

thm_2Elist_2EALL_DISTINCT_REVERSE (TMMHrMgSagoQA75vvSkCB5Q2bGd2CTcK2JU)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Let $ty_2Elist_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty_2Elist_2Elist A0) \quad (1)$$

Let $c_2Elist_2EAPPEND : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Elist_2EAPPEND A_27a \in (((ty_2Elist_2Elist A_27a)^{(ty_2Elist_2Elist A_27a)})^{(ty_2Elist_2Elist A_27a)}) \quad (2)$$

Let $c_2Elist_2EREVERSE : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Elist_2EREVERSE A_27a \in ((ty_2Elist_2Elist A_27a)^{(ty_2Elist_2Elist A_27a)}) \quad (3)$$

Let $c_2Elist_2ENIL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Elist_2ENIL A_27a \in (ty_2Elist_2Elist A_27a) \quad (4)$$

Let $c_2Elist_2ECONS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Elist_2ECONS A_27a \in (((ty_2Elist_2Elist A_27a)^{(ty_2Elist_2Elist A_27a)})^{A_27a}) \quad (5)$$

Let $c_2Elist_2EFILTER : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Elist_2EFILTER A_27a \in (((ty_2Elist_2Elist A_27a)^{(ty_2Elist_2Elist A_27a)})^{(2^{A_27a})}) \quad (6)$$

Let $c_2Elist_2ELIST_TO_SET : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Elist_2ELIST_TO_SET A_27a \in ((2^{A_27a})^{(ty_2Elist_2Elist A_27a)}) \quad (7)$$

Definition 3 We define c_2Ebool_2EIN to be $\lambda A_27a : \iota. (\lambda V0x \in A_27a. (\lambda V1f \in (2^{A_27a}). (ap\ V1f\ V0x)))$

Let $c_2Elist_2EALL_DISTINCT : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2EALL_DISTINCT\ A_27a \in (2^{(ty_2Elist_2Elist\ A_27a)}) \quad (8)$$

Definition 4 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (ap\ (ap\ (c_2Emin_2E_3D\ (2^{A_27a})))$

Definition 5 We define c_2Ebool_2EF to be $(ap\ (c_2Ebool_2E_21\ 2))\ (\lambda V0t \in 2.V0t)$.

Definition 6 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2. \lambda Q \in 2. inj_o\ (p\ P \Rightarrow p\ Q)$ of type ι .

Definition 7 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap\ (c_2Ebool_2E_21\ 2))\ (\lambda V2t \in 2. (ap\ (c_2Emin_2E_3D_3D_3E\ V0t1\ V2t))\ V1t2))))$

Definition 8 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap\ (c_2Ebool_2E_21\ 2))\ (\lambda V2t \in 2. (ap\ (c_2Emin_2E_3D_3D_3E\ V0t1\ V2t))\ V1t2))))$

Definition 9 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2. (ap\ (ap\ c_2Emin_2E_3D_3D_3E\ V0t)\ c_2Ebool_2EF))$

Assume the following.

$$True \quad (9)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \quad (10)$$

Assume the following.

$$(\forall V0t \in 2. (((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((p\ V0t) \Rightarrow False) \Leftrightarrow (\neg (p\ V0t)))))) \quad (11)$$

Assume the following.

$$((\forall V0t \in 2. ((\neg (\neg (p\ V0t))) \Leftrightarrow (p\ V0t)) \wedge ((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \quad (12)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. ((V0x = V0x) \Leftrightarrow True)) \quad (13)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in A_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (14)$$

Assume the following.

$$(\forall V0t \in 2. (((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg (p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg (p\ V0t)))))) \quad (15)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p V0t1) \Rightarrow (p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \quad (16)$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1x_{.27} \in 2.(\forall V2y \in 2.(\forall V3y_{.27} \in 2.(((p V0x) \Leftrightarrow (p V1x_{.27})) \wedge ((p V1x_{.27}) \Rightarrow ((p V2y) \Leftrightarrow (p V3y_{.27})))))) \Rightarrow (((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x_{.27}) \Rightarrow (p V3y_{.27})))))) \quad (17)$$

Assume the following.

$$\begin{aligned} \forall A_{.27a}.nonempty A_{.27a} \Rightarrow & ((\forall V0l \in (ty_2Elist_2Elist \\ A_{.27a}).((ap (ap (c_2Elist_2EAPPEND A_{.27a}) (c_2Elist_2ENIL A_{.27a})) \\ V0l) = V0l)) \wedge (\forall V1l1 \in (ty_2Elist_2Elist A_{.27a}).(\forall V2l2 \in \\ (ty_2Elist_2Elist A_{.27a}).(\forall V3h \in A_{.27a}.((ap (ap (c_2Elist_2EAPPEND \\ A_{.27a}) (ap (ap (c_2Elist_2ECONS A_{.27a}) V3h) V1l1)) V2l2) = (ap (ap \\ (c_2Elist_2ECONS A_{.27a}) V3h) (ap (ap (c_2Elist_2EAPPEND A_{.27a}) \\ V1l1) V2l2))))))) \end{aligned} \quad (18)$$

Assume the following.

$$\begin{aligned} \forall A_{.27a}.nonempty A_{.27a} \Rightarrow & (((ap (c_2Elist_2EREVERSE A_{.27a}) \\ (c_2Elist_2ENIL A_{.27a})) = (c_2Elist_2ENIL A_{.27a})) \wedge (\forall V0h \in \\ A_{.27a}.(\forall V1t \in (ty_2Elist_2Elist A_{.27a}).((ap (c_2Elist_2EREVERSE \\ A_{.27a}) (ap (ap (c_2Elist_2ECONS A_{.27a}) V0h) V1t)) = (ap (ap (c_2Elist_2EAPPEND \\ A_{.27a}) (ap (c_2Elist_2EREVERSE A_{.27a}) V1t)) (ap (ap (c_2Elist_2ECONS \\ A_{.27a}) V0h) (c_2Elist_2ENIL A_{.27a}))))))) \end{aligned} \quad (19)$$

Assume the following.

$$\forall A_{.27a}.nonempty A_{.27a} \Rightarrow (\forall V0l \in (ty_2Elist_2Elist A_{.27a}).((ap (c_2Elist_2EREVERSE A_{.27a}) (ap (c_2Elist_2EREVERSE A_{.27a}) V0l)) = V0l)) \quad (20)$$

Assume the following.

$$\begin{aligned} \forall A_{.27a}.nonempty A_{.27a} \Rightarrow & (\forall V0l \in (ty_2Elist_2Elist \\ A_{.27a}).(\forall V1x \in A_{.27a}.((p (ap (ap (c_2Ebool_2EIN A_{.27a}) V1x) \\ (ap (c_2Elist_2ELIST_TO_SET A_{.27a}) (ap (c_2Elist_2EREVERSE \\ A_{.27a}) V0l)))) \Leftrightarrow (p (ap (ap (c_2Ebool_2EIN A_{.27a}) V1x) (ap (c_2Elist_2ELIST_TO_SET \\ A_{.27a}) V0l)))))) \end{aligned} \quad (21)$$

Assume the following.

$$\begin{aligned} \forall A_{.27a}.nonempty A_{.27a} \Rightarrow & (\forall V0l \in (ty_2Elist_2Elist \\ A_{.27a}).(\forall V1P \in (2^{A_{.27a}}).((ap (ap (c_2Elist_2EFILTER A_{.27a}) \\ V1P) (ap (c_2Elist_2EREVERSE A_{.27a}) V0l)) = (ap (c_2Elist_2EREVERSE \\ A_{.27a}) (ap (ap (c_2Elist_2EFILTER A_{.27a}) V1P) V0l)))))) \end{aligned} \quad (22)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0l \in (ty_2Elist_2Elist \\ & A.27a).(p\ (ap\ (c.2Elist_2EALL_DISTINCT\ A.27a)\ V0l)) \Leftrightarrow (\forall V1x \in \\ & A.27a.(p\ (ap\ (ap\ (c.2Ebool_2EIN\ A.27a)\ V1x)\ (ap\ (c.2Elist_2ELIST_TO_SET \\ & A.27a)\ V0l))) \Rightarrow ((ap\ (ap\ (c.2Elist_2EFILTER\ A.27a)\ (ap\ (c.2Emin_2E_3D \\ & A.27a)\ V1x))\ V0l) = (ap\ (ap\ (c.2Elist_2ECONS\ A.27a)\ V1x)\ (c.2Elist_2ENIL \\ & A.27a)))))) \end{aligned} \quad (23)$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \quad (24)$$

Assume the following.

$$(\forall V0A \in 2.((p\ V0A) \Rightarrow ((\neg(p\ V0A)) \Rightarrow False))) \quad (25)$$

Assume the following.

$$\begin{aligned} & (\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p\ V0A) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\ & ((p\ V0A) \Rightarrow False) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False)))) \end{aligned} \quad (26)$$

Assume the following.

$$\begin{aligned} & (\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p\ V0A)) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\ & ((p\ V0A) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False)))) \end{aligned} \quad (27)$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p\ V0A)) \Rightarrow False) \Rightarrow (((p\ V0A) \Rightarrow False) \Rightarrow False))) \quad (28)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p) \Leftrightarrow (\\ & (p\ V1q) \Leftrightarrow (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee ((p\ V1q) \vee (p\ V2r))) \wedge (((p\ V0p) \vee ((\neg(\\ & p\ V2r)) \vee (\neg(p\ V1q)))) \wedge ((p\ V1q) \vee ((\neg(p\ V2r)) \vee (\neg(p\ V0p)))) \wedge ((p\ V2r) \vee \\ & ((\neg(p\ V1q)) \vee (\neg(p\ V0p)))))))))) \end{aligned} \quad (29)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p) \Leftrightarrow (\\ & (p\ V1q) \Rightarrow (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee (p\ V1q)) \wedge (((p\ V0p) \vee (\neg(p\ V2r))) \wedge ((\\ & \neg(p\ V1q)) \vee ((p\ V2r) \vee (\neg(p\ V0p)))))))))) \end{aligned} \quad (30)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p\ V0p) \Rightarrow (p\ V1q))) \Rightarrow (p\ V0p))) \quad (31)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p\ V0p) \Rightarrow (p\ V1q))) \Rightarrow (\neg(p\ V1q)))) \quad (32)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V0p)))))) \quad (33)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V1q)))))) \quad (34)$$

Assume the following.

$$(\forall V0p \in 2. ((\neg(\neg(p V0p))) \Rightarrow (p V0p))) \quad (35)$$

Theorem 1

$$\forall A_{27a}. \text{nonempty } A_{27a} \Rightarrow (\forall V0l \in (ty_2Elist_2Elist \\ A_{27a}). ((p (ap (c_2Elist_2EALL_DISTINCT A_{27a}) (ap (c_2Elist_2EREVERSE \\ A_{27a}) V0l))) \Leftrightarrow (p (ap (c_2Elist_2EALL_DISTINCT A_{27a}) V0l))))$$