

thm_2Elist_2EALL_DISTINCT_ZIP_SWAP
 (TMVBZGqwkwn-
 NUeH8Mb4WndfgZ9H9f5ntMue)

October 26, 2020

Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_T$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Epair_2Eprod A0 A1) \quad (1)$$

Let $ty_2Elist_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty_2Elist_2Elist A0) \quad (2)$$

Let $c_2Elist_2EZIP : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Elist_2EZIP A_27a A_27b \in ((ty_2Elist_2Elist (ty_2Epair_2Eprod A_27a A_27b))^{(ty_2Epair_2Eprod (ty_2Elist_2Elist A_27a) A_27b)}) \quad (3)$$

Let $ty_2Eenum_2Eenum : \iota$ be given. Assume the following.

$$nonempty ty_2Eenum_2Eenum \quad (4)$$

Let $c_2Elist_2EEEL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Elist_2EEEL A_27a \in ((A_27a^{(ty_2Elist_2Elist A_27a)})^{ty_2Eenum_2Eenum}) \quad (5)$$

Let $c_2Elist_2ELENGTH : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Elist_2ELENGTH A_27a \in (ty_2Eenum_2Eenum^{(ty_2Elist_2Elist A_27a)}) \quad (6)$$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (ap (ap (c_2Emin_2E_3D (2^{A_27a})))$

Definition 4 We define c_2Ebool_2EF to be $(ap (c_2Ebool_2E_21\ 2) (\lambda V0t \in 2.V0t))$.

Definition 5 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 6 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2EF$

Definition 7 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21\ 2) (\lambda V2t \in 2$

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \quad (7)$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \quad (8)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \quad (9)$$

Definition 8 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap c_2Enum_2EABS_num$

Definition 9 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.\mathbf{if} (\exists x \in A.p (ap P x)) \mathbf{then} (the (\lambda x.x \in A \wedge p$ of type $\iota \Rightarrow \iota$.

Definition 10 We define $c_2Ebool_2E_3F$ to be $\lambda A.\lambda 27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap V0P (ap (c_2Emin_2E_40$

Definition 11 We define $c_2Eprim_rec_2E_3C$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Let $c_2Elist_2EALL_DISTINCT : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A.\lambda 27a.nonempty\ A.\lambda 27a \Rightarrow c_2Elist_2EALL_DISTINCT\ A.\lambda 27a \in (2^{(ty_2Elist_2Elist\ A.\lambda 27a)}) \quad (10)$$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A.\lambda 27a.nonempty\ A.\lambda 27a \Rightarrow \forall A.\lambda 27b.nonempty\ A.\lambda 27b \Rightarrow c_2Epair_2EABS_prod\ A.\lambda 27a\ A.\lambda 27b \in ((ty_2Epair_2Eprod\ A.\lambda 27a\ A.\lambda 27b)^{(2^{A-27b})^{A-27a}}) \quad (11)$$

Definition 12 We define $c_2Epair_2E_2C$ to be $\lambda A.\lambda 27a : \iota.\lambda A.\lambda 27b : \iota.\lambda V0x \in A.\lambda V1y \in A.\lambda V1z \in A.\lambda V1w \in A.(ap (c_2$

Definition 13 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21\ 2) (\lambda V2t \in 2$

Assume the following.

$$True \quad (12)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \quad (13)$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \quad (14)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow False) \Leftrightarrow (\neg (p V0t)))))) \quad (15)$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t)) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True)))) \quad (16)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.(V0x = V0x)) \quad (17)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \quad (18)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (19)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg (p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg (p V0t)))))) \quad (20)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.(((p V1B) \wedge (p V2C)) \vee (p V0A)) \Leftrightarrow (((p V1B) \vee (p V0A)) \wedge ((p V2C) \vee (p V0A)))))) \quad (21)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p V0t1) \Rightarrow ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \quad (22)$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1x_27 \in 2.(\forall V2y \in 2.(\forall V3y_27 \in 2.(((p V0x) \Leftrightarrow (p V1x_27)) \wedge ((p V1x_27) \Rightarrow ((p V2y) \Leftrightarrow (p V3y_27)))))) \Rightarrow (((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x_27) \Rightarrow (p V3y_27)))))) \quad (23)$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \quad \forall V0l1 \in (ty_2Elist_2Elist\ A_27a).(\forall V1l2 \in (ty_2Elist_2Elist \\
& \quad A_27b).(((ap\ (c_2Elist_2ELENGTH\ A_27a)\ V0l1) = (ap\ (c_2Elist_2ELENGTH \\
& \quad A_27b)\ V1l2)) \Rightarrow (((ap\ (c_2Elist_2ELENGTH\ (ty_2Epair_2Eprod\ A_27a \\
& \quad A_27b))\ (ap\ (c_2Elist_2EZIP\ A_27a\ A_27b))\ (ap\ (ap\ (c_2Epair_2E_2C \\
& \quad (ty_2Elist_2Elist\ A_27a)\ (ty_2Elist_2Elist\ A_27b))\ V0l1)\ V1l2)))) = \\
& \quad (ap\ (c_2Elist_2ELENGTH\ A_27a)\ V0l1)) \wedge ((ap\ (c_2Elist_2ELENGTH \\
& \quad (ty_2Epair_2Eprod\ A_27a\ A_27b))\ (ap\ (c_2Elist_2EZIP\ A_27a\ A_27b)) \\
& \quad (ap\ (ap\ (c_2Epair_2E_2C\ (ty_2Elist_2Elist\ A_27a)\ (ty_2Elist_2Elist \\
& \quad A_27b))\ V0l1)\ V1l2)))) = (ap\ (c_2Elist_2ELENGTH\ A_27b)\ V1l2)))))) \\
& \hspace{15em} (24)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \quad \forall V0l1 \in (ty_2Elist_2Elist\ A_27a).(\forall V1l2 \in (ty_2Elist_2Elist \\
& \quad A_27b).(\forall V2n \in ty_2Enum_2Enum.((((ap\ (c_2Elist_2ELENGTH \\
& \quad A_27a)\ V0l1) = (ap\ (c_2Elist_2ELENGTH\ A_27b)\ V1l2)) \wedge (p\ (ap\ (ap\ c_2Eprim_rec_2E_3C \\
& \quad V2n)\ (ap\ (c_2Elist_2ELENGTH\ A_27a)\ V0l1)))) \Rightarrow ((ap\ (ap\ (c_2Elist_2EEL \\
& \quad (ty_2Epair_2Eprod\ A_27a\ A_27b))\ V2n)\ (ap\ (c_2Elist_2EZIP\ A_27a \\
& \quad A_27b)\ (ap\ (ap\ (c_2Epair_2E_2C\ (ty_2Elist_2Elist\ A_27a)\ (ty_2Elist_2Elist \\
& \quad A_27b))\ V0l1)\ V1l2))) = (ap\ (ap\ (c_2Epair_2E_2C\ A_27a\ A_27b))\ (ap \\
& \quad (ap\ (c_2Elist_2EEL\ A_27a)\ V2n)\ V0l1))\ (ap\ (ap\ (c_2Elist_2EEL\ A_27b) \\
& \quad V2n)\ V1l2)))))) \\
& \hspace{15em} (25)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0l \in (ty_2Elist_2Elist \\
& \quad A_27a).(p\ (ap\ (c_2Elist_2EALL_DISTINCT\ A_27a)\ V0l)) \Leftrightarrow (\forall V1n1 \in \\
& \quad ty_2Enum_2Enum.(\forall V2n2 \in ty_2Enum_2Enum.(((p\ (ap\ (ap\ c_2Eprim_rec_2E_3C \\
& \quad V1n1)\ (ap\ (c_2Elist_2ELENGTH\ A_27a)\ V0l))) \wedge (p\ (ap\ (ap\ c_2Eprim_rec_2E_3C \\
& \quad V2n2)\ (ap\ (c_2Elist_2ELENGTH\ A_27a)\ V0l)))) \Rightarrow (((ap\ (ap\ (c_2Elist_2EEL \\
& \quad A_27a)\ V1n1)\ V0l) = (ap\ (ap\ (c_2Elist_2EEL\ A_27a)\ V2n2)\ V0l)) \Leftrightarrow (V1n1 = \\
& \quad V2n2)))))) \\
& \hspace{15em} (26)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \quad \forall V0x \in A_27a.(\forall V1y \in A_27b.(\forall V2a \in A_27a.(\forall V3b \in \\
& \quad A_27b.(((ap\ (ap\ (c_2Epair_2E_2C\ A_27a\ A_27b)\ V0x)\ V1y) = (ap\ (ap \\
& \quad (c_2Epair_2E_2C\ A_27a\ A_27b)\ V2a)\ V3b))) \Leftrightarrow ((V0x = V2a) \wedge (V1y = V3b)))))) \\
& \hspace{15em} (27)
\end{aligned}$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \quad (28)$$

Assume the following.

$$(\forall V0A \in 2.((p\ V0A) \Rightarrow ((\neg(p\ V0A)) \Rightarrow False))) \quad (29)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p V0A) \vee (p V1B))) \Rightarrow False) \Leftrightarrow ((p V0A) \Rightarrow False) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \quad (30)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg(\neg(p V0A)) \vee (p V1B))) \Rightarrow False) \Leftrightarrow ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \quad (31)$$

Assume the following.

$$(\forall V0A \in 2. (((\neg(p V0A)) \Rightarrow False) \Rightarrow ((p V0A) \Rightarrow False) \Rightarrow False)) \quad (32)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (p V1q) \Leftrightarrow (p V2r)) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee (\neg(p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee ((\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee ((\neg(p V1q)) \vee (\neg(p V0p)))))))))) \quad (33)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (p V1q) \wedge (p V2r)) \Leftrightarrow (((p V0p) \vee ((\neg(p V1q)) \vee (\neg(p V2r)))) \wedge (((p V1q) \vee (\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p)))))))) \quad (34)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (p V1q) \vee (p V2r)) \Leftrightarrow (((p V0p) \vee (\neg(p V1q))) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((p V1q) \vee ((p V2r) \vee (\neg(p V0p)))))))))) \quad (35)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (p V1q) \Rightarrow (p V2r)) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((\neg(p V1q)) \vee ((p V2r) \vee (\neg(p V0p)))))))))) \quad (36)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p)))))) \quad (37)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (p V0p)))) \quad (38)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (\neg(p V1q)))) \quad (39)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V0p)))))) \quad (40)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V1q)))))) \quad (41)$$

Assume the following.

$$(\forall V0p \in 2.((\neg(\neg(p V0p))) \Rightarrow (p V0p))) \quad (42)$$

Theorem 1

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & \forall V0l1 \in (ty_2Elist_2Elist\ A_27a).(\forall V1l2 \in (ty_2Elist_2Elist \\ & A_27b).((p\ (ap\ (c_2Elist_2EALL_DISTINCT\ (ty_2Epair_2Eprod \\ & A_27a\ A_27b))\ (ap\ (c_2Elist_2EZIP\ A_27a\ A_27b))\ (ap\ (ap\ (c_2Epair_2E_2C \\ & (ty_2Elist_2Elist\ A_27a)\ (ty_2Elist_2Elist\ A_27b))\ V0l1\ V1l2)))))) \wedge \\ & ((ap\ (c_2Elist_2ELENGTH\ A_27a)\ V0l1) = (ap\ (c_2Elist_2ELENGTH \\ & A_27b)\ V1l2))) \Rightarrow (p\ (ap\ (c_2Elist_2EALL_DISTINCT\ (ty_2Epair_2Eprod \\ & A_27b\ A_27a))\ (ap\ (c_2Elist_2EZIP\ A_27b\ A_27a))\ (ap\ (ap\ (c_2Epair_2E_2C \\ & (ty_2Elist_2Elist\ A_27b)\ (ty_2Elist_2Elist\ A_27a))\ V1l2)\ V0l1)))))) \end{aligned}$$