

thm_2Elist_2ECONS (TMFqb- bQw4H86WfJDjq8ZRQmGZmYdm4up8FD)

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Definition 1 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 2 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 3 We define $c_2Ebool_2E_2T$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 4 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a}))$

Definition 5 We define $c_2Ebool_2E_2F$ to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Let $ty_2Elist_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty_2Elist_2Elist A0) \quad (1)$$

Let $c_2Elist_2EHD : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Elist_2EHD A_27a \in (A_27a^{(ty_2Elist_2Elist A_27a)}) \quad (2)$$

Let $c_2Elist_2ETL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Elist_2ETL A_27a \in ((ty_2Elist_2Elist A_27a)^{(ty_2Elist_2Elist A_27a)}) \quad (3)$$

Definition 6 We define $c_2Ebool_2E_2E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_2F))$

Let $c_2Elist_2ENULL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Elist_2ENULL A_27a \in (2^{(ty_2Elist_2Elist A_27a)}) \quad (4)$$

Definition 7 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t)))$

Let $c_2Elist_2ECONS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Elist_2ECONS A_27a \in (((ty_2Elist_2Elist A_27a)^{(ty_2Elist_2Elist A_27a)})^{A_27a}) \quad (5)$$

Definition 8 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.$ **if** $(\exists x \in A.p (ap P x))$ **then** (the $(\lambda x.x \in A \wedge p$ of type $\iota \Rightarrow \iota$).

Definition 9 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap V0P (ap (c_2Emin_2E_40 A$

Let $c_2Elist_2ENIL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Elist_2ENIL A_27a \in (ty_2Elist_2Elist A_27a) \quad (6)$$

Definition 10 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in$

Assume the following.

$$True \quad (7)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \quad (8)$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p V0t))) \quad (9)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge ((p V0t) \Rightarrow False) \Leftrightarrow (\neg (p V0t)))))) \quad (10)$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \quad (11)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \quad (12)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p V0t)))))) \quad (13)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0h \in A_27a.(\forall V1t \in (ty_2Elist_2Elist A_27a).(ap (c_2Elist_2EHD A_27a) (ap (ap (c_2Elist_2ECONS A_27a) V0h) V1t)) = V0h))) \quad (14)$$

Assume the following.

$$\forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow (\forall V0h \in A_{.27a}.(\forall V1t \in (ty_2Elist_2Elist\ A_{.27a}).((ap\ (c_2Elist_2ETL\ A_{.27a})\ (ap\ (ap\ (c_2Elist_2ECONS\ A_{.27a})\ V0h)\ V1t)) = V1t))) \quad (15)$$

Assume the following.

$$\forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow ((p\ (ap\ (c_2Elist_2ENULL\ A_{.27a})\ (c_2Elist_2ENIL\ A_{.27a}))) \wedge (\forall V0h \in A_{.27a}.(\forall V1t \in (ty_2Elist_2Elist\ A_{.27a}).(\neg(p\ (ap\ (c_2Elist_2ENULL\ A_{.27a})\ (ap\ (ap\ (c_2Elist_2ECONS\ A_{.27a})\ V0h)\ V1t))))))) \quad (16)$$

Assume the following.

$$\forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow (\forall V0l \in (ty_2Elist_2Elist\ A_{.27a}).((V0l = (c_2Elist_2ENIL\ A_{.27a})) \vee (\exists V1h \in A_{.27a}.(\exists V2t \in (ty_2Elist_2Elist\ A_{.27a}).(V0l = (ap\ (ap\ (c_2Elist_2ECONS\ A_{.27a})\ V1h)\ V2t)))))) \quad (17)$$

Theorem 1

$$\forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow (\forall V0l \in (ty_2Elist_2Elist\ A_{.27a}).((\neg(p\ (ap\ (c_2Elist_2ENULL\ A_{.27a})\ V0l))) \Rightarrow ((ap\ (ap\ (c_2Elist_2ECONS\ A_{.27a})\ (ap\ (c_2Elist_2EHD\ A_{.27a})\ V0l))\ (ap\ (c_2Elist_2ETL\ A_{.27a})\ V0l)) = V0l)))$$