thm_2Elist_2EEL_LENGTH_dropWhile_REVERSE (TMLuitwYZKr7Ym1dZg4NEe1Zh8G19rBkWaS)

October 26, 2020

Definition 1 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.$**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge p$
of type $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$
of type $\iota \Rightarrow \iota$.

Definition 3 *We define* c_2Ebool_2E_3F *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ V0P\ (ap\ ($c_2Emin_2E_40$\ $

Definition 4 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$
of type ι.

Definition 5 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ ($c_2Emin_2E_3D$\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1$

Definition 6 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ ($c_2Emin_2E_3D$\ (2^{A\_27}$

Definition 7 *We define* c_2Ebool_2E_5C_2F *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ ($c_2Ebool_2E_21$\ 2)\ (\lambda V2t \in 2$

Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \qquad (1)$$

Let $ty\_2Elist\_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Elist\_2Elist\ A0) \qquad (2)$$

Let $c\_2Elist\_2ELENGTH : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ELENGTH\ A\_27a \in (ty\_2Enum\_2Enum^{(ty\_2Elist\_2Elist\ A\_27a)}) \qquad (3)$$

Let $c\_2Elist\_2EHD : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2EHD\ A\_27a \in (A\_27a^{(ty\_2Elist\_2Elist\ A\_27a)}) \qquad (4)$$

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Let $c\_2Elist\_2EEL : \iota{\Rightarrow}\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a{\Rightarrow}c\_2Elist\_2EEL\ A\_27a \in ((A\_27a^{(ty\_2Elist\_2Elist\ A\_27a)})^{ty\_2Enum\_2Enum})$$

(5)

Let $c\_2Elist\_2EREVERSE : \iota{\Rightarrow}\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a{\Rightarrow}c\_2Elist\_2EREVERSE\ A\_27a \in ((ty\_2Elist\_2Elist \\ A\_27a)^{(ty\_2Elist\_2Elist\ A\_27a)})$$

(6)

Definition 8 *We define* c$\_2$Ebool$\_2$EF *to be* $(ap\ ($c$\_2$Ebool$\_2$E$\_21\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 9 *We define* c$\_2$Ebool$\_2$E$\_2$F$\_5$C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c\_2$Ebool$\_2E\_21\ 2)\ (\lambda V2t \in 2$

Definition 10 *We define* c$\_2$Ebool$\_2$ECOND *to be* $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.($

Let $c\_2Elist\_2ECONS : \iota{\Rightarrow}\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a{\Rightarrow}c\_2Elist\_2ECONS\ A\_27a \in (((ty\_2Elist\_2Elist \\ A\_27a)^{(ty\_2Elist\_2Elist\ A\_27a)})^{A\_27a})$$

(7)

Let $c\_2Elist\_2ENIL : \iota{\Rightarrow}\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a{\Rightarrow}c\_2Elist\_2ENIL\ A\_27a \in (ty\_2Elist\_2Elist \\ A\_27a)$$

(8)

Let $c\_2Elist\_2EEVERY : \iota{\Rightarrow}\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a{\Rightarrow}c\_2Elist\_2EEVERY\ A\_27a \in ((2^{(ty\_2Elist\_2Elist\ A\_27a)})^{(2^{A\_27a})})$$

(9)

Let $c\_2Elist\_2EAPPEND : \iota{\Rightarrow}\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a{\Rightarrow}c\_2Elist\_2EAPPEND\ A\_27a \in (((ty\_2Elist\_2Elist \\ A\_27a)^{(ty\_2Elist\_2Elist\ A\_27a)})^{(ty\_2Elist\_2Elist\ A\_27a)})$$

(10)

Let $c\_2Elist\_2EdropWhile : \iota{\Rightarrow}\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a{\Rightarrow}c\_2Elist\_2EdropWhile\ A\_27a \in ((\\ (ty\_2Elist\_2Elist\ A\_27a)^{(ty\_2Elist\_2Elist\ A\_27a)})^{(2^{A\_27a})})$$

(11)

Definition 11 *We define* c$\_2$Ecombin$\_2$Eo *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda V0f \in (A\_27b^{A\_27c}).\lambda V1g$

Let $c\_2Elist\_2EEXISTS : \iota{\Rightarrow}\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a{\Rightarrow}c\_2Elist\_2EEXISTS\ A\_27a \in ((2^{(ty\_2Elist\_2Elist\ A\_27a)})^{(2^{A\_27a})})$$

(12)

Let $c\_2Earithmetic\_2EEVEN : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2EEVEN \in (2^{ty\_2Enum\_2Enum})$$

(13)

Let $c\_2Earithmetic\_2EODD : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2EODD \in (2^{ty\_2Enum\_2Enum})$$

(14)

Definition 12 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap\ $c_2Emin_2E_3D_3D_3E$\ V0t)\ $c_2Ebool_2E

Let $c\_2Enum\_2EREP\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EREP\_\_num \in (omega^{ty\_2Enum\_2Enum}) \qquad (15)$$

Let $c\_2Enum\_2ESUC\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2ESUC\_\_REP \in (omega^{omega}) \qquad (16)$$

Let $c\_2Enum\_2EABS\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EABS\_\_num \in (ty\_2Enum\_2Enum^{omega}) \qquad (17)$$

Definition 13 *We define* c_2Enum_2ESUC *to be* $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_\_num$

Definition 14 *We define* c_2Eprim__rec_2E_3C *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enu$

Definition 15 *We define* c_2Earithmetic_2E_3E *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2En$

Definition 16 *We define* c_2Earithmetic_2E_3E_3D *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2$

Let $c\_2Enum\_2EZERO\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2EZERO\_\_REP \in omega \qquad (18)$$

Definition 17 *We define* c_2Enum_2E0 *to be* $(ap\ c\_2Enum\_2EABS\_\_num\ c\_2Enum\_2EZERO\_\_REP).$

Definition 18 *We define* c_2Eprim__rec_2EPRE *to be* $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ (ap\ (ap\ ($c_2Ebool_2E

Let $c\_2Earithmetic\_2EEXP : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2EEXP \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum})$$
$$(19)$$

Let $c\_2Earithmetic\_2E\_2D : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2D \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum})$$
$$(20)$$

Let $c\_2Earithmetic\_2E\_2A : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2A \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum})$$
$$(21)$$

Definition 19 *We define* c_2Enumeral_2EiZ *to be* $\lambda V0x \in ty\_2Enum\_2Enum.V0x.$

Definition 20 *We define* c_2Earithmetic_2ENUMERAL *to be* $\lambda V0x \in ty\_2Enum\_2Enum.V0x.$

Let $c\_2Earithmetic\_2E\_2B : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum})$$
$$(22)$$

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Definition 21 *We define* c_2Earithmetic_2EBIT2 *to be* $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmeti$

Definition 22 *We define* c_2Earithmetic_2EBIT1 *to be* $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmeti$

Definition 23 *We define* c_2Earithmetic_2EZERO *to be* c_2Enum_2E0.

Definition 24 *We define* c_2Earithmetic_2E_3C_3D *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m)$$
$$\text{c\_2Enum\_2E0}) = V0m)) \tag{23}$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.($$
$$(ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ V1n) = (ap\ (ap\ c\_2Earithmetic\_2E\_2B$$
$$V1n)\ V0m)))) \tag{24}$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.((V0m = \text{c\_2Enum\_2E0}) \vee (\exists V1n \in$$
$$ty\_2Enum\_2Enum.(V0m = (ap\ \text{c\_2Enum\_2ESUC}\ V1n))))) \tag{25}$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.($$
$$(p\ (ap\ (ap\ \text{c\_2Eprim\_\_rec\_2E\_3C}\ (ap\ \text{c\_2Enum\_2ESUC}\ V0m))\ (ap\ \text{c\_2Enum\_2ESUC}$$
$$V1n)))\Leftrightarrow(p\ (ap\ (ap\ \text{c\_2Eprim\_\_rec\_2E\_3C}\ V0m)\ V1n))))) \tag{26}$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.(p\ (ap\ (ap\ \text{c\_2Earithmetic\_2E\_3C\_3D}$$
$$\text{c\_2Enum\_2E0})\ V0n))) \tag{27}$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.((p\ (ap\ (ap\ \text{c\_2Earithmetic\_2E\_3C\_3D}$$
$$V0n)\ \text{c\_2Enum\_2E0}))\Leftrightarrow(V0n = \text{c\_2Enum\_2E0}))) \tag{28}$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.((ap\ \text{c\_2Enum\_2ESUC}\ V0m) = (ap\ (ap$$
$$c\_2Earithmetic\_2E\_2B\ V0m)\ (ap\ \text{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \text{c\_2Earithmetic\_2EBIT1}$$
$$\text{c\_2Earithmetic\_2EZERO}))))) \tag{29}$$

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Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.(\\
((ap\ (ap\ c\_2Earithmetic\_2E\_2A\ \textsf{c\_2Enum\_2E0})\ V0m) = \textsf{c\_2Enum\_2E0})\wedge\\
(((ap\ (ap\ c\_2Earithmetic\_2E\_2A\ V0m)\ \textsf{c\_2Enum\_2E0}) = \textsf{c\_2Enum\_2E0})\wedge\\
(((ap\ (ap\ c\_2Earithmetic\_2E\_2A\ (ap\ \textsf{c\_2Earithmetic\_2ENUMERAL}\\
(ap\ \textsf{c\_2Earithmetic\_2EBIT1}\ \textsf{c\_2Earithmetic\_2EZERO})))\ V0m) = V0m)\wedge\\
(((ap\ (ap\ c\_2Earithmetic\_2E\_2A\ V0m)\ (ap\ \textsf{c\_2Earithmetic\_2ENUMERAL}\\
(ap\ \textsf{c\_2Earithmetic\_2EBIT1}\ \textsf{c\_2Earithmetic\_2EZERO}))) = V0m)\wedge(\\
((ap\ (ap\ c\_2Earithmetic\_2E\_2A\ (ap\ \textsf{c\_2Enum\_2ESUC}\ V0m))\ V1n) = (ap\\
(ap\ c\_2Earithmetic\_2E\_2B\ (ap\ (ap\ c\_2Earithmetic\_2E\_2A\ V0m)\ V1n))\\
V1n))\wedge((ap\ (ap\ c\_2Earithmetic\_2E\_2A\ V0m)\ (ap\ \textsf{c\_2Enum\_2ESUC}\ V1n)) =\\
(ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ (ap\ (ap\ c\_2Earithmetic\_2E\_2A\\
V0m)\ V1n)))))))))))$$

(30)

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.(\\
\forall V2p \in ty\_2Enum\_2Enum.(((p\ (ap\ (ap\ \textsf{c\_2Earithmetic\_2E\_3C\_3D}\\
V0m)\ V1n))\wedge(p\ (ap\ (ap\ \textsf{c\_2Earithmetic\_2E\_3C\_3D}\ V1n)\ V2p)))\Rightarrow(p\ (\\
ap\ (ap\ \textsf{c\_2Earithmetic\_2E\_3C\_3D}\ V0m)\ V2p))))))$$

(31)

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.(\\
\forall V2p \in ty\_2Enum\_2Enum.((p\ (ap\ (ap\ \textsf{c\_2Earithmetic\_2E\_3C\_3D}\\
(ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ V1n))\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\\
V0m)\ V2p)))\Leftrightarrow(p\ (ap\ (ap\ \textsf{c\_2Earithmetic\_2E\_3C\_3D}\ V1n)\ V2p))))))$$

(32)

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.(\\
(\neg(p\ (ap\ (ap\ \textsf{c\_2Earithmetic\_2E\_3C\_3D}\ V0m)\ V1n)))\Leftrightarrow(p\ (ap\ (ap\ \textsf{c\_2Earithmetic\_2E\_3C\_3D}\\
(ap\ \textsf{c\_2Enum\_2ESUC}\ V1n))\ V0m)))))$$

(33)

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.((ap\ \textsf{c\_2Enum\_2ESUC}\ V0n) = (ap\ (ap\\
c\_2Earithmetic\_2E\_2B\ (ap\ \textsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \textsf{c\_2Earithmetic\_2EBIT1}\\
\textsf{c\_2Earithmetic\_2EZERO}))\ V0n)))$$

(34)

Assume the following.
$$True \qquad\qquad (35)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1)\Rightarrow(p\ V1t2))\Rightarrow(((p\\
V1t2)\Rightarrow(p\ V0t1))\Rightarrow((p\ V0t1)\Leftrightarrow(p\ V1t2)))))))$$

(36)

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Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p\ V0t))) \tag{37}$$

Assume the following.

$$(\forall V0t \in 2.((p\ V0t) \lor (\neg(p\ V0t)))) \tag{38}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in \\ A\_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \tag{39}$$

Assume the following.

$$(\forall V0t \in 2.(((True \land (p\ V0t)) \Leftrightarrow (p\ V0t)) \land ((((p\ V0t) \land True) \Leftrightarrow \\ (p\ V0t)) \land (((False \land (p\ V0t)) \Leftrightarrow False) \land ((((p\ V0t) \land False) \Leftrightarrow False) \land \\ (((p\ V0t) \land (p\ V0t)) \Leftrightarrow (p\ V0t))))))) \tag{40}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \land ((((p\ V0t) \Rightarrow True) \Leftrightarrow \\ True) \land (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \land ((((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \land ((\\ (p\ V0t) \Rightarrow False) \Leftrightarrow (\neg(p\ V0t))))))))) \tag{41}$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \land (((\neg True) \Leftrightarrow False) \land \\ ((\neg False) \Leftrightarrow True))) \tag{42}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow \\ True)) \tag{43}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in \\ A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \tag{44}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \land ((((p\ V0t) \Leftrightarrow True) \Leftrightarrow \\ (p\ V0t)) \land (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg(p\ V0t))) \land (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg(\\ p\ V0t)))))))) \tag{45}$$

Assume the following.

$$(\forall V0t \in 2.(((p\ V0t) \Rightarrow False) \Leftrightarrow ((p\ V0t) \Leftrightarrow False))) \tag{46}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p\ V0t1)\Rightarrow \\ ((p\ V1t2)\Rightarrow(p\ V2t3)))\Leftrightarrow(((p\ V0t1)\wedge(p\ V1t2))\Rightarrow(p\ V2t3))))))) \tag{47}$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1x\_27 \in 2.(\forall V2y \in 2.(\forall V3y\_27 \in \\ 2.((((p\ V0x)\Leftrightarrow(p\ V1x\_27))\wedge((p\ V1x\_27)\Rightarrow((p\ V2y)\Leftrightarrow(p\ V3y\_27))))\Rightarrow \\ (((p\ V0x)\Rightarrow(p\ V2y))\Leftrightarrow((p\ V1x\_27)\Rightarrow(p\ V3y\_27)))))))))) \tag{48}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0P \in 2.(\forall V1Q \in 2. \\ (\forall V2x \in A\_27a.(\forall V3x\_27 \in A\_27a.(\forall V4y \in A\_27a. \\ (\forall V5y\_27 \in A\_27a.((((p\ V0P)\Leftrightarrow(p\ V1Q))\wedge(((p\ V1Q)\Rightarrow(V2x = V3x\_27))\wedge \\ ((\neg(p\ V1Q))\Rightarrow(V4y = V5y\_27))))\Rightarrow((ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ A\_27a) \\ V0P)\ V2x)\ V4y) = (ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ A\_27a)\ V1Q)\ V3x\_27) \\ V5y\_27)))))))))) \tag{49}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0h \in A\_27a.(\forall V1t \in \\ (ty\_2Elist\_2Elist\ A\_27a).((ap\ (c\_2Elist\_2EHD\ A\_27a)\ (ap\ (ap\ (\\ c\_2Elist\_2ECONS\ A\_27a)\ V0h)\ V1t)) = V0h))) \tag{50}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(((ap\ (c\_2Elist\_2ELENGTH\ A\_27a) \\ (c\_2Elist\_2ENIL\ A\_27a)) = \mathsf{c\_2Enum\_2E0})\wedge(\forall V0h \in A\_27a.(\\ \forall V1t \in (ty\_2Elist\_2Elist\ A\_27a).((ap\ (c\_2Elist\_2ELENGTH \\ A\_27a)\ (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V0h)\ V1t)) = (ap\ \mathsf{c\_2Enum\_2ESUC} \\ (ap\ (c\_2Elist\_2ELENGTH\ A\_27a)\ V1t)))))) \tag{51}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow((\forall V0P \in (2^{A\_27a}).((p\ (ap \\ (ap\ (c\_2Elist\_2EEVERY\ A\_27a)\ V0P)\ (c\_2Elist\_2ENIL\ A\_27a)))\Leftrightarrow True))\wedge \\ (\forall V1P \in (2^{A\_27a}).(\forall V2h \in A\_27a.(\forall V3t \in (ty\_2Elist\_2Elist \\ A\_27a).((p\ (ap\ (ap\ (c\_2Elist\_2EEVERY\ A\_27a)\ V1P)\ (ap\ (ap\ (c\_2Elist\_2ECONS \\ A\_27a)\ V2h)\ V3t)))\Leftrightarrow((p\ (ap\ V1P\ V2h))\wedge(p\ (ap\ (ap\ (c\_2Elist\_2EEVERY \\ A\_27a)\ V1P)\ V3t)))))))))) \tag{52}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0P \in (2^{(ty\_2Elist\_2Elist\ A\_27a)}). \\ (((p\ (ap\ V0P\ (c\_2Elist\_2ENIL\ A\_27a)))\wedge(\forall V1t \in (ty\_2Elist\_2Elist \\ A\_27a).((p\ (ap\ V0P\ V1t))\Rightarrow(\forall V2h \in A\_27a.(p\ (ap\ V0P\ (ap\ (ap\ (\\ c\_2Elist\_2ECONS\ A\_27a)\ V2h)\ V1t)))))))\Rightarrow(\forall V3l \in (ty\_2Elist\_2Elist \\ A\_27a).(p\ (ap\ V0P\ V3l))))) \tag{53}$$

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Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0l1 \in (ty\_2Elist\_2Elist \\ A\_27a).(\forall V1l2 \in (ty\_2Elist\_2Elist\ A\_27a).((ap\ (c\_2Elist\_2ELENGTH \\ A\_27a)\ (ap\ (ap\ (c\_2Elist\_2EAPPEND\ A\_27a)\ V0l1)\ V1l2)) = (ap\ (ap\ c\_2Earithmetic\_2E\_2B \\ (ap\ (c\_2Elist\_2ELENGTH\ A\_27a)\ V0l1))\ (ap\ (c\_2Elist\_2ELENGTH\ A\_27a) \\ V1l2)))))$$

(54)

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}).(\forall V1l1 \in \\ (ty\_2Elist\_2Elist\ A\_27a).(\forall V2l2 \in (ty\_2Elist\_2Elist\ A\_27a). \\ ((p\ (ap\ (ap\ (c\_2Elist\_2EEVERY\ A\_27a)\ V0P)\ (ap\ (ap\ (c\_2Elist\_2EAPPEND \\ A\_27a)\ V1l1)\ V2l2))) \Leftrightarrow ((p\ (ap\ (ap\ (c\_2Elist\_2EEVERY\ A\_27a)\ V0P) \\ V1l1)) \wedge (p\ (ap\ (ap\ (c\_2Elist\_2EEVERY\ A\_27a)\ V0P)\ V2l2)))))))$$

(55)

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}).(\forall V1l \in \\ (ty\_2Elist\_2Elist\ A\_27a).((\neg (p\ (ap\ (ap\ (c\_2Elist\_2EEVERY\ A\_27a) \\ V0P)\ V1l))) \Leftrightarrow (p\ (ap\ (ap\ (c\_2Elist\_2EEXISTS\ A\_27a)\ (ap\ (ap\ \mathsf{c\_2Ecombin\_2Eo} \\ A\_27a\ 2\ 2)\ \mathsf{c\_2Ebool\_2E\_7E})\ V0P))\ V1l)))))$$

(56)

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0l \in (ty\_2Elist\_2Elist \\ A\_27a).(((ap\ (c\_2Elist\_2ELENGTH\ A\_27a)\ V0l) = \mathsf{c\_2Enum\_2E0}) \Leftrightarrow (\\ V0l = (c\_2Elist\_2ENIL\ A\_27a))))$$

(57)

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\\ \forall V0n \in ty\_2Enum\_2Enum.(\forall V1l \in A\_27b.(\forall V2ls \in \\ (ty\_2Elist\_2Elist\ A\_27b).(((ap\ (c\_2Elist\_2EEL\ A\_27a)\ \mathsf{c\_2Enum\_2E0}) = \\ (c\_2Elist\_2EHD\ A\_27a)) \wedge ((ap\ (ap\ (c\_2Elist\_2EEL\ A\_27b)\ (ap\ \mathsf{c\_2Enum\_2ESUC} \\ V0n))\ (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27b)\ V1l)\ V2ls)) = (ap\ (ap\ (c\_2Elist\_2EEL \\ A\_27b)\ V0n)\ V2ls))))))$$

(58)

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (((ap\ (c\_2Elist\_2EREVERSE\ A\_27a) \\ (c\_2Elist\_2ENIL\ A\_27a)) = (c\_2Elist\_2ENIL\ A\_27a)) \wedge (\forall V0h \in \\ A\_27a.(\forall V1t \in (ty\_2Elist\_2Elist\ A\_27a).((ap\ (c\_2Elist\_2EREVERSE \\ A\_27a)\ (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V0h)\ V1t)) = (ap\ (ap\ (c\_2Elist\_2EAPPEND \\ A\_27a)\ (ap\ (c\_2Elist\_2EREVERSE\ A\_27a)\ V1t))\ (ap\ (ap\ (c\_2Elist\_2ECONS \\ A\_27a)\ V0h)\ (c\_2Elist\_2ENIL\ A\_27a)))))))$$

(59)

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow ((\forall V0P \in (2^{A\_27a}).((ap\ (ap\ (c\_2Elist\_2EdropWhile\ A\_27a)\ V0P)\ (c\_2Elist\_2ENIL\ A\_27a)) = (c\_2Elist\_2ENIL\ A\_27a))) \wedge (\forall V1P \in (2^{A\_27a}).(\forall V2h \in A\_27a.(\forall V3t \in (ty\_2Elist\_2Elist\ A\_27a).((ap\ (ap\ (c\_2Elist\_2EdropWhile\ A\_27a)\ V1P)\ (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V2h)\ V3t)) = (ap\ (ap\ (ap\ (c\_2Ebool\_2ECOND\ (ty\_2Elist\_2Elist\ A\_27a))\ (ap\ V1P\ V2h))\ (ap\ (ap\ (c\_2Elist\_2EdropWhile\ A\_27a)\ V1P)\ V3t))\ (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V2h)\ V3t)))))))$$

(60)

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}).(\forall V1ls \in (ty\_2Elist\_2Elist\ A\_27a).(((ap\ (ap\ (c\_2Elist\_2EdropWhile\ A\_27a)\ V0P)\ V1ls) = (c\_2Elist\_2ENIL\ A\_27a)) \Leftrightarrow (p\ (ap\ (ap\ (c\_2Elist\_2EEVERY\ A\_27a)\ V0P)\ V1ls)))))$$

(61)

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}).(\forall V1l1 \in (ty\_2Elist\_2Elist\ A\_27a).(\forall V2l2 \in (ty\_2Elist\_2Elist\ A\_27a).((p\ (ap\ (ap\ (c\_2Elist\_2EEVERY\ A\_27a)\ V0P)\ V1l1)) \Rightarrow ((ap\ (ap\ (c\_2Elist\_2EdropWhile\ A\_27a)\ V0P)\ (ap\ (ap\ (c\_2Elist\_2EAPPEND\ A\_27a)\ V1l1)\ V2l2)) = (ap\ (ap\ (c\_2Elist\_2EdropWhile\ A\_27a)\ V0P)\ V2l2)))))))$$

(62)

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}).(\forall V1l1 \in (ty\_2Elist\_2Elist\ A\_27a).(\forall V2l2 \in (ty\_2Elist\_2Elist\ A\_27a).((p\ (ap\ (ap\ (c\_2Elist\_2EEXISTS\ A\_27a)\ (ap\ (ap\ (c\_2Ecombin\_2Eo\ A\_27a\ 2\ 2)\ c\_2Ebool\_2E\_7E)\ V0P))\ V1l1)) \Rightarrow ((ap\ (ap\ (c\_2Elist\_2EdropWhile\ A\_27a)\ V0P)\ (ap\ (ap\ (c\_2Elist\_2EAPPEND\ A\_27a)\ V1l1)\ V2l2)) = (ap\ (ap\ (c\_2Elist\_2EAPPEND\ A\_27a)\ (ap\ (ap\ (c\_2Elist\_2EdropWhile\ A\_27a)\ V0P)\ V1l1))\ V2l2)))))))$$

(63)

Assume the following.

$((\forall V0n \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2B$
c_2Enum_2E0) $V0n) = V0n))\wedge((\forall V1n \in ty\_2Enum\_2Enum.((ap$
$(ap\ c\_2Earithmetic\_2E\_2B\ V1n)$ c_2Enum_2E0) $= V1n))\wedge((\forall V2n \in$
$ty\_2Enum\_2Enum.(\forall V3m \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2B$
$(ap$ c_2Earithmetic_2ENUMERAL $V2n))\ (ap$ c_2Earithmetic_2ENUMERAL
$V3m)) = (ap$ c_2Earithmetic_2ENUMERAL $(ap$ c_2Enumeral_2EiZ $(ap$
$(ap\ c\_2Earithmetic\_2E\_2B\ V2n)\ V3m))))))\wedge((\forall V4n \in ty\_2Enum\_2Enum.$
$((ap\ (ap\ c\_2Earithmetic\_2E\_2A$ c_2Enum_2E0) $V4n) = $ c_2Enum_2E0)$)\wedge$
$((\forall V5n \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2A$
$V5n)$ c_2Enum_2E0) $= $ c_2Enum_2E0)$)\wedge((\forall V6n \in ty\_2Enum\_2Enum.$
$(\forall V7m \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2A\ ($
ap c_2Earithmetic_2ENUMERAL $V6n))\ (ap$ c_2Earithmetic_2ENUMERAL
$V7m)) = (ap$ c_2Earithmetic_2ENUMERAL $(ap\ (ap\ c\_2Earithmetic\_2E\_2A$
$V6n)\ V7m)))))\wedge((\forall V8n \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2D$
c_2Enum_2E0) $V8n) = $ c_2Enum_2E0)$)\wedge((\forall V9n \in ty\_2Enum\_2Enum.$
$((ap\ (ap\ c\_2Earithmetic\_2E\_2D\ V9n)$ c_2Enum_2E0) $= V9n))\wedge((\forall V10n \in$
$ty\_2Enum\_2Enum.(\forall V11m \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2D$
$(ap$ c_2Earithmetic_2ENUMERAL $V10n))\ (ap$ c_2Earithmetic_2ENUMERAL
$V11m)) = (ap$ c_2Earithmetic_2ENUMERAL $(ap\ (ap\ c\_2Earithmetic\_2E\_2D$
$V10n)\ V11m)))))\wedge((\forall V12n \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2EEXP$
c_2Enum_2E0) $(ap$ c_2Earithmetic_2ENUMERAL $(ap$ c_2Earithmetic_2EBIT1
$V12n))) = $ c_2Enum_2E0)$)\wedge((\forall V13n \in ty\_2Enum\_2Enum.((ap$
$(ap\ c\_2Earithmetic\_2EEXP$ c_2Enum_2E0) $(ap$ c_2Earithmetic_2ENUMERAL
$(ap$ c_2Earithmetic_2EBIT2 $V13n))) = $ c_2Enum_2E0)$)\wedge((\forall V14n \in$
$ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2EEXP\ V14n)$ c_2Enum_2E0) $=$
$(ap$ c_2Earithmetic_2ENUMERAL $(ap$ c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)$)))\wedge$
$((\forall V15n \in ty\_2Enum\_2Enum.(\forall V16m \in ty\_2Enum\_2Enum.$
$((ap\ (ap\ c\_2Earithmetic\_2EEXP\ (ap$ c_2Earithmetic_2ENUMERAL $V15n))$
$(ap$ c_2Earithmetic_2ENUMERAL $V16m)) = (ap$ c_2Earithmetic_2ENUMERAL
$(ap\ (ap\ c\_2Earithmetic\_2EEXP\ V15n)\ V16m)))))\wedge(((ap$ c_2Enum_2ESUC
c_2Enum_2E0) $= (ap$ c_2Earithmetic_2ENUMERAL $(ap$ c_2Earithmetic_2EBIT1
c_2Earithmetic_2EZERO)$))\wedge((\forall V17n \in ty\_2Enum\_2Enum.($
$(ap$ c_2Enum_2ESUC $(ap$ c_2Earithmetic_2ENUMERAL $V17n)) = (ap$ c_2Earithmetic_2ENUMERAL
$(ap$ c_2Enum_2ESUC $V17n))))\wedge(((ap$ c_2Eprim__rec_2EPRE c_2Enum_2E0) $=$
c_2Enum_2E0)$\wedge((\forall V18n \in ty\_2Enum\_2Enum.((ap$ c_2Eprim__rec_2EPRE
$(ap$ c_2Earithmetic_2ENUMERAL $V18n)) = (ap$ c_2Earithmetic_2ENUMERAL
$(ap$ c_2Eprim__rec_2EPRE $V18n))))\wedge((\forall V19n \in ty\_2Enum\_2Enum.$
$(((ap$ c_2Earithmetic_2ENUMERAL $V19n) = $ c_2Enum_2E0)$\Leftrightarrow(V19n = $ c_2Earithmetic_2EZERO)$))\wedge$
$((\forall V20n \in ty\_2Enum\_2Enum.(($c_2Enum_2E0 $= (ap$ c_2Earithmetic_2ENUMERAL
$V20n))\Leftrightarrow(V20n = $ c_2Earithmetic_2EZERO)$))\wedge((\forall V21n \in ty\_2Enum\_2Enum.$
$(\forall V22m \in ty\_2Enum\_2Enum.(((ap$ c_2Earithmetic_2ENUMERAL
$V21n) = (ap$ c_2Earithmetic_2ENUMERAL $V22m))\Leftrightarrow(V21n = V22m))))\wedge$
$((\forall V23n \in ty\_2Enum\_2Enum.((p\ (ap\ (ap\ c\_2Eprim\_\_rec\_2E\_3C$
$V23n)$ c_2Enum_2E0)$)\Leftrightarrow False))\wedge((\forall V24n \in ty\_2Enum\_2Enum.$
$((p\ (ap\ (ap\ c\_2Eprim\_\_rec\_2E\_3C$ c_2Enum_2E0) $(ap$ c_2Earithmetic_2ENUMERAL
$V24n)))\Leftrightarrow(p\ (ap\ (ap\ c\_2Eprim\_\_rec\_2E\_3C$ c_2Earithmetic_2EZERO)
$V24n))))\wedge((\forall V25n \in ty\_2Enum\_2Enum.(\forall V26m \in ty\_2Enum\_2Enum.$
$((p\ (ap\ (ap\ c\_2Eprim\_\_rec\_2E\_3C\ (ap$ c_2Earithmetic_2ENUMERAL
$V25n))\ (ap$ c_2Earithmetic_2ENUMERAL $V26m)))\Leftrightarrow(p\ (ap\ (ap\ c\_2Eprim\_\_rec\_2E\_3C$
$V25n)\ V26m)))))\wedge((\forall V27n \in ty\_2Enum\_2Enum.((p\ (ap\ (ap\ c\_2Earithmetic\_2E\_3E$
c_2Enum_2E0) $V27n))\Leftrightarrow False))\wedge((\forall V28n \in ty\_2Enum\_2Enum.$
$((p\ (ap\ (ap\ c\_2Earithmetic\_2E\_3E\ (ap$ c_2Earithmetic_2ENUMERAL
$V28n))$ c_2Enum_2E0)$)\Leftrightarrow(p\ (ap\ (ap\ c\_2Eprim\_\_rec\_2E\_3C$ c_2Earithmetic_2EZERO)
$V28n))))\wedge((\forall V29n \in ty\_2Enum\_2Enum.(\forall V30m \in ty\_2Enum\_2Enum.$
$((p\ (ap\ (ap\ c\_2Earithmetic\_2E\_3E\ (ap$ c_2Earithmetic_2ENUMERAL
$V29n))\ (ap$ c_2Earithmetic_2ENUMERAL $V30m)))\Leftrightarrow(p\ (ap\ (ap\ c\_2Eprim\_\_rec\_2E\_3C$
$V30m)\ V29n)))))\wedge((\forall V31n \in ty\_2Enum\_2Enum.((p\ (ap\ (ap\ c\_2Earithmetic\_2E\_3C\_3D$
c_2Enum_2E0) $V31n))\Leftrightarrow True))\wedge((\forall V32n \in ty\_2Enum\_2Enum.$
$((p\ (ap\ (ap\ c\_2Earithmetic\_2E\_3C\_3D\ (ap\ c\_2Earithmetic\_2ENUMERAL$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.(\forall V1m \in ty\_2Enum\_2Enum.($$
$$((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ \mathsf{c\_2Earithmetic\_2EZERO})\ V0n)){\Leftrightarrow}$$
$$True){\wedge}(((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}$$
$$V0n))\ \mathsf{c\_2Earithmetic\_2EZERO})){\Leftrightarrow}False){\wedge}(((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}$$
$$(ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V0n))\ \mathsf{c\_2Earithmetic\_2EZERO})){\Leftrightarrow}False){\wedge}$$
$$(((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}$$
$$V0n))\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V1m))){\Leftrightarrow}(p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}$$
$$V0n)\ V1m))){\wedge}(((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}$$
$$V0n))\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V1m))){\Leftrightarrow}(p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}$$
$$V0n)\ V1m))){\wedge}(((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}$$
$$V0n))\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V1m))){\Leftrightarrow}({\neg}(p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}$$
$$V1m)\ V0n)))){\wedge}((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}$$
$$V0n))\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V1m))){\Leftrightarrow}(p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}$$
$$V0n)\ V1m))))))))))))$$

$$(65)$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.({\neg}(p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}$$
$$V0n)\ \mathsf{c\_2Enum\_2E0}))))$$

$$(66)$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.(p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ \mathsf{c\_2Enum\_2E0})$$
$$(ap\ \mathsf{c\_2Enum\_2ESUC}\ V0n))))$$

$$(67)$$

Theorem 1

$$\forall A\_27a.nonempty\ A\_27a{\Rightarrow}(\forall V0P \in (2^{A\_27a}).(\forall V1ls \in$$
$$(ty\_2Elist\_2Elist\ A\_27a).(\forall V2k \in ty\_2Enum\_2Enum.(((p$$
$$(ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ (ap\ (c\_2Elist\_2ELENGTH\ A\_27a)$$
$$(ap\ (ap\ (c\_2Elist\_2EdropWhile\ A\_27a)\ V0P)\ (ap\ (c\_2Elist\_2EREVERSE$$
$$A\_27a)\ V1ls))))\ V2k)){\wedge}(p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ V2k)\ (ap\ ($$
$$c\_2Elist\_2ELENGTH\ A\_27a)\ V1ls)))){\Rightarrow}(p\ (ap\ V0P\ (ap\ (ap\ (c\_2Elist\_2EEL$$
$$A\_27a)\ V2k)\ V1ls)))))))$$