkKhx3MAyRQvGqqK7FCprN)

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Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o\ (x = y)$ *of type* $\iota{\Rightarrow}\iota$.

Definition 2 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1$

Definition 3 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

Definition 4 *We define* c_2Ebool_2EF *to be* $(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V0t \in 2.V0t))$.

Let $c\_2Enum\_2EZERO\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2EZERO\_\_REP \in omega \tag{1}$$

Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{2}$$

Let $c\_2Enum\_2EABS\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EABS\_\_num \in (ty\_2Enum\_2Enum^{omega}) \tag{3}$$

Definition 5 *We define* c_2Enum_2E0 *to be* $(ap\ c\_2Enum\_2EABS\_\_num\ c\_2Enum\_2EZERO\_\_REP)$.

Let $ty\_2Erealax\_2Ereal : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Erealax\_2Ereal \tag{4}$$

Let $c\_2Ereal\_2Ereal\_\_of\_\_num : \iota$ be given. Assume the following.

$$c\_2Ereal\_2Ereal\_\_of\_\_num \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}) \tag{5}$$

Let $ty\_2Ehreal\_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Ehreal\_2Ehreal \tag{6}$$

Let $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \quad (7)$$

Let $c\_2Erealax\_2Ereal\_\_REP\_\_CLASS : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Ereal\_\_REP\_\_CLASS \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{ty\_2Erealax} \quad (8)$$

Definition 6 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.$**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge p$ *of type* $\iota \Rightarrow \iota$.

Definition 7 *We define* c_2Erealax_2Ereal__REP *to be* $\lambda V0a \in ty\_2Erealax\_2Ereal.(ap$ (c_2Emin_2E_40 $(ty$

Let $c\_2Erealax\_2Etreal\_\_add : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Etreal\_\_add \in (((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehr} \quad (9)$$

Let $c\_2Erealax\_2Etreal\_\_eq : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Etreal\_\_eq \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Eh} \quad (10)$$

Let $c\_2Erealax\_2Ereal\_\_ABS\_\_CLASS : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Ereal\_\_ABS\_\_CLASS \in (ty\_2Erealax\_2Ereal^{(2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehr} \quad (11)$$

Definition 8 *We define* c_2Erealax_2Ereal__ABS *to be* $\lambda V0r \in (ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_$

Definition 9 *We define* c_2Erealax_2Ereal__add *to be* $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Erealax$

Let $c\_2Erealax\_2Etreal\_\_neg : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Etreal\_\_neg \in ((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)}) \quad (12)$$

Definition 10 *We define* c_2Erealax_2Ereal__neg *to be* $\lambda V0T1 \in ty\_2Erealax\_2Ereal.(ap$ c_2Erealax_2Ereal

Definition 11 *We define* c_2Ereal_2Ereal__sub *to be* $\lambda V0x \in ty\_2Erealax\_2Ereal.\lambda V1y \in ty\_2Erealax\_2E$

Let $c\_2Erealax\_2Etreal\_\_inv : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Etreal\_\_inv \in ((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)}) \quad (13)$$

Definition 12 *We define* c_2Erealax_2Einv *to be* $\lambda V0T1 \in ty\_2Erealax\_2Ereal.(ap$ c_2Erealax_2Ereal__ABS

Let $c\_2Erealax\_2Etreal\_\_mul : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Etreal\_\_mul \in (((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehr} \quad (14)$$

Definition 13 *We define* c_2Erealax_2Ereal__mul *to be* $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Ereala$

Definition 14 *We define* c_2Ereal_2E_2F *to be* $\lambda V0x \in ty\_2Erealax\_2Ereal.\lambda V1y \in ty\_2Erealax\_2Ereal.($

Let $c\_2Erealax\_2Etreal\_\_lt : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Etreal\_\_lt \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Ehr} \quad (15)$$

Definition 15 *We define* c_2Erealax_2Ereal__lt *to be* $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Erealax\_$

Definition 16 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 17 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap$ c_2Emin_2E_3D_3D_3E $V0t)$ c_2Ebool_2E

Definition 18 *We define* c_2Ereal_2Ereal__lte *to be* $\lambda V0x \in ty\_2Erealax\_2Ereal.\lambda V1y \in ty\_2Erealax\_2Er$

Definition 19 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap$ (c_2Ebool_2E_21 2) $(\lambda V2t \in$

Definition 20 *We define* c_2Ebool_2ECOND *to be* $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.($

Definition 21 *We define* c_2Ereal_2Eabs *to be* $\lambda V0x \in ty\_2Erealax\_2Ereal.(ap\ (ap\ (ap$ (c_2Ebool_2ECONI

Let $c\_2Epair\_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2ESND\ A\_27a\ A\_27b \in (A\_27b^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}) \quad (16)$$

Let $c\_2Epair\_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EFST\ A\_27a\ A\_27b \in (A\_27a^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}) \quad (17)$$

Definition 22 *We define* c_2Epair_2EUNCURRY *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda V0f \in ((A\_27c^{A\_27}$

Let $ty\_2Emetric\_2Emetric : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Emetric\_2Emetric\ A0) \quad (18)$$

Let $c\_2Emetric\_2Emetric : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Emetric\_2Emetric\ A\_27a \in ((ty\_2Emetric\_2Emetric\ A\_27a)^{(ty\_2Erealax\_2Ereal^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27a)})}) \quad (19)$$

Definition 23 *We define* $\mathsf{c\_2Emetric\_2Emr1}$ *to be* $(ap\ (c\_2Emetric\_2Emetric\ ty\_2Erealax\_2Ereal)\ (ap\ (\mathsf{c}$

Let $c\_2Epair\_2EABS\_\_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EABS\_\_prod\ A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{((2^{A\_27b})^{A\_27a})}) \quad (20)$$

Definition 24 *We define* $\mathsf{c\_2Epair\_2E\_2C}$ *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap\ (c\_2$

Let $c\_2Enets\_2Etendsto : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Enets\_2Etendsto\ A\_27a \in (((2^{A\_27a})^{A\_27a})^{(ty\_2Epair\_2Eprod\ (ty\_2Emetric\_2Em} \quad (21)$$

Let $c\_2Emetric\_2Edist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Emetric\_2Edist\ A\_27a \in ((ty\_2Erealax\_2Ereal^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27} \quad (22)$$

Definition 25 *We define* $\mathsf{c\_2Ebool\_2E\_3F}$ *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ V0P\ (ap\ (\mathsf{c\_2Emin\_2E\_40}$

Let $ty\_2Etopology\_2Etopology : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Etopology\_2Etopology\ A0) \quad (23)$$

Let $c\_2Etopology\_2Etopology : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Etopology\_2Etopology\ A\_27a \in ((ty\_2Etopology\_2Etopology\ A\_27a)^{(2^{(2^{A\_27a})})}) \quad (24)$$

Definition 26 *We define* $\mathsf{c\_2Emetric\_2Emtop}$ *to be* $\lambda A\_27a : \iota.\lambda V0m \in (ty\_2Emetric\_2Emetric\ A\_27a).(ap$

Let $c\_2Enets\_2Etends : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Enets\_2Etends\ A\_27a\ A\_27b \in (((2^{(ty\_2Epair\_2Eprod\ (ty\_2Etopology\_2Etopology\ A\_27a)\ ((2^{A\_27b})^{A\_27b}))})^{A\_27a})^{(A\_27a^{A\_27b})}) \quad (25)$$

Definition 27 *We define* $\mathsf{c\_2Elim\_2Etends\_\_real\_\_real}$ *to be* $\lambda V0f \in (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}).$

Definition 28 *We define* $\mathsf{c\_2Elim\_2Ediffl}$ *to be* $\lambda V0f \in (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}).\lambda V1l \in ty\_2$

Definition 29 *We define* $\mathsf{c\_2Elim\_2Edifferentiable}$ *to be* $\lambda V0f \in (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}).\lambda V$

Definition 30 *We define* $\mathsf{c\_2Elim\_2Econtl}$ *to be* $\lambda V0f \in (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}).\lambda V1x \in ty$

Definition 31 *We define* $\mathsf{c\_2Earithmetic\_2EZERO}$ *to be* $\mathsf{c\_2Enum\_2E0}$.

Let $c\_2Enum\_2EREP\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EREP\_\_num \in (omega^{ty\_2Enum\_2Enum}) \tag{26}$$

Let $c\_2Enum\_2ESUC\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2ESUC\_\_REP \in (omega^{omega}) \tag{27}$$

Definition 32 *We define* c_2Enum_2ESUC *to be* $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_\_num$

Let $c\_2Earithmetic\_2E\_2B : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{28}$$

Definition 33 *We define* c_2Earithmetic_2EBIT1 *to be* $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic$

Definition 34 *We define* c_2Earithmetic_2ENUMERAL *to be* $\lambda V0x \in ty\_2Enum\_2Enum.V0x.$

Assume the following.

$$True \tag{29}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1)\Rightarrow(p\ V1t2))\Rightarrow(((p\ V1t2)\Rightarrow(p\ V0t1))\Rightarrow((p\ V0t1)\Leftrightarrow(p\ V1t2)))))) \tag{30}$$

Assume the following.

$$(\forall V0t \in 2.(False\Rightarrow(p\ V0t))) \tag{31}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow\forall A\_27b.nonempty\ A\_27b\Rightarrow(\forall V0t1 \in A\_27a.(\forall V1t2 \in A\_27b.((ap\ (\lambda V2x \in A\_27b.V0t1)\ V1t2) = V0t1))) \tag{32}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0t \in 2.((\forall V1x \in A\_27a.(p\ V0t))\Leftrightarrow(p\ V0t))) \tag{33}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p\ V0t1)\wedge((p\ V1t2)\wedge(p\ V2t3)))\Leftrightarrow(((p\ V0t1)\wedge(p\ V1t2))\wedge(p\ V2t3)))))) \tag{34}$$

Assume the following.

$$(\forall V0t \in 2.(((True\wedge(p\ V0t))\Leftrightarrow(p\ V0t))\wedge((((p\ V0t)\wedge True)\Leftrightarrow(p\ V0t))\wedge(((False\wedge(p\ V0t))\Leftrightarrow False)\wedge((((p\ V0t)\wedge False)\Leftrightarrow False)\wedge(((p\ V0t)\wedge(p\ V0t))\Leftrightarrow(p\ V0t))))))) \tag{35}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow False) \Leftrightarrow (\neg (p\ V0t)))))))) \tag{36}$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \tag{37}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \tag{38}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \tag{39}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg (p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg (p\ V0t))))))) \tag{40}$$

Assume the following.

$$(\forall V0f \in (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}).(\forall V1l \in ty\_2Erealax\_2Ereal.(\forall V2x \in ty\_2Erealax\_2Ereal.((p\ (ap\ (ap\ (ap\ \mathsf{c\_2Elim\_2Ediffl}\ V0f)\ V1l)\ V2x)) \Rightarrow (p\ (ap\ (ap\ \mathsf{c\_2Elim\_2Econtl}\ V0f)\ V2x)))))) \tag{41}$$

Assume the following.

$$(\forall V0k \in ty\_2Erealax\_2Ereal.(\forall V1x \in ty\_2Erealax\_2Ereal.(p\ (ap\ (ap\ \mathsf{c\_2Elim\_2Econtl}\ (\lambda V2x \in ty\_2Erealax\_2Ereal.V0k))\ V1x)))) \tag{42}$$

Assume the following.

$$(\forall V0f \in (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}).(\forall V1g \in (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}).(\forall V2x \in ty\_2Erealax\_2Ereal.(((p\ (ap\ (ap\ \mathsf{c\_2Elim\_2Econtl}\ V0f)\ V2x)) \wedge (p\ (ap\ (ap\ \mathsf{c\_2Elim\_2Econtl}\ V1g)\ V2x))) \Rightarrow (p\ (ap\ (ap\ \mathsf{c\_2Elim\_2Econtl}\ (\lambda V3x \in ty\_2Erealax\_2Ereal.(ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_mul}\ (ap\ V0f\ V3x))\ (ap\ V1g\ V3x))))\ V2x)))))) \tag{43}$$

Assume the following.

$$(\forall V0f \in (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}).(\forall V1g \in (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}).(\forall V2x \in ty\_2Erealax\_2Ereal. (((p\ (ap\ (ap\ \mathsf{c\_2Elim\_2Econtl}\ V0f)\ V2x)) \wedge (p\ (ap\ (ap\ \mathsf{c\_2Elim\_2Econtl}\ V1g)\ V2x))) \Rightarrow (p\ (ap\ (ap\ \mathsf{c\_2Elim\_2Econtl}\ (\lambda V3x \in ty\_2Erealax\_2Ereal. (ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_sub}\ (ap\ V0f\ V3x))\ (ap\ V1g\ V3x))))\ V2x)))))) \quad (44)$$

Assume the following.

$$(\forall V0f \in (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}).(\forall V1g \in (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}).(\forall V2l \in ty\_2Erealax\_2Ereal. (\forall V3m \in ty\_2Erealax\_2Ereal.(\forall V4x \in ty\_2Erealax\_2Ereal. (((p\ (ap\ (ap\ (ap\ \mathsf{c\_2Elim\_2Ediffl}\ V0f)\ V2l)\ V4x)) \wedge (p\ (ap\ (ap\ (ap\ \mathsf{c\_2Elim\_2Ediffl}\ V1g)\ V3m)\ V4x))) \Rightarrow (p\ (ap\ (ap\ (ap\ \mathsf{c\_2Elim\_2Ediffl}\ (\lambda V5x \in ty\_2Erealax\_2Ereal. (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_add}\ (ap\ V0f\ V5x))\ (ap\ V1g\ V5x))))\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_add}\ V2l)\ V3m))\ V4x)))))))) \quad (45)$$

Assume the following.

$$(\forall V0f \in (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}).(\forall V1c \in ty\_2Erealax\_2Ereal.(\forall V2l \in ty\_2Erealax\_2Ereal.(\forall V3x \in ty\_2Erealax\_2Ereal.((p\ (ap\ (ap\ (ap\ \mathsf{c\_2Elim\_2Ediffl}\ V0f)\ V2l)\ V3x)) \Rightarrow (p\ (ap\ (ap\ (ap\ \mathsf{c\_2Elim\_2Ediffl}\ (\lambda V4x \in ty\_2Erealax\_2Ereal.(ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_mul}\ V1c)\ (ap\ V0f\ V4x))))\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_mul}\ V1c)\ V2l))\ V3x))))))) \quad (46)$$

Assume the following.

$$(\forall V0f \in (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}).(\forall V1g \in (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}).(\forall V2l \in ty\_2Erealax\_2Ereal. (\forall V3m \in ty\_2Erealax\_2Ereal.(\forall V4x \in ty\_2Erealax\_2Ereal. (((p\ (ap\ (ap\ (ap\ \mathsf{c\_2Elim\_2Ediffl}\ V0f)\ V2l)\ V4x)) \wedge (p\ (ap\ (ap\ (ap\ \mathsf{c\_2Elim\_2Ediffl}\ V1g)\ V3m)\ V4x))) \Rightarrow (p\ (ap\ (ap\ (ap\ \mathsf{c\_2Elim\_2Ediffl}\ (\lambda V5x \in ty\_2Erealax\_2Ereal. (ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_sub}\ (ap\ V0f\ V5x))\ (ap\ V1g\ V5x))))\ (ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_sub}\ V2l)\ V3m))\ V4x)))))))) \quad (47)$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.(p\ (ap\ (ap\ (ap\ \mathsf{c\_2Elim\_2Ediffl}\ (\lambda V1x \in ty\_2Erealax\_2Ereal.V1x))\ (ap\ c\_2Ereal\_2Ereal\_\_of\_\_num\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO}))))\ V0x))) \quad (48)$$

Assume the following.

$$(\forall V0f \in (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}).(\forall V1a \in ty\_2Erealax\_2Ereal.(\forall V2b \in ty\_2Erealax\_2Ereal.(((p\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_lt}\ V1a)\ V2b))\wedge(((ap\ V0f\ V1a) = (ap\ V0f\ V2b))\wedge((\forall V3x \in ty\_2Erealax\_2Ereal.(((p\ (ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_lte}\ V1a)\ V3x))\wedge(p\ (ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_lte}\ V3x)\ V2b)))\Rightarrow(p\ (ap\ (ap\ \mathsf{c\_2Elim\_2Econtl}\ V0f)\ V3x))))\wedge(\forall V4x \in ty\_2Erealax\_2Ereal.(((p\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_lt}\ V1a)\ V4x))\wedge(p\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_lt}\ V4x)\ V2b)))\Rightarrow(p\ (ap\ (ap\ \mathsf{c\_2Elim\_2Edifferentiable}\ V0f)\ V4x)))))))\Rightarrow(\exists V5z \in ty\_2Erealax\_2Ereal.((p\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_lt}\ V1a)\ V5z))\wedge((p\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_lt}\ V5z)\ V2b))\wedge(p\ (ap\ (ap\ (ap\ \mathsf{c\_2Elim\_2Ediffl}\ V0f)\ (ap\ c\_2Ereal\_2Ereal\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0}))\ V5z)))))))))) \tag{49}$$

Assume the following.

$$(\forall V0f \in (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}).(\forall V1a \in ty\_2Erealax\_2Ereal.(\forall V2b \in ty\_2Erealax\_2Ereal.((ap\ (\lambda V3x \in ty\_2Erealax\_2Ereal.(ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_sub}\ (ap\ V0f\ V3x))\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_mul}\ (ap\ (ap\ \mathsf{c\_2Ereal\_2E\_2F}\ (ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_sub}\ (ap\ V0f\ V2b))\ (ap\ V0f\ V1a)))\ (ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_sub}\ V2b)\ V1a)))\ V3x)))\ V1a) = (ap\ (\lambda V4x \in ty\_2Erealax\_2Ereal.(ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_sub}\ (ap\ V0f\ V4x))\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_mul}\ (ap\ (ap\ \mathsf{c\_2Ereal\_2E\_2F}\ (ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_sub}\ (ap\ V0f\ V2b))\ (ap\ V0f\ V1a)))\ (ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_sub}\ V2b)\ V1a)))\ V4x)))\ V2b))))) \tag{50}$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.((ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_add}\ (ap\ c\_2Ereal\_2Ereal\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0}))\ V0x) = V0x)) \tag{51}$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.(\neg(p\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_lt}\ V0x)\ V0x)))) \tag{52}$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.((ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_mul}\ V0x)\ (ap\ c\_2Ereal\_2Ereal\_\_of\_\_num\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO})))) = V0x)) \tag{53}$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.(\forall V1y \in ty\_2Erealax\_2Ereal.((ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_add}\ (ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_sub}\ V0x)\ V1y))\ V1y) = V0x))) \tag{54}$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.(\forall V1y \in ty\_2Erealax\_2Ereal.\\ (((ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_sub}\ V0x)\ V1y) = (ap\ c\_2Ereal\_2Ereal\_\_of\_\_num\\ \mathsf{c\_2Enum\_2E0}))\Leftrightarrow(V0x = V1y)))) \tag{55}$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.(\forall V1y \in ty\_2Erealax\_2Ereal.\\ ((\neg(V1y = (ap\ c\_2Ereal\_2Ereal\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0})))\Rightarrow((ap\ (\\ ap\ \mathsf{c\_2Erealax\_2Ereal\_\_mul}\ V1y)\ (ap\ (ap\ \mathsf{c\_2Ereal\_2E\_2F}\ V0x)\ V1y)) =\\ V0x)))) \tag{56}$$

Theorem 1

$$(\forall V0f \in (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}).(\forall V1a \in\\ ty\_2Erealax\_2Ereal.(\forall V2b \in ty\_2Erealax\_2Ereal.(((p\ (\\ ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_lt}\ V1a)\ V2b))\wedge((\forall V3x \in ty\_2Erealax\_2Ereal.\\ (((p\ (ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_lte}\ V1a)\ V3x))\wedge(p\ (ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_lte}\\ V3x)\ V2b)))\Rightarrow(p\ (ap\ (ap\ \mathsf{c\_2Elim\_2Econtl}\ V0f)\ V3x))))\wedge(\forall V4x \in\\ ty\_2Erealax\_2Ereal.(((p\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_lt}\ V1a)\ V4x))\wedge\\ (p\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_lt}\ V4x)\ V2b)))\Rightarrow(p\ (ap\ (ap\ \mathsf{c\_2Elim\_2Edifferentiable}\\ V0f)\ V4x))))))\Rightarrow(\exists V5l \in ty\_2Erealax\_2Ereal.(\exists V6z \in\\ ty\_2Erealax\_2Ereal.((p\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_lt}\ V1a)\ V6z))\wedge\\ ((p\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_lt}\ V6z)\ V2b))\wedge((p\ (ap\ (ap\ (ap\ \mathsf{c\_2Elim\_2Ediffl}\\ V0f)\ V5l)\ V6z))\wedge((ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_sub}\ (ap\ V0f\ V2b))\ (ap\\ V0f\ V1a)) = (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_mul}\ (ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_sub}\\ V2b)\ V1a))\ V5l)))))))))))$$