

thm_2Elist_2EALL_DISTINCT_FLAT_REVERSE (TMH75osaL78F9L5nZirTzq9CZuKHGAEB8Yo)

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Definition 1 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.$ if $(\exists x \in A.p (ap P x))$ then (the $(\lambda x.x \in A \wedge p x)$ of type $\iota \Rightarrow \iota$).

Definition 2 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 3 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap V0P (ap (c_2Emin_2E_40 A_27a) P)))$

Let $ty_2Elist_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty_2Elist_2Elist A0) \quad (1)$$

Let $c_2Elist_2EFLAT : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Elist_2EFLAT A_27a \in ((ty_2Elist_2Elist A_27a)^{(ty_2Elist_2Elist (ty_2Elist_2Elist A_27a))}) \quad (2)$$

Let $c_2Elist_2EREVERSE : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Elist_2EREVERSE A_27a \in ((ty_2Elist_2Elist A_27a)^{(ty_2Elist_2Elist A_27a)}) \quad (3)$$

Let $c_2Elist_2ECONS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Elist_2ECONS A_27a \in (((ty_2Elist_2Elist A_27a)^{(ty_2Elist_2Elist A_27a)})^{A_27a}) \quad (4)$$

Definition 4 We define $c_2Ebool_2E_2ET$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Let $c_2Elist_2ENIL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Elist_2ENIL A_27a \in (ty_2Elist_2Elist A_27a) \quad (5)$$

Let $c_2Elist_2ELIST_TO_SET : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2ELIST_TO_SET\ A_27a \in \left((2^{A_27a})^{(ty_2Elist_2Elist\ A_27a)} \right) \quad (6)$$

Definition 5 We define c_2Ebool_2EIN to be $\lambda A_27a : \iota. (\lambda V0x \in A_27a. (\lambda V1f \in (2^{A_27a}). (ap\ V1f\ V0x)))$

Let $c_2Elist_2EAPPEND : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2EAPPEND\ A_27a \in \left(((ty_2Elist_2Elist\ A_27a)^{(ty_2Elist_2Elist\ A_27a)})^{(ty_2Elist_2Elist\ A_27a)} \right) \quad (7)$$

Let $c_2Elist_2EALL_DISTINCT : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2EALL_DISTINCT\ A_27a \in \left(2^{(ty_2Elist_2Elist\ A_27a)} \right) \quad (8)$$

Definition 6 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (ap\ (ap\ (c_2Emin_2E_3D\ (2^{A_27a})))$

Definition 7 We define c_2Ebool_2EF to be $(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 8 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2. \lambda Q \in 2. inj_o\ (p\ P \Rightarrow p\ Q)$ of type ι .

Definition 9 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in 2. (ap\ (c_2Emin_2E_3D_3D_3E\ V0t1)\ V2t))))$

Definition 10 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in 2. (ap\ (c_2Emin_2E_3D_3D_3E\ V0t1)\ V2t))))$

Definition 11 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2. (ap\ (ap\ c_2Emin_2E_3D_3D_3E\ V0t)\ c_2Ebool_2E_5C_2F))$

Assume the following.

$$True \quad (9)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \quad (10)$$

Assume the following.

$$(\forall V0t \in 2. (False \Rightarrow (p\ V0t))) \quad (11)$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(p\ V0t)) \Rightarrow ((p\ V0t) \Rightarrow False))) \quad (12)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Rightarrow True) \Leftrightarrow \\ & True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((\\ & (p\ V0t) \Rightarrow False) \Leftrightarrow (\neg(p\ V0t)))))) \end{aligned} \quad (13)$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \quad (14)$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in A_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (15)$$

Assume the following.

$$(\forall V0t \in 2. (((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p V0t))))) \quad (16)$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0P \in (2^{A_27a}). (\neg(\exists V1x \in A_27a. (p (ap V0P V1x)))) \Leftrightarrow (\forall V2x \in A_27a. (\neg(p (ap V0P V2x))))) \quad (17)$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0Q \in 2. (\forall V1P \in (2^{A_27a}). ((\forall V2x \in A_27a. ((p (ap V1P V2x)) \vee (p V0Q))) \Leftrightarrow ((\forall V3x \in A_27a. (p (ap V1P V3x)) \vee (p V0Q))))) \quad (18)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p V0A) \wedge (p V1B))) \Leftrightarrow ((\neg(p V0A)) \vee (\neg(p V1B)))) \wedge (((\neg((p V0A) \vee (p V1B))) \Leftrightarrow ((\neg(p V0A)) \wedge (\neg(p V1B))))) \quad (19)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p V0t1) \Rightarrow ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))) \quad (20)$$

Assume the following.

$$(\forall V0x \in 2. (\forall V1x_27 \in 2. (\forall V2y \in 2. (\forall V3y_27 \in 2. (((p V0x) \Leftrightarrow (p V1x_27)) \wedge ((p V1x_27) \Rightarrow ((p V2y) \Leftrightarrow (p V3y_27)))) \Rightarrow (((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x_27) \Rightarrow (p V3y_27))))) \quad (21)$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0P \in (2^{A_27a}). (\forall V1Q \in 2. (((\exists V2x \in A_27a. (p (ap V0P V2x))) \Rightarrow (p V1Q)) \Leftrightarrow (\forall V3x \in A_27a. ((p (ap V0P V3x)) \Rightarrow (p V1Q)))) \wedge (((\exists V4x \in A_27a. (p (ap V0P V4x)) \wedge (p V1Q)) \Leftrightarrow (\exists V5x \in A_27a. ((p (ap V0P V5x)) \wedge (p V1Q)))) \wedge (((p V1Q) \wedge (\exists V6x \in A_27a. (p (ap V0P V6x)))) \Leftrightarrow (\exists V7x \in A_27a. ((p V1Q) \wedge (p (ap V0P V7x))))) \quad (22)$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (((ap\ (c_2Elist_2EFLAT\ A_27a)\ (\\
& \quad c_2Elist_2ENIL\ (ty_2Elist_2Elist\ A_27a))) = (c_2Elist_2ENIL \\
& \quad A_27a)) \wedge (\forall V0h \in (ty_2Elist_2Elist\ A_27a).(\forall V1t \in \\
& \quad (ty_2Elist_2Elist\ (ty_2Elist_2Elist\ A_27a)).((ap\ (c_2Elist_2EFLAT \\
& \quad A_27a)\ (ap\ (ap\ (c_2Elist_2ECONS\ (ty_2Elist_2Elist\ A_27a))\ V0h) \\
& \quad V1t)) = (ap\ (ap\ (c_2Elist_2EAPPEND\ A_27a)\ V0h)\ (ap\ (c_2Elist_2EFLAT \\
& \quad A_27a)\ V1t))))))
\end{aligned} \tag{23}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in (2^{(ty_2Elist_2Elist\ A_27a)}). \\
& \quad (((p\ (ap\ V0P\ (c_2Elist_2ENIL\ A_27a))) \wedge (\forall V1t \in (ty_2Elist_2Elist \\
& \quad A_27a).((p\ (ap\ V0P\ V1t)) \Rightarrow (\forall V2h \in A_27a.(p\ (ap\ V0P\ (ap\ (ap\ (\\
& \quad c_2Elist_2ECONS\ A_27a)\ V2h)\ V1t)))))) \Rightarrow (\forall V3l \in (ty_2Elist_2Elist \\
& \quad A_27a).(p\ (ap\ V0P\ V3l))))))
\end{aligned} \tag{24}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0l \in (ty_2Elist_2Elist \\
& \quad A_27a).((ap\ (ap\ (c_2Elist_2EAPPEND\ A_27a)\ V0l)\ (c_2Elist_2ENIL \\
& \quad A_27a)) = V0l))
\end{aligned} \tag{25}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1L \in \\
& \quad (ty_2Elist_2Elist\ (ty_2Elist_2Elist\ A_27a)).((p\ (ap\ (ap\ (c_2Ebool_2EIN \\
& \quad A_27a)\ V0x)\ (ap\ (c_2Elist_2ELIST_TO_SET\ A_27a)\ (ap\ (c_2Elist_2EFLAT \\
& \quad A_27a)\ V1L)))) \Leftrightarrow (\exists V2l \in (ty_2Elist_2Elist\ A_27a).((p\ (ap \\
& \quad (ap\ (c_2Ebool_2EIN\ (ty_2Elist_2Elist\ A_27a))\ V2l)\ (ap\ (c_2Elist_2ELIST_TO_SET \\
& \quad (ty_2Elist_2Elist\ A_27a)\ V1L))) \wedge (p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a) \\
& \quad V0x)\ (ap\ (c_2Elist_2ELIST_TO_SET\ A_27a)\ V2l))))))
\end{aligned} \tag{26}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0l1 \in (ty_2Elist_2Elist \\
& \quad (ty_2Elist_2Elist\ A_27a)).(\forall V1l2 \in (ty_2Elist_2Elist \\
& \quad (ty_2Elist_2Elist\ A_27a)).((ap\ (c_2Elist_2EFLAT\ A_27a)\ (ap\ (\\
& \quad ap\ (c_2Elist_2EAPPEND\ (ty_2Elist_2Elist\ A_27a))\ V0l1)\ V1l2)) = \\
& \quad (ap\ (ap\ (c_2Elist_2EAPPEND\ A_27a)\ (ap\ (c_2Elist_2EFLAT\ A_27a) \\
& \quad V0l1))\ (ap\ (c_2Elist_2EFLAT\ A_27a)\ V1l2))))
\end{aligned} \tag{27}$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow & (((ap\ (c.2Elist_2EREVERSE\ A.27a) \\ & (c.2Elist_2ENIL\ A.27a)) = (c.2Elist_2ENIL\ A.27a)) \wedge (\forall V0h \in \\ & A.27a. (\forall V1t \in (ty_2Elist_2Elist\ A.27a). ((ap\ (c.2Elist_2EREVERSE \\ & A.27a)\ (ap\ (ap\ (c.2Elist_2ECONS\ A.27a)\ V0h)\ V1t)) = (ap\ (ap\ (c.2Elist_2EAPPEND \\ & A.27a)\ (ap\ (c.2Elist_2EREVERSE\ A.27a)\ V1t))\ (ap\ (ap\ (c.2Elist_2ECONS \\ & A.27a)\ V0h)\ (c.2Elist_2ENIL\ A.27a)))))) \end{aligned} \quad (28)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow & (\forall V0l \in (ty_2Elist_2Elist \\ & A.27a). (\forall V1x \in A.27a. ((p\ (ap\ (ap\ (c.2Ebool_2EIN\ A.27a)\ V1x) \\ & (ap\ (c.2Elist_2ELIST_TO_SET\ A.27a)\ (ap\ (c.2Elist_2EREVERSE \\ & A.27a)\ V0l)))) \Leftrightarrow (p\ (ap\ (ap\ (c.2Ebool_2EIN\ A.27a)\ V1x)\ (ap\ (c.2Elist_2ELIST_TO_SET \\ & A.27a)\ V0l)))))) \end{aligned} \quad (29)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow & (((p\ (ap\ (c.2Elist_2EALL_DISTINCT \\ & A.27a)\ (c.2Elist_2ENIL\ A.27a))) \Leftrightarrow True) \wedge (\forall V0h \in A.27a. (\\ \forall V1t \in (ty_2Elist_2Elist\ A.27a). ((p\ (ap\ (c.2Elist_2EALL_DISTINCT \\ & A.27a)\ (ap\ (ap\ (c.2Elist_2ECONS\ A.27a)\ V0h)\ V1t))) \Leftrightarrow ((\neg(p\ (ap\ (ap \\ & (c.2Ebool_2EIN\ A.27a)\ V0h)\ (ap\ (c.2Elist_2ELIST_TO_SET\ A.27a) \\ & V1t)))) \wedge (p\ (ap\ (c.2Elist_2EALL_DISTINCT\ A.27a)\ V1t)))))) \end{aligned} \quad (30)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow & (\forall V0l1 \in (ty_2Elist_2Elist \\ & A.27a). (\forall V1l2 \in (ty_2Elist_2Elist\ A.27a). ((p\ (ap\ (c.2Elist_2EALL_DISTINCT \\ & A.27a)\ (ap\ (ap\ (c.2Elist_2EAPPEND\ A.27a)\ V0l1)\ V1l2))) \Leftrightarrow ((p\ (ap \\ & (c.2Elist_2EALL_DISTINCT\ A.27a)\ V0l1)) \wedge ((p\ (ap\ (c.2Elist_2EALL_DISTINCT \\ & A.27a)\ V1l2)) \wedge (\forall V2e \in A.27a. ((p\ (ap\ (ap\ (c.2Ebool_2EIN\ A.27a) \\ & V2e)\ (ap\ (c.2Elist_2ELIST_TO_SET\ A.27a)\ V0l1))) \Rightarrow (\neg(p\ (ap\ (ap \\ & (c.2Ebool_2EIN\ A.27a)\ V2e)\ (ap\ (c.2Elist_2ELIST_TO_SET\ A.27a) \\ & V1l2)))))))))) \end{aligned} \quad (31)$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \quad (32)$$

Assume the following.

$$(\forall V0A \in 2. ((p\ V0A) \Rightarrow ((\neg(p\ V0A)) \Rightarrow False))) \quad (33)$$

Assume the following.

$$\begin{aligned} (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p\ V0A) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\ ((p\ V0A) \Rightarrow False) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False)))) \end{aligned} \quad (34)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg(\neg(p V0A)) \vee (p V1B))) \Rightarrow False) \Leftrightarrow (p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \quad (35)$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p V0A)) \Rightarrow False) \Rightarrow (((p V0A) \Rightarrow False) \Rightarrow False))) \quad (36)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \Leftrightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee ((\neg(p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee ((\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee ((\neg(p V1q)) \vee (\neg(p V0p)))))))))) \quad (37)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \wedge (p V2r)) \Leftrightarrow (((p V0p) \vee ((\neg(p V1q)) \vee (\neg(p V2r)))) \wedge (((p V1q) \vee (\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p)))))))) \quad (38)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \vee (p V2r)) \Leftrightarrow (((p V0p) \vee (\neg(p V1q))) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((p V1q) \vee ((p V2r) \vee (\neg(p V0p)))))))) \quad (39)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \Rightarrow (p V2r)) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((\neg(p V1q)) \vee ((p V2r) \vee (\neg(p V0p)))))))) \quad (40)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p)))))) \quad (41)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (p V0p)))) \quad (42)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (\neg(p V1q)))) \quad (43)$$

Theorem 1

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0xs \in (ty_2Elist_2Elist (ty_2Elist_2Elist A_27a)).((p (ap (c_2Elist_2EALL_DISTINCT A_27a) (ap (c_2Elist_2EFLAT A_27a) (ap (c_2Elist_2EREVERSE (ty_2Elist_2Elist A_27a)) V0xs)))) \Leftrightarrow (p (ap (c_2Elist_2EALL_DISTINCT A_27a) (ap (c_2Elist_2EFLAT A_27a) V0xs))))))$$