thm_2Elist_2EEVERY2__mono (TMRoWkx-ENM4QWHUXtvhvrxqeycjUa41GvLm)

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Let $ty\_2Elist\_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Elist\_2Elist\ A0) \tag{1}$$

Let $c\_2Elist\_2ELIST\_\_REL : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Elist\_2ELIST\_\_REL\ A\_27a\ A\_27b \in (((2^{(ty\_2Elist\_2Elist\ A\_27b)})^{(ty\_2Elist\_2Elist\ A\_27a)})^{((2^{A\_27b})^{A\_27a})}) \tag{2}$$

Definition 1 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 2 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 3 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\text{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1$

Definition 4 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\text{c\_2Emin\_2E\_3D}\ (2^{A\_27$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0R1 \in ((2^{A\_27b})^{A\_27a}).(\forall V1R2 \in ((2^{A\_27b})^{A\_27a}).(\forall V2l1 \in (ty\_2Elist\_2Elist\ A\_27a).(\forall V3l2 \in (ty\_2Elist\_2Elist\ A\_27b).((\forall V4x \in A\_27a.(\forall V5y \in A\_27b.((p\ (ap\ (ap\ V0R1\ V4x)\ V5y)) \Rightarrow (p\ (ap\ (ap\ V1R2\ V4x)\ V5y))))) \Rightarrow ((p\ (ap\ (ap\ (ap\ (c\_2Elist\_2ELIST\_\_REL\ A\_27a\ A\_27b)\ V0R1)\ V2l1)\ V3l2)) \Rightarrow (p\ (ap\ (ap\ (ap\ (c\_2Elist\_2ELIST\_\_REL\ A\_27a\ A\_27b)\ V1R2)\ V2l1)\ V3l2)))))))) \tag{3}$$

Theorem 1

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0R1 \in ((2^{A\_27b})^{A\_27a}).(\forall V1R2 \in ((2^{A\_27b})^{A\_27a}).(\forall V2l1 \in (ty\_2Elist\_2Elist\ A\_27a).(\forall V3l2 \in (ty\_2Elist\_2Elist\ A\_27b).((\forall V4x \in A\_27a.(\forall V5y \in A\_27b.((p\ (ap\ (ap\ V0R1\ V4x)\ V5y)) \Rightarrow (p\ (ap\ (ap\ V1R2\ V4x)\ V5y))))) \Rightarrow ((p\ (ap\ (ap\ (ap\ (c\_2Elist\_2ELIST\_\_REL\ A\_27a\ A\_27b)\ V0R1)\ V2l1)\ V3l2)) \Rightarrow (p\ (ap\ (ap\ (ap\ (c\_2Elist\_2ELIST\_\_REL\ A\_27a\ A\_27b)\ V1R2)\ V2l1)\ V3l2)))))))))$$