thm_2Elist_2EDROP__splitAtPki (TML6BHjmngjVNbsLtu3VpdDirxHVd419Rcy)

October 26, 2020

Definition 1 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o \ (p \ P \Rightarrow p \ Q)$ *of type* ι.

Definition 2 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o \ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 3 *We define* c_2Ebool_2ET *to be* $(ap \ (ap \ (\mathsf{c\_2Emin\_2E\_3D} \ (2^2)) \ (\lambda V0x \in 2.V0x)) \ (\lambda V1x \in 2.V1$

Definition 4 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap \ (ap \ (\mathsf{c\_2Emin\_2E\_3D} \ (2^{A\_27}$

Definition 5 *We define* c_2Ebool_2E_5C_2F *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap \ (\mathsf{c\_2Ebool\_2E\_21} \ 2) \ (\lambda V2t \in 2$

Definition 6 *We define* c_2Ecombin_2Eo *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda V0f \in (A\_27b^{A\_27c}).\lambda V1g \in$

Definition 7 *We define* c_2Ecombin_2EK *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.(\lambda V0x \in A\_27a.(\lambda V1y \in A\_27b.V0x))$

Definition 8 *We define* c_2Ecombin_2ES *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.(\lambda V0f \in ((A\_27c^{A\_27b})^{A\_27a}$

Definition 9 *We define* c_2Ecombin_2EI *to be* $\lambda A\_27a : \iota.(ap \ (ap \ (\mathsf{c\_2Ecombin\_2ES} \ A\_27a \ (A\_27a^{A\_27a}) \ A\_2$

Let $ty\_2Elist\_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty \ A0 \Rightarrow nonempty \ (ty\_2Elist\_2Elist \ A0) \tag{1}$$

Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty \ ty\_2Enum\_2Enum \tag{2}$$

Let $c\_2Elist\_2ETAKE : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty \ A\_27a \Rightarrow c\_2Elist\_2ETAKE \ A\_27a \in (((ty\_2Elist\_2Elist \\ A\_27a)^{(ty\_2Elist\_2Elist \ A\_27a)})^{ty\_2Enum\_2Enum}) \tag{3}$$

Let $c\_2Elist\_2EEL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty \ A\_27a \Rightarrow c\_2Elist\_2EEL \ A\_27a \in ((A\_27a^{(ty\_2Elist\_2Elist \ A\_27a)})^{ty\_2Enum\_2Enum}) \tag{4}$$

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Let $c\_2Elist\_2EsplitAtPki : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Elist\_2EsplitAtPki$$
$$A\_27a\ A\_27b \in (((A\_27a^{(ty\_2Elist\_2Elist\ A\_27b)})^{((A\_27a^{(ty\_2Elist\_2Elist\ A\_27b)})^{(ty\_2Elist\_2Elist\ A\_27b)})})((2^{A\_27b})^{ty\_2En}$$

$$(5)$$

Let $c\_2Elist\_2ENIL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ENIL\ A\_27a \in (ty\_2Elist\_2Elist$$
$$A\_27a)$$

$$(6)$$

Let $c\_2Elist\_2EDROP : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2EDROP\ A\_27a \in (((ty\_2Elist\_2Elist$$
$$A\_27a)^{(ty\_2Elist\_2Elist\ A\_27a)})^{ty\_2Enum\_2Enum})$$

$$(7)$$

Let $c\_2Elist\_2ELENGTH : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ELENGTH\ A\_27a \in (ty\_2Enum\_2Enum^{(ty\_2Elist\_2Elist\ A\_27a)})$$

$$(8)$$

Let $c\_2Earithmetic\_2EEVEN : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2EEVEN \in (2^{ty\_2Enum\_2Enum})$$

$$(9)$$

Let $c\_2Earithmetic\_2EODD : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2EODD \in (2^{ty\_2Enum\_2Enum})$$

$$(10)$$

Definition 10 *We define* c_2Ebool_2EF *to be* $(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 11 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap\ \mathsf{c\_2Emin\_2E\_3D\_3D\_3E}\ V0t)\ \mathsf{c\_2Ebool\_2E}$

Definition 12 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in$

Let $c\_2Enum\_2EREP\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EREP\_num \in (omega^{ty\_2Enum\_2Enum})$$

$$(11)$$

Let $c\_2Enum\_2ESUC\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2ESUC\_REP \in (omega^{omega})$$

$$(12)$$

Let $c\_2Enum\_2EABS\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum^{omega})$$

$$(13)$$

Definition 13 *We define* c_2Enum_2ESUC *to be* $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_num$

Definition 14 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.\mathbf{if}\ (\exists x \in A.p\ (ap\ P\ x))\ \mathbf{then}\ (the\ (\lambda x.x \in A \wedge$
of type $\iota \Rightarrow \iota$.

Definition 15 *We define* c_2Ebool_2E_3F *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ V0P\ (ap\ (\mathsf{c\_2Emin\_2E\_40}$

Definition 16 *We define* c_2Eprim__rec_2E_3C *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enu$

Definition 17 *We define* c_2Earithmetic_2E_3E *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2En$

Definition 18 *We define* c_2Earithmetic_2E_3E_3D *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2$

Let $c\_2Enum\_2EZERO\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2EZERO\_\_REP \in omega \tag{14}$$

Definition 19 *We define* c_2Enum_2E0 *to be* $(ap\ c\_2Enum\_2EABS\_\_num\ c\_2Enum\_2EZERO\_\_REP).$

Definition 20 *We define* c_2Ebool_2ECOND *to be* $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.($

Definition 21 *We define* c_2Eprim__rec_2EPRE *to be* $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2E}$

Let $c\_2Earithmetic\_2EEXP : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2EEXP \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{15}$$

Let $c\_2Earithmetic\_2E\_2D : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2D \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{16}$$

Let $c\_2Earithmetic\_2E\_2A : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2A \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{17}$$

Definition 22 *We define* c_2Enumeral_2EiZ *to be* $\lambda V0x \in ty\_2Enum\_2Enum.V0x.$

Definition 23 *We define* c_2Earithmetic_2ENUMERAL *to be* $\lambda V0x \in ty\_2Enum\_2Enum.V0x.$

Let $c\_2Earithmetic\_2E\_2B : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{18}$$

Definition 24 *We define* c_2Earithmetic_2EBIT2 *to be* $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic$

Definition 25 *We define* c_2Earithmetic_2EBIT1 *to be* $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic$

Definition 26 *We define* c_2Earithmetic_2EZERO *to be* c_2Enum_2E0.

Definition 27 *We define* c_2Earithmetic_2E_3C_3D *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2$

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Let $ty\_2Eoption\_2Eoption : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Eoption\_2Eoption\ A0) \qquad (19)$$

Let $c\_2Eoption\_2Eoption\_\_CASE : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Eoption\_2Eoption\_\_CASE$$
$$A\_27a\ A\_27b \in (((A\_27b^{(A\_27b^{A\_27a})})^{A\_27b})^{(ty\_2Eoption\_2Eoption\ A\_27a)})$$
$$(20)$$

Let $ty\_2Eone\_2Eone : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Eone\_2Eone \qquad (21)$$

Definition 28 *We define* c_2Eone_2Eone *to be* $(ap\ ($c_2Emin_2E_40 $ty\_2Eone\_2Eone)\ (\lambda V0x \in ty\_2Eone\_2$

Let $ty\_2Esum\_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Esum\_2Esum$$
$$A0\ A1) \qquad (22)$$

Let $c\_2Esum\_2EABS\_\_sum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Esum\_2EABS\_\_sum$$
$$A\_27a\ A\_27b \in ((ty\_2Esum\_2Esum\ A\_27a\ A\_27b)^{((2^{A\_27b})^{A\_27a})^2)}) \qquad (23)$$

Definition 29 *We define* c_2Esum_2EINR *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0e \in A\_27b.(ap\ (c\_2Esum\_2EABS$

Let $c\_2Eoption\_2Eoption\_\_ABS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Eoption\_2Eoption\_\_ABS\ A\_27a \in$$
$$((ty\_2Eoption\_2Eoption\ A\_27a)^{(ty\_2Esum\_2Esum\ A\_27a\ ty\_2Eone\_2Eone)}) \qquad (24)$$

Definition 30 *We define* c_2Eoption_2ENONE *to be* $\lambda A\_27a : \iota.(ap\ (c\_2Eoption\_2Eoption\_\_ABS\ A\_27a)\ ($

Let $c\_2Ewhile\_2EWHILE : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ewhile\_2EWHILE\ A\_27a \in (((A\_27a^{A\_27a})^{(A\_27a^{A\_27a})})^{(2^{A\_27a})}) \qquad (25)$$

Definition 31 *We define* c_2Ewhile_2ELEAST *to be* $\lambda V0P \in (2^{ty\_2Enum\_2Enum}).(ap\ (ap\ (ap\ (c\_2Ewhile\_2$

Definition 32 *We define* c_2Esum_2EINL *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0e \in A\_27a.(ap\ (c\_2Esum\_2EABS$

Definition 33 *We define* c_2Eoption_2ESOME *to be* $\lambda A\_27a : \iota.\lambda V0x \in A\_27a.(ap\ (c\_2Eoption\_2Eoption\_\_$

Definition 34 *We define* c_2Ewhile_2EOLEAST *to be* $\lambda V0P \in (2^{ty\_2Enum\_2Enum}).(ap\ (ap\ (ap\ (c_2Ebool_2

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Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.($$
$$((ap\ (ap\ c\_2Earithmetic\_2E\_2B\ \mathsf{c\_2Enum\_2E0})\ V0m) = V0m)\wedge(((ap\ ($$
$$ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ \mathsf{c\_2Enum\_2E0}) = V0m)\wedge(((ap\ (ap\ c\_2Earithmetic\_2E\_2B$$
$$(ap\ \mathsf{c\_2Enum\_2ESUC}\ V0m))\ V1n) = (ap\ \mathsf{c\_2Enum\_2ESUC}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B$$
$$V0m)\ V1n)))\wedge((ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ (ap\ \mathsf{c\_2Enum\_2ESUC}$$
$$V1n)) = (ap\ \mathsf{c\_2Enum\_2ESUC}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ V1n))))))))$$
(26)

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.($$
$$(ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ V1n) = (ap\ (ap\ c\_2Earithmetic\_2E\_2B$$
$$V1n)\ V0m))))$$
(27)

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.(p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}$$
$$\mathsf{c\_2Enum\_2E0})\ V0n)))$$
(28)

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.($$
$$(\neg(p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ V0m)\ V1n)))\Leftrightarrow(p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}$$
$$V1n)\ V0m)))))$$
(29)

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.($$
$$((ap\ (ap\ c\_2Earithmetic\_2E\_2A\ \mathsf{c\_2Enum\_2E0})\ V0m) = \mathsf{c\_2Enum\_2E0})\wedge$$
$$(((ap\ (ap\ c\_2Earithmetic\_2E\_2A\ V0m)\ \mathsf{c\_2Enum\_2E0}) = \mathsf{c\_2Enum\_2E0})\wedge$$
$$(((ap\ (ap\ c\_2Earithmetic\_2E\_2A\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}$$
$$(ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO})))\ V0m) = V0m)\wedge$$
$$(((ap\ (ap\ c\_2Earithmetic\_2E\_2A\ V0m)\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}$$
$$(ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO}))) = V0m)\wedge($$
$$((ap\ (ap\ c\_2Earithmetic\_2E\_2A\ (ap\ \mathsf{c\_2Enum\_2ESUC}\ V0m))\ V1n) = (ap$$
$$(ap\ c\_2Earithmetic\_2E\_2B\ (ap\ (ap\ c\_2Earithmetic\_2E\_2A\ V0m)\ V1n))$$
$$V1n))\wedge((ap\ (ap\ c\_2Earithmetic\_2E\_2A\ V0m)\ (ap\ \mathsf{c\_2Enum\_2ESUC}\ V1n)) =$$
$$(ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ (ap\ (ap\ c\_2Earithmetic\_2E\_2A$$
$$V0m)\ V1n)))))))))))$$
(30)

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.($$
$$\forall V2p \in ty\_2Enum\_2Enum.(((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}$$
$$V0m)\ V1n))\wedge(p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ V1n)\ V2p)))\Rightarrow(p\ ($$
$$ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ V0m)\ V2p))))))$$
(31)

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.(\\
\forall V2p \in ty\_2Enum\_2Enum.((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\\
(ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_2B}\ V0m)\ V1n))\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_2B}\\
V0m)\ V2p)))\Leftrightarrow(p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ V1n)\ V2p))))))) \tag{32}$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.(\\
(\neg(p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ V0m)\ V1n)))\Leftrightarrow(p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\\
(ap\ \mathsf{c\_2Enum\_2ESUC}\ V1n))\ V0m))))) \tag{33}$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.((ap\ \mathsf{c\_2Enum\_2ESUC}\ V0n) = (ap\ (ap\\
c\_2Earithmetic\_2E\_2B\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\\
\mathsf{c\_2Earithmetic\_2EZERO}))\ V0n))) \tag{34}$$

Assume the following.

$$True \tag{35}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0t \in 2.((\forall V1x \in\\
A\_27a.(p\ V0t))\Leftrightarrow(p\ V0t))) \tag{36}$$

Assume the following.

$$(\forall V0t \in 2.(((True\Rightarrow(p\ V0t))\Leftrightarrow(p\ V0t))\wedge((((p\ V0t)\Rightarrow True)\Leftrightarrow\\
True)\wedge(((False\Rightarrow(p\ V0t))\Leftrightarrow True)\wedge((((p\ V0t)\Rightarrow(p\ V0t))\Leftrightarrow True)\wedge((\\
(p\ V0t)\Rightarrow False)\Leftrightarrow(\neg(p\ V0t)))))))) \tag{37}$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p\ V0t)))\Leftrightarrow(p\ V0t)))\wedge(((\neg True)\Leftrightarrow False)\wedge\\
((\neg False)\Leftrightarrow True))) \tag{38}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0x \in A\_27a.((V0x = V0x)\Leftrightarrow\\
True)) \tag{39}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0x \in A\_27a.(\forall V1y \in\\
A\_27a.((V0x = V1y)\Leftrightarrow(V1y = V0x)))) \tag{40}$$

Assume the following.

$$(\forall V0t \in 2.(((True\Leftrightarrow(p\ V0t))\Leftrightarrow(p\ V0t))\wedge((((p\ V0t)\Leftrightarrow True)\Leftrightarrow\\
(p\ V0t))\wedge(((False\Leftrightarrow(p\ V0t))\Leftrightarrow(\neg(p\ V0t)))\wedge(((p\ V0t)\Leftrightarrow False)\Leftrightarrow(\neg(\\
p\ V0t)))))) \tag{41}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.(((p\ V0A)\vee(\\ (p\ V1B)\vee(p\ V2C)))\Leftrightarrow(((p\ V0A)\vee(p\ V1B))\vee(p\ V2C)))))) \tag{42}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((p\ V0A)\vee(p\ V1B))\Leftrightarrow((p\ V1B)\vee \\ (p\ V0A))))) \tag{43}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p\ V0A)\wedge(p\ V1B)))\Leftrightarrow((\neg(\\ p\ V0A))\vee(\neg(p\ V1B))))\wedge((\neg((p\ V0A)\vee(p\ V1B)))\Leftrightarrow((\neg(p\ V0A))\wedge(\neg(p\ V1B))))))) \tag{44}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((p\ V0A)\Rightarrow(p\ V1B))\Leftrightarrow((\neg(p\ V0A))\vee \\ (p\ V1B))))) \tag{45}$$

Assume the following.

$$(\forall V0t \in 2.(((p\ V0t)\Rightarrow False)\Leftrightarrow((p\ V0t)\Leftrightarrow False))) \tag{46}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p\ V0t1)\Rightarrow \\ ((p\ V1t2)\Rightarrow(p\ V2t3)))\Leftrightarrow(((p\ V0t1)\wedge(p\ V1t2))\Rightarrow(p\ V2t3)))))) \tag{47}$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1x\_27 \in 2.(\forall V2y \in 2.(\forall V3y\_27 \in \\ 2.(((p\ V0x)\Leftrightarrow(p\ V1x\_27))\wedge((p\ V1x\_27)\Rightarrow((p\ V2y)\Leftrightarrow(p\ V3y\_27))))\Rightarrow \\ (((p\ V0x)\Rightarrow(p\ V2y))\Leftrightarrow((p\ V1x\_27)\Rightarrow(p\ V3y\_27))))))))) \tag{48}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0f \in (2^{A\_27a}).(\forall V1v \in \\ A\_27a.((\forall V2x \in A\_27a.((V2x = V1v)\Rightarrow(p\ (ap\ V0f\ V2x))))\Leftrightarrow(p\ (\\ ap\ V0f\ V1v))))) \tag{49}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow\forall A\_27b.nonempty\ A\_27b\Rightarrow\forall A\_27c. \\ nonempty\ A\_27c\Rightarrow(\forall V0f \in (A\_27b^{A\_27a}).(\forall V1g \in (A\_27a^{A\_27c}). \\ (\forall V2x \in A\_27c.((ap\ (ap\ (ap\ (\mathsf{c\_2Ecombin\_2Eo}\ A\_27c\ A\_27b\ A\_27a) \\ V0f)\ V1g)\ V2x) = (ap\ V0f\ (ap\ V1g\ V2x)))))) \tag{50}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow\forall A\_27b.nonempty\ A\_27b\Rightarrow(\\ \forall V0x \in A\_27a.(\forall V1y \in A\_27b.((ap\ (ap\ (\mathsf{c\_2Ecombin\_2EK} \\ A\_27a\ A\_27b)\ V0x)\ V1y) = V0x))) \tag{51}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((ap\ (\mathsf{c\_2Ecombin\_2EI}$$
$$A\_27a)\ V0x) = V0x)) \tag{52}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ($$
$$\forall V0P \in ((2^{A\_27b})^{ty\_2Enum\_2Enum}).(\forall V1k \in ((A\_27a^{(ty\_2Elist\_2Elist\ A\_27b)})^{(ty\_2Elist\_2Elist\ A\_27b)}).$$
$$(\forall V2l \in (ty\_2Elist\_2Elist\ A\_27b).((ap\ (ap\ (ap\ (c\_2Elist\_2EsplitAtPki$$
$$A\_27a\ A\_27b)\ V0P)\ V1k)\ V2l) = (ap\ (ap\ (ap\ (c\_2Eoption\_2Eoption\_\_CASE$$
$$ty\_2Enum\_2Enum\ A\_27a)\ (ap\ \mathsf{c\_2Ewhile\_2EOLEAST}\ (\lambda V3i \in ty\_2Enum\_2Enum.$$
$$(ap\ (ap\ \mathsf{c\_2Ebool\_2E\_2F\_5C}\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ V3i)\ (ap$$
$$(c\_2Elist\_2ELENGTH\ A\_27b)\ V2l)))\ (ap\ (ap\ V0P\ V3i)\ (ap\ (ap\ (c\_2Elist\_2EEL$$
$$A\_27b)\ V3i)\ V2l))))))\ (ap\ (ap\ V1k\ V2l)\ (c\_2Elist\_2ENIL\ A\_27b)))$$
$$(\lambda V4i \in ty\_2Enum\_2Enum.(ap\ (ap\ V1k\ (ap\ (ap\ (c\_2Elist\_2ETAKE$$
$$A\_27b)\ V4i)\ V2l))\ (ap\ (ap\ (c\_2Elist\_2EDROP\ A\_27b)\ V4i)\ V2l)))))))))$$
$$\tag{53}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0l \in (ty\_2Elist\_2Elist$$
$$A\_27a).(\forall V1n \in ty\_2Enum\_2Enum.((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}$$
$$(ap\ (c\_2Elist\_2ELENGTH\ A\_27a)\ V0l))\ V1n)) \Rightarrow ((ap\ (ap\ (c\_2Elist\_2EDROP$$
$$A\_27a)\ V1n)\ V0l) = (c\_2Elist\_2ENIL\ A\_27a)))))$$
$$\tag{54}$$

Assume the following.

$((\forall V0n \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2B$
$\text{c\_2Enum\_2E0})\ V0n) = V0n))\land((\forall V1n \in ty\_2Enum\_2Enum.((ap$
$(ap\ c\_2Earithmetic\_2E\_2B\ V1n)\ \text{c\_2Enum\_2E0}) = V1n))\land((\forall V2n \in$
$ty\_2Enum\_2Enum.(\forall V3m \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2B$
$(ap\ \text{c\_2Earithmetic\_2ENUMERAL}\ V2n))\ (ap\ \text{c\_2Earithmetic\_2ENUMERAL}$
$V3m)) = (ap\ \text{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \text{c\_2Enumeral\_2EiZ}\ (ap$
$(ap\ c\_2Earithmetic\_2E\_2B\ V2n)\ V3m))))))\land((\forall V4n \in ty\_2Enum\_2Enum.$
$((ap\ (ap\ c\_2Earithmetic\_2E\_2A\ \text{c\_2Enum\_2E0})\ V4n) = \text{c\_2Enum\_2E0}))\land$
$((\forall V5n \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2A$
$V5n)\ \text{c\_2Enum\_2E0}) = \text{c\_2Enum\_2E0}))\land((\forall V6n \in ty\_2Enum\_2Enum.$
$(\forall V7m \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2A\ ($
$ap\ \text{c\_2Earithmetic\_2ENUMERAL}\ V6n))\ (ap\ \text{c\_2Earithmetic\_2ENUMERAL}$
$V7m)) = (ap\ \text{c\_2Earithmetic\_2ENUMERAL}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2A$
$V6n)\ V7m)))))\land((\forall V8n \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2D$
$\text{c\_2Enum\_2E0})\ V8n) = \text{c\_2Enum\_2E0}))\land((\forall V9n \in ty\_2Enum\_2Enum.$
$((ap\ (ap\ c\_2Earithmetic\_2E\_2D\ V9n)\ \text{c\_2Enum\_2E0}) = V9n))\land((\forall V10n \in$
$ty\_2Enum\_2Enum.(\forall V11m \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2D$
$(ap\ \text{c\_2Earithmetic\_2ENUMERAL}\ V10n))\ (ap\ \text{c\_2Earithmetic\_2ENUMERAL}$
$V11m)) = (ap\ \text{c\_2Earithmetic\_2ENUMERAL}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2D$
$V10n)\ V11m))))) \land((\forall V12n \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2EEXP$
$\text{c\_2Enum\_2E0})\ (ap\ \text{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \text{c\_2Earithmetic\_2EBIT1}$
$V12n))) = \text{c\_2Enum\_2E0}))\land((\forall V13n \in ty\_2Enum\_2Enum.((ap$
$(ap\ c\_2Earithmetic\_2EEXP\ \text{c\_2Enum\_2E0})\ (ap\ \text{c\_2Earithmetic\_2ENUMERAL}$
$(ap\ \text{c\_2Earithmetic\_2EBIT2}\ V13n))) = \text{c\_2Enum\_2E0}))\land((\forall V14n \in$
$ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2EEXP\ V14n)\ \text{c\_2Enum\_2E0}) =$
$(ap\ \text{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \text{c\_2Earithmetic\_2EBIT1}\ \text{c\_2Earithmetic\_2EZERO}))))\land$
$((\forall V15n \in ty\_2Enum\_2Enum.(\forall V16m \in ty\_2Enum\_2Enum.$
$((ap\ (ap\ c\_2Earithmetic\_2EEXP\ (ap\ \text{c\_2Earithmetic\_2ENUMERAL}\ V15n))$
$(ap\ \text{c\_2Earithmetic\_2ENUMERAL}\ V16m)) = (ap\ \text{c\_2Earithmetic\_2ENUMERAL}$
$(ap\ (ap\ c\_2Earithmetic\_2EEXP\ V15n)\ V16m)))))\land(((ap\ \text{c\_2Enum\_2ESUC}$
$\text{c\_2Enum\_2E0}) = (ap\ \text{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \text{c\_2Earithmetic\_2EBIT1}$
$\text{c\_2Earithmetic\_2EZERO})))\land((\forall V17n \in ty\_2Enum\_2Enum.($
$(ap\ \text{c\_2Enum\_2ESUC}\ (ap\ \text{c\_2Earithmetic\_2ENUMERAL}\ V17n)) = (ap\ \text{c\_2Earithmetic\_2ENUMERAL}$
$(ap\ \text{c\_2Enum\_2ESUC}\ V17n))))\land(((ap\ \text{c\_2Eprim\_\_rec\_2EPRE}\ \text{c\_2Enum\_2E0}) =$
$\text{c\_2Enum\_2E0})\land((\forall V18n \in ty\_2Enum\_2Enum.((ap\ \text{c\_2Eprim\_\_rec\_2EPRE}$
$(ap\ \text{c\_2Earithmetic\_2ENUMERAL}\ V18n)) = (ap\ \text{c\_2Earithmetic\_2ENUMERAL}$
$(ap\ \text{c\_2Eprim\_\_rec\_2EPRE}\ V18n))))\land((\forall V19n \in ty\_2Enum\_2Enum.$
$(((ap\ \text{c\_2Earithmetic\_2ENUMERAL}\ V19n) = \text{c\_2Enum\_2E0}) \Leftrightarrow (V19n = \text{c\_2Earithmetic\_2EZERO})))\land$
$((\forall V20n \in ty\_2Enum\_2Enum.((\text{c\_2Enum\_2E0} = (ap\ \text{c\_2Earithmetic\_2ENUMERAL}$
$V20n)) \Leftrightarrow (V20n = \text{c\_2Earithmetic\_2EZERO})))\land((\forall V21n \in ty\_2Enum\_2Enum.$
$(\forall V22m \in ty\_2Enum\_2Enum.(((ap\ \text{c\_2Earithmetic\_2ENUMERAL}$
$V21n) = (ap\ \text{c\_2Earithmetic\_2ENUMERAL}\ V22m)) \Leftrightarrow (V21n = V22m))))\land$
$((\forall V23n \in ty\_2Enum\_2Enum.((p\ (ap\ (ap\ \text{c\_2Eprim\_\_rec\_2E\_3C}$
$V23n)\ \text{c\_2Enum\_2E0})) \Leftrightarrow False))\land((\forall V24n \in ty\_2Enum\_2Enum.$
$((p\ (ap\ (ap\ \text{c\_2Eprim\_\_rec\_2E\_3C}\ \text{c\_2Enum\_2E0})\ (ap\ \text{c\_2Earithmetic\_2ENUMERAL}$
$V24n))) \Leftrightarrow (p\ (ap\ (ap\ \text{c\_2Eprim\_\_rec\_2E\_3C}\ \text{c\_2Earithmetic\_2EZERO})$
$V24n))))\land((\forall V25n \in ty\_2Enum\_2Enum.(\forall V26m \in ty\_2Enum\_2Enum.$
$((p\ (ap\ (ap\ \text{c\_2Eprim\_\_rec\_2E\_3C}\ (ap\ \text{c\_2Earithmetic\_2ENUMERAL}$
$V25n))\ (ap\ \text{c\_2Earithmetic\_2ENUMERAL}\ V26m))) \Leftrightarrow (p\ (ap\ (ap\ \text{c\_2Eprim\_\_rec\_2E\_3C}$
$V25n)\ V26m)))))\land((\forall V27n \in ty\_2Enum\_2Enum.((p\ (ap\ (ap\ \text{c\_2Earithmetic\_2E\_3E}$
$\text{c\_2Enum\_2E0})\ V27n)) \Leftrightarrow False))\land((\forall V28n \in ty\_2Enum\_2Enum.$
$((p\ (ap\ (ap\ \text{c\_2Earithmetic\_2E\_3E}\ (ap\ \text{c\_2Earithmetic\_2ENUMERAL}$
$V28n))\ \text{c\_2Enum\_2E0})) \Leftrightarrow (p\ (ap\ (ap\ \text{c\_2Eprim\_\_rec\_2E\_3C}\ \text{c\_2Earithmetic\_2EZERO})$
$V28n))))\land((\forall V29n \in ty\_2Enum\_2Enum.(\forall V30m \in ty\_2Enum\_2Enum.$
$((p\ (ap\ (ap\ \text{c\_2Earithmetic\_2E\_3E}\ (ap\ \text{c\_2Earithmetic\_2ENUMERAL}$
$V29n))\ (ap\ \text{c\_2Earithmetic\_2ENUMERAL}\ V30m))) \Leftrightarrow (p\ (ap\ (ap\ \text{c\_2Eprim\_\_rec\_2E\_3C}$
$V30m)\ V29n)))))\land((\forall V31n \in ty\_2Enum\_2Enum.((p\ (ap\ (ap\ \text{c\_2Earithmetic\_2E\_3C\_3D}$
$\text{c\_2Enum\_2E0})\ V31n)) \Leftrightarrow True))\land((\forall V32n \in ty\_2Enum\_2Enum.$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.(\forall V1m \in ty\_2Enum\_2Enum.($$
$$((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ \mathsf{c\_2Earithmetic\_2EZERO})\ V0n)) \Leftrightarrow$$
$$True) \wedge (((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}$$
$$V0n))\ \mathsf{c\_2Earithmetic\_2EZERO})) \Leftrightarrow False) \wedge (((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}$$
$$(ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V0n))\ \mathsf{c\_2Earithmetic\_2EZERO})) \Leftrightarrow False) \wedge$$
$$(((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}$$
$$V0n))\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V1m))) \Leftrightarrow (p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}$$
$$V0n)\ V1m))) \wedge (((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}$$
$$V0n))\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V1m))) \Leftrightarrow (p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}$$
$$V0n)\ V1m))) \wedge (((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}$$
$$V0n))\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V1m))) \Leftrightarrow (\neg(p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}$$
$$V1m)\ V0n)))) \wedge ((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}$$
$$V0n))\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V1m))) \Leftrightarrow (p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}$$
$$V0n)\ V1m))))))))))))$$

$$(56)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ($$
$$(\forall V0v \in A\_27b.(\forall V1f \in (A\_27b^{A\_27a}).((ap\ (ap\ (ap\ (c\_2Eoption\_2Eoption\_CASE$$
$$A\_27a\ A\_27b)\ (\mathsf{c\_2Eoption\_2ENONE}\ A\_27a))\ V0v)\ V1f) = V0v))) \wedge (\forall V2x \in$$
$$A\_27a.(\forall V3v \in A\_27b.(\forall V4f \in (A\_27b^{A\_27a}).((ap\ (ap$$
$$(ap\ (c\_2Eoption\_2Eoption\_CASE\ A\_27a\ A\_27b)\ (ap\ (\mathsf{c\_2Eoption\_2ESOME}$$
$$A\_27a)\ V2x))\ V3v)\ V4f) = (ap\ V4f\ V2x))))))$$

$$(57)$$

Assume the following.

$$(\forall V0P \in (2^{ty\_2Enum\_2Enum}).(\forall V1Q \in (2^{(ty\_2Eoption\_2Eoption\ ty\_2Enum\_2Enum)}).$$
$$(((((\forall V2n \in ty\_2Enum\_2Enum.(\neg(p\ (ap\ V0P\ V2n)))) \Rightarrow (p\ (ap\ V1Q$$
$$(\mathsf{c\_2Eoption\_2ENONE}\ ty\_2Enum\_2Enum)))) \wedge (\forall V3n \in ty\_2Enum\_2Enum.$$
$$(((p\ (ap\ V0P\ V3n)) \wedge (\forall V4m \in ty\_2Enum\_2Enum.((p\ (ap\ (ap\ \mathsf{c\_2Eprim\_rec\_2E\_3C}$$
$$V4m)\ V3n)) \Rightarrow (\neg(p\ (ap\ V0P\ V4m)))))) \Rightarrow (p\ (ap\ V1Q\ (ap\ (\mathsf{c\_2Eoption\_2ESOME}$$
$$ty\_2Enum\_2Enum)\ V3n)))))) \Rightarrow (p\ (ap\ V1Q\ (ap\ \mathsf{c\_2Ewhile\_2EOLEAST}\ V0P)))))))$$

$$(58)$$

Theorem 1

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0n \in ty\_2Enum\_2Enum.($$
$$\forall V1l \in (ty\_2Elist\_2Elist\ A\_27a).((ap\ (ap\ (c\_2Elist\_2EDROP$$
$$A\_27a)\ V0n)\ V1l) = (ap\ (ap\ (ap\ (c\_2Elist\_2EsplitAtPki\ (ty\_2Elist\_2Elist$$
$$A\_27a)\ A\_27a)\ (ap\ (ap\ (\mathsf{c\_2Ecombin\_2Eo}\ ty\_2Enum\_2Enum\ (2^{A\_27a})$$
$$2)\ (\mathsf{c\_2Ecombin\_2EK}\ 2\ A\_27a))\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ ty\_2Enum\_2Enum)$$
$$V0n)))\ (ap\ (\mathsf{c\_2Ecombin\_2EK}\ ((ty\_2Elist\_2Elist\ A\_27a)^{(ty\_2Elist\_2Elist\ A\_27a)})$$
$$(ty\_2Elist\_2Elist\ A\_27a))\ (\mathsf{c\_2Ecombin\_2EI}\ (ty\_2Elist\_2Elist$$
$$A\_27a))))\ V1l))))$$