

thm_2Elist_2EAPPEND__SNOC
(TMWa2sqzW2WHCmQNu4n7zqr3AnHgPLNNbE5)

October 26, 2020

Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Let $ty_2Elist_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty_2Elist_2Elist A0) \quad (1)$$

Let $c_2Elist_2EAPPEND : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Elist_2EAPPEND A_27a \in (((ty_2Elist_2Elist A_27a)(ty_2Elist_2Elist A_27a))(ty_2Elist_2Elist A_27a)) \quad (2)$$

Definition 3 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Let $c_2Elist_2ECONS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Elist_2ECONS A_27a \in (((ty_2Elist_2Elist A_27a)(ty_2Elist_2Elist A_27a))A_27a) \quad (3)$$

Let $c_2Elist_2ENIL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Elist_2ENIL A_27a \in (ty_2Elist_2Elist A_27a) \quad (4)$$

Let $c_2Elist_2ESNOC : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Elist_2ESNOC A_27a \in (((ty_2Elist_2Elist A_27a)(ty_2Elist_2Elist A_27a))A_27a) \quad (5)$$

Definition 4 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a})))$

Definition 5 We define $c_Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_Ebool_2E_21 2) (\lambda V2t \in 2.$

Assume the following.

$$True \tag{6}$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A_27a.(p V0t)) \Leftrightarrow (p V0t))) \tag{7}$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \tag{8}$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty A_27a \Rightarrow & ((\forall V0l \in (ty_2Elist_2Elist \\ & A_27a).((ap (ap (c_2Elist_2EAPPEND A_27a) (c_2Elist_2ENIL A_27a)) \\ & V0l) = V0l)) \wedge (\forall V1l1 \in (ty_2Elist_2Elist A_27a).(\forall V2l2 \in \\ & (ty_2Elist_2Elist A_27a).(\forall V3h \in A_27a.((ap (ap (c_2Elist_2EAPPEND \\ & A_27a) (ap (ap (c_2Elist_2ECONS A_27a) V3h) V1l1)) V2l2) = (ap (ap \\ & (c_2Elist_2ECONS A_27a) V3h) (ap (ap (c_2Elist_2EAPPEND A_27a) \\ & V1l1) V2l2)))))))))) \end{aligned} \tag{9}$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty A_27a \Rightarrow & (\forall V0P \in (2^{(ty_2Elist_2Elist A_27a)}). \\ & (((p (ap V0P (c_2Elist_2ENIL A_27a))) \wedge (\forall V1t \in (ty_2Elist_2Elist \\ & A_27a).((p (ap V0P V1t)) \Rightarrow (\forall V2h \in A_27a.(p (ap V0P (ap (ap \\ & c_2Elist_2ECONS A_27a) V2h) V1t)))))) \Rightarrow (\forall V3l \in (ty_2Elist_2Elist \\ & A_27a).(p (ap V0P V3l)))))) \end{aligned} \tag{10}$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty A_27a \Rightarrow & ((\forall V0x \in A_27a.((ap (ap (c_2Elist_2ESNOC \\ & A_27a) V0x) (c_2Elist_2ENIL A_27a)) = (ap (ap (c_2Elist_2ECONS \\ & A_27a) V0x) (c_2Elist_2ENIL A_27a)))) \wedge (\forall V1x \in A_27a.(\forall V2x.27 \in \\ & A_27a.(\forall V3l \in (ty_2Elist_2Elist A_27a).((ap (ap (c_2Elist_2ESNOC \\ & A_27a) V1x) (ap (ap (c_2Elist_2ECONS A_27a) V2x.27) V3l)) = (ap (\\ & ap (c_2Elist_2ECONS A_27a) V2x.27) (ap (ap (c_2Elist_2ESNOC A_27a) \\ & V1x) V3l)))))))))) \end{aligned} \tag{11}$$

Theorem 1

$$\begin{aligned} \forall A_27a.nonempty A_27a \Rightarrow & (\forall V0l1 \in (ty_2Elist_2Elist \\ & A_27a).(\forall V1x \in A_27a.(\forall V2l2 \in (ty_2Elist_2Elist \\ & A_27a).((ap (ap (c_2Elist_2EAPPEND A_27a) V0l1) (ap (ap (c_2Elist_2ESNOC \\ & A_27a) V1x) V2l2)) = (ap (ap (c_2Elist_2ESNOC A_27a) V1x) (ap (ap \\ & (c_2Elist_2EAPPEND A_27a) V0l1) V2l2)))))) \end{aligned}$$