thm_2ElistRange_2ElistRangeINC__SING (TM-NRsWT8ZEEGExqpzt5pBxZWQ8Gq3LxFF9j)

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Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1$

Definition 3 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

Definition 4 *We define* c_2Ebool_2EF *to be* $(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V0t \in 2.V0t))$.

Let $c\_2Enum\_2EZERO\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2EZERO\_\_REP \in omega \tag{1}$$

Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{2}$$

Let $c\_2Enum\_2EABS\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EABS\_\_num \in (ty\_2Enum\_2Enum^{omega}) \tag{3}$$

Definition 5 *We define* c_2Enum_2E0 *to be* $(ap\ c\_2Enum\_2EABS\_\_num\ c\_2Enum\_2EZERO\_\_REP)$.

Let $c\_2Enum\_2EREP\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EREP\_\_num \in (omega^{ty\_2Enum\_2Enum}) \tag{4}$$

Let $c\_2Enum\_2ESUC\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2ESUC\_\_REP \in (omega^{omega}) \tag{5}$$

Definition 6 *We define* c_2Enum_2ESUC *to be* $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_\_num\ ($

Let $c\_2Earithmetic\_2E\_2B : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{6}$$

Definition 7 *We define* c_2Earithmetic_2EBIT2 *to be* $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic\_$

Let $ty\_2Elist\_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Elist\_2Elist\ A0) \tag{7}$$

Let $c\_2Elist\_2ECONS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ECONS\ A\_27a \in (((ty\_2Elist\_2Elist\ A\_27a)^{(ty\_2Elist\_2Elist\ A\_27a)})^{A\_27a}) \tag{8}$$

Let $c\_2Elist\_2EGENLIST\_\_AUX : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2EGENLIST\_\_AUX\ A\_27a \in ((((ty\_2Elist\_2Elist\ A\_27a)^{(ty\_2Elist\_2Elist\ A\_27a)})^{ty\_2Enum\_2Enum})^{(A\_27a^{ty\_2Enum\_2Enum})}) \tag{9}$$

Let $c\_2Elist\_2ENIL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ENIL\ A\_27a \in (ty\_2Elist\_2Elist\ A\_27a) \tag{10}$$

Definition 8 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 9 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in 2$

Definition 10 *We define* c_2Earithmetic_2EZERO *to be* c_2Enum_2E0.

Definition 11 *We define* c_2Earithmetic_2EBIT1 *to be* $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic$

Definition 12 *We define* c_2Earithmetic_2ENUMERAL *to be* $\lambda V0x \in ty\_2Enum\_2Enum.V0x$.

Let $c\_2Earithmetic\_2E\_2D : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2D \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{11}$$

Let $c\_2Elist\_2EGENLIST : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2EGENLIST\ A\_27a \in (((ty\_2Elist\_2Elist\ A\_27a)^{ty\_2Enum\_2Enum})^{(A\_27a^{ty\_2Enum\_2Enum})}) \tag{12}$$

Definition 13 *We define* c_2ElistRange_2ElistRangeINC *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2En$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ \mathsf{c\_2Enum\_2E0}) = V0m)) \tag{13}$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ V1n) = (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V1n)\ V0m)))) \tag{14}$$

Assume the following.

$$(\forall V0a \in ty\_2Enum\_2Enum.(\forall V1c \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2D\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0a)\ V1c))\ V1c) = V0a))) \tag{15}$$

Assume the following.

$$(\forall V0c \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2D\ V0c)\ V0c) = \mathsf{c\_2Enum\_2E0})) \tag{16}$$

Assume the following.

$$True \tag{17}$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \wedge True) \Leftrightarrow (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge ((((p\ V0t) \wedge False) \Leftrightarrow False) \wedge (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t))))))) \tag{18}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \tag{19}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0a0 \in A\_27a.(\forall V1a1 \in (ty\_2Elist\_2Elist\ A\_27a).(\forall V2a0\_27 \in A\_27a.(\forall V3a1\_27 \in (ty\_2Elist\_2Elist\ A\_27a).(((ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V0a0)\ V1a1) = (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V2a0\_27)\ V3a1\_27)) \Leftrightarrow ((V0a0 = V2a0\_27) \wedge (V1a1 = V3a1\_27))))))) \tag{20}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow ((\forall V0f \in (A\_27a^{ty\_2Enum\_2Enum}). (\forall V1l \in (ty\_2Elist\_2Elist\ A\_27a).((ap\ (ap\ (ap\ (c\_2Elist\_2EGENLIST\_\_AUX\ A\_27a)\ V0f)\ \mathsf{c\_2Enum\_2E0})\ V1l) = V1l))) \land ((\forall V2f \in (A\_27a^{ty\_2Enum\_2Enum}). (\forall V3n \in ty\_2Enum\_2Enum.(\forall V4l \in (ty\_2Elist\_2Elist\ A\_27a).((ap\ (ap\ (ap\ (c\_2Elist\_2EGENLIST\_\_AUX\ A\_27a)\ V2f)\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V3n)))\ V4l) = (ap\ (ap\ (ap\ (c\_2Elist\_2EGENLIST\_\_AUX\ A\_27a)\ V2f)\ (ap\ (ap\ c\_2Earithmetic\_2E\_2D\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V3n)))\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO}))))\ (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ (ap\ V2f\ (ap\ (ap\ c\_2Earithmetic\_2E\_2D\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V3n)))\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO})))))\ V4l)))))) \land (\forall V5f \in (A\_27a^{ty\_2Enum\_2Enum}).(\forall V6n \in ty\_2Enum\_2Enum.(\forall V7l \in (ty\_2Elist\_2Elist\ A\_27a).((ap\ (ap\ (ap\ (c\_2Elist\_2EGENLIST\_\_AUX\ A\_27a)\ V5f)\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V6n)))\ V7l) = (ap\ (ap\ (ap\ (c\_2Elist\_2EGENLIST\_\_AUX\ A\_27a)\ V5f)\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V6n)))\ (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ (ap\ V5f\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V6n))))\ V7l))))))))) \tag{21}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0f \in (A\_27a^{ty\_2Enum\_2Enum}). (\forall V1n \in ty\_2Enum\_2Enum.(((ap\ (ap\ (c\_2Elist\_2EGENLIST\ A\_27a)\ V0f)\ \mathsf{c\_2Enum\_2E0}) = (c\_2Elist\_2ENIL\ A\_27a)) \land ((ap\ (ap\ (c\_2Elist\_2EGENLIST\ A\_27a)\ V0f)\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V1n)) = (ap\ (ap\ (ap\ (c\_2Elist\_2EGENLIST\_\_AUX\ A\_27a)\ V0f)\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V1n))\ (c\_2Elist\_2ENIL\ A\_27a)))))) \tag{22}$$

Theorem 1

$$(\forall V0m \in ty\_2Enum\_2Enum.((ap\ (ap\ \mathsf{c\_2ElistRange\_2ElistRangeINC}\ V0m)\ V0m) = (ap\ (ap\ (c\_2Elist\_2ECONS\ ty\_2Enum\_2Enum)\ V0m)\ (c\_2Elist\_2ENIL\ ty\_2Enum\_2Enum))))$$