

thm_2Elist_2ECARD_LIST_TO_SET_ALL_DISTINCT
 (TMbjkzD-
 KvCimmWwc4PeqBzXsyJopbhVM9K5)

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Definition 1 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 2 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 3 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 4 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a}))$

Definition 5 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2$

Definition 6 We define c_2Ebool_2EF to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Let $ty_2Elist_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty_2Elist_2Elist A0) \quad (1)$$

Let $c_2Elist_2ECONS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Elist_2ECONS A_27a \in (((ty_2Elist_2Elist A_27a)^{(ty_2Elist_2Elist A_27a)})^{A_27a}) \quad (2)$$

Let $c_2Elist_2ENIL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Elist_2ENIL A_27a \in (ty_2Elist_2Elist A_27a) \quad (3)$$

Let $c_2Elist_2EALL_DISTINCT : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Elist_2EALL_DISTINCT A_27a \in (2^{(ty_2Elist_2Elist A_27a)}) \quad (4)$$

Definition 7 We define $c_Ebool_E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_Ebool_E_21\ 2) (\lambda V2t \in 2))$
 Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \quad (5)$$

Let $c_2Elist_2ELENGTH : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2ELENGTH\ A_27a \in (ty_2Enum_2Enum^{(ty_2Elist_2Elist\ A_27a)}) \quad (6)$$

Let $c_2Elist_2ELIST_TO_SET : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2ELIST_TO_SET\ A_27a \in ((2^{A_27a})^{(ty_2Elist_2Elist\ A_27a)}) \quad (7)$$

Definition 8 We define $c_Ebool_E_7E$ to be $(\lambda V0t \in 2.(ap (ap\ c_2Emin_2E_3D_3D_3E\ V0t)\ c_Ebool_2EF$

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \quad (8)$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \quad (9)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \quad (10)$$

Definition 9 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap\ c_2Enum_2EABS_num\ m)$

Definition 10 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.\mathbf{if} (\exists x \in A.p (ap\ P\ x)) \mathbf{then} (the\ (\lambda x.x \in A)\ P)$
 of type $\iota \Rightarrow \iota$.

Definition 11 We define $c_Ebool_E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ V0P (ap (c_2Emin_2E_40$

Definition 12 We define $c_2Eprim_rec_2E_3C$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Definition 13 We define $c_2Earithmetic_2E_3C_3D$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \quad (11)$$

Definition 14 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Definition 15 We define $c_2Epred_set_2EEMPTY$ to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.c_Ebool_2EF)$.

Definition 16 We define c_Ebool_2EIN to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.(\lambda V1f \in (2^{A_27a}).(ap\ V1f\ V0x)))$

Definition 17 We define c_Ebool_2ECOND to be $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a.(\lambda V3t3 \in 2$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \quad (12)$$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EABS_prod\ A_27a\ A_27b \in ((ty_2Epair_2Eprod\ A_27a\ A_27b)^{(2^{A_27b}})^{A_27a}) \quad (13)$$

Definition 18 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap\ (c_2Epair_2EABS_prod\ A_27a\ A_27b)\ V0x\ V1y)$

Let $c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epred_set_2EGSPEC\ A_27a\ A_27b \in ((2^{A_27a})^{((ty_2Epair_2Eprod\ A_27a\ 2)^{A_27b})}) \quad (14)$$

Definition 19 We define $c_2Epred_set_2EINSERT$ to be $\lambda A_27a : \iota.\lambda V0x \in A_27a.\lambda V1s \in (2^{A_27a}).(ap\ (c_2Epair_2EABS_prod\ A_27a\ A_27a)\ V0x\ V1s)$

Let $c_2Epred_set_2ECARD : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Epred_set_2ECARD\ A_27a \in (ty_2Enum_2Enum^{(2^{A_27a})}) \quad (15)$$

Definition 20 We define $c_2Epred_set_2EFINITE$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).(ap\ (c_2Ebool_2E_21\ 2)\ V0s)$

Assume the following.

$$(\forall V0n \in ty_2Enum_2Enum.(\forall V1m \in ty_2Enum_2Enum.(p\ (ap\ (ap\ c_2Earithmetic_2E_3C_3D\ V0n)\ V1m)) \Rightarrow (p\ (ap\ (ap\ c_2Eprim_rec_2E_3C\ V0n)\ (ap\ c_2Enum_2ESUC\ V1m)))))) \quad (16)$$

Assume the following.

$$True \quad (17)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \quad (18)$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p\ V0t))) \quad (19)$$

Assume the following.

$$(\forall V0t \in 2.((p\ V0t) \vee \neg(p\ V0t))) \quad (20)$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow \\
& (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge \\
& (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \quad (21)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow \\
& True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge ((\\
& (p V0t) \Rightarrow False) \Leftrightarrow (\neg (p V0t)))))) \quad (22)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& ((\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t)) \wedge ((\neg True) \Leftrightarrow False) \wedge \\
& ((\neg False) \Leftrightarrow True))) \quad (23)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow \\
& True)) \quad (24)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in \\
& A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (25)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow \\
& (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg (p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(\\
& p V0t)))))) \quad (26)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty A_27a \Rightarrow (\forall V0t1 \in A_27a.(\forall V1t2 \in \\
& A_27a.(((ap (ap (ap (c_2Ebool_2ECOND A_27a) c_2Ebool_2ET) V0t1) \\
& V1t2) = V0t1) \wedge ((ap (ap (ap (c_2Ebool_2ECOND A_27a) c_2Ebool_2EF) \\
& V0t1) V1t2) = V1t2)))) \quad (27)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p V0t1) \Rightarrow \\
& ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \quad (28)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in 2.(\forall V1x_27 \in 2.(\forall V2y \in 2.(\forall V3y_27 \in \\
& 2.(((p V0x) \Leftrightarrow (p V1x_27)) \wedge ((p V1x_27) \Rightarrow ((p V2y) \Leftrightarrow (p V3y_27)))))) \Rightarrow \\
& (((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x_27) \Rightarrow (p V3y_27)))))) \quad (29)
\end{aligned}$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow & (((ap\ (c.2Elist.2ELENGTH\ A.27a) \\ & (c.2Elist.2ENIL\ A.27a)) = c.2Enum.2E0) \wedge (\forall V0h \in A.27a. (\\ & \forall V1t \in (ty.2Elist.2Elist\ A.27a). ((ap\ (c.2Elist.2ELENGTH \\ A.27a)\ (ap\ (ap\ (c.2Elist.2ECONS\ A.27a)\ V0h)\ V1t)) = (ap\ c.2Enum.2ESUC \\ & (ap\ (c.2Elist.2ELENGTH\ A.27a)\ V1t)))))) \end{aligned} \quad (30)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow & (\\ & \forall V0h \in A.27b. (\forall V1t \in (ty.2Elist.2Elist\ A.27b). (\\ & (ap\ (c.2Elist.2ELIST_TO_SET\ A.27a)\ (c.2Elist.2ENIL\ A.27a)) = \\ & (c.2Epred_set.2EEMPTY\ A.27a)) \wedge ((ap\ (c.2Elist.2ELIST_TO_SET \\ A.27b)\ (ap\ (ap\ (c.2Elist.2ECONS\ A.27b)\ V0h)\ V1t)) = (ap\ (ap\ (c.2Epred_set.2EINSERT \\ & A.27b)\ V0h)\ (ap\ (c.2Elist.2ELIST_TO_SET\ A.27b)\ V1t)))))) \end{aligned} \quad (31)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow & (\forall V0P \in (2^{(ty.2Elist.2Elist\ A.27a)}). \\ & (((p\ (ap\ V0P\ (c.2Elist.2ENIL\ A.27a))) \wedge (\forall V1t \in (ty.2Elist.2Elist \\ & A.27a). ((p\ (ap\ V0P\ V1t)) \Rightarrow (\forall V2h \in A.27a. (p\ (ap\ V0P\ (ap\ (ap\ (\\ & c.2Elist.2ECONS\ A.27a)\ V2h)\ V1t)))))) \Rightarrow (\forall V3l \in (ty.2Elist.2Elist \\ & A.27a). (p\ (ap\ V0P\ V3l)))))) \end{aligned} \quad (32)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow & (((p\ (ap\ (c.2Elist.2EALL_DISTINCT \\ & A.27a)\ (c.2Elist.2ENIL\ A.27a))) \Leftrightarrow True) \wedge (\forall V0h \in A.27a. (\\ & \forall V1t \in (ty.2Elist.2Elist\ A.27a). ((p\ (ap\ (c.2Elist.2EALL_DISTINCT \\ & A.27a)\ (ap\ (ap\ (c.2Elist.2ECONS\ A.27a)\ V0h)\ V1t))) \Leftrightarrow ((\neg (p\ (ap\ (ap \\ & (c.2Ebool.2EIN\ A.27a)\ V0h)\ (ap\ (c.2Elist.2ELIST_TO_SET\ A.27a) \\ & V1t)))) \wedge (p\ (ap\ (c.2Elist.2EALL_DISTINCT\ A.27a)\ V1t)))))) \end{aligned} \quad (33)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow & (\forall V0l \in (ty.2Elist.2Elist \\ & A.27a). (p\ (ap\ (c.2Epred_set.2EFINITE\ A.27a)\ (ap\ (c.2Elist.2ELIST_TO_SET \\ & A.27a)\ V0l)))) \end{aligned} \quad (34)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow & (\forall V0ls \in (ty.2Elist.2Elist \\ & A.27a). (p\ (ap\ (ap\ c.2Earithmic.2E_3C.3D\ (ap\ (c.2Epred_set.2ECARD \\ & A.27a)\ (ap\ (c.2Elist.2ELIST_TO_SET\ A.27a)\ V0ls)))\ (ap\ (c.2Elist.2ELENGTH \\ & A.27a)\ V0ls)))) \end{aligned} \quad (35)$$

Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow ((ap\ (c.2Epred_set.2ECARD\ A.27a)\ (c.2Epred_set.2EEMPTY\ A.27a)) = c.2Enum.2E0) \quad (36)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0s \in (2^{A.27a}).((p\ (ap \\ (c.2Epred_set.2EFINITE\ A.27a)\ V0s)) \Rightarrow (\forall V1x \in A.27a.((\\ ap\ (c.2Epred_set.2ECARD\ A.27a)\ (ap\ (ap\ (c.2Epred_set.2EINSERT \\ A.27a)\ V1x)\ V0s)) = (ap\ (ap\ (ap\ (c.2Ebool.2ECOND\ ty.2Enum.2Enum) \\ (ap\ (ap\ (c.2Ebool.2EIN\ A.27a)\ V1x)\ V0s))\ (ap\ (c.2Epred_set.2ECARD \\ A.27a)\ V0s))\ (ap\ c.2Enum.2ESUC\ (ap\ (c.2Epred_set.2ECARD\ A.27a) \\ V0s))))))) \end{aligned} \quad (37)$$

Assume the following.

$$(\forall V0m \in ty.2Enum.2Enum.(\forall V1n \in ty.2Enum.2Enum.((ap\ c.2Enum.2ESUC\ V0m) = (ap\ c.2Enum.2ESUC\ V1n)) \Leftrightarrow (V0m = V1n)))) \quad (38)$$

Assume the following.

$$(\forall V0m \in ty.2Enum.2Enum.(\forall V1n \in ty.2Enum.2Enum.((p\ (ap\ (ap\ c.2Eprim_rec.2E_3C\ V0m)\ V1n)) \Rightarrow (\neg(V0m = V1n))))) \quad (39)$$

Theorem 1

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0ls \in (ty.2Elist.2Elist \\ A.27a).(((ap\ (c.2Epred_set.2ECARD\ A.27a)\ (ap\ (c.2Elist.2ELIST_TO_SET \\ A.27a)\ V0ls)) = (ap\ (c.2Elist.2ELENGTH\ A.27a)\ V0ls)) \Rightarrow (p\ (ap\ (c.2Elist.2EALL_DISTINCT \\ A.27a)\ V0ls)))) \end{aligned}$$