

thm_2Elist_2EEVERY_APPEND

(TMYzBMe3ym6PgFpFBFye3STPTQsyXiY2hFy)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_2T$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a}))$

Definition 4 We define $c_2Ebool_2E_2F$ to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Let $ty_2Elist_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty_2Elist_2Elist A0) \quad (1)$$

Let $c_2Elist_2EAPPEND : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Elist_2EAPPEND A_27a \in (((ty_2Elist_2Elist A_27a)^{(ty_2Elist_2Elist A_27a)})^{(ty_2Elist_2Elist A_27a)}) \quad (2)$$

Let $c_2Elist_2EEVERY : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Elist_2EEVERY A_27a \in ((2^{(ty_2Elist_2Elist A_27a)})^{(2^{A_27a})}) \quad (3)$$

Let $c_2Elist_2ECONS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Elist_2ECONS A_27a \in (((ty_2Elist_2Elist A_27a)^{(ty_2Elist_2Elist A_27a)})^{A_27a}) \quad (4)$$

Let $c_2Elist_2ENIL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Elist_2ENIL A_27a \in (ty_2Elist_2Elist A_27a) \quad (5)$$

Definition 5 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 6 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.$

Assume the following.

$$True \tag{6}$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A_27a.(p V0t)) \Leftrightarrow (p V0t))) \tag{7}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p V0t1) \wedge (p V1t2) \wedge (p V2t3)) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \wedge (p V2t3)))))) \tag{8}$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \tag{9}$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \tag{10}$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty A_27a \Rightarrow & ((\forall V0l \in (ty_2Elist_2Elist A_27a).((ap (ap (c_2Elist_2EAPPEND A_27a) (c_2Elist_2ENIL A_27a)) \\ & V0l) = V0l) \wedge (\forall V1l1 \in (ty_2Elist_2Elist A_27a).(\forall V2l2 \in \\ & (ty_2Elist_2Elist A_27a).(\forall V3h \in A_27a.((ap (ap (c_2Elist_2EAPPEND \\ & A_27a) (ap (ap (c_2Elist_2ECONS A_27a) V3h) V1l1)) V2l2) = (ap (ap \\ & (c_2Elist_2ECONS A_27a) V3h) (ap (ap (c_2Elist_2EAPPEND A_27a) \\ & V1l1) V2l2)))))))))) \end{aligned} \tag{11}$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty A_27a \Rightarrow & ((\forall V0P \in (2^{A_27a}).((p (ap \\ & (ap (c_2Elist_2EEVERY A_27a) V0P) (c_2Elist_2ENIL A_27a))) \Leftrightarrow True)) \wedge \\ & (\forall V1P \in (2^{A_27a}).(\forall V2h \in A_27a.(\forall V3t \in (ty_2Elist_2Elist \\ & A_27a).((p (ap (ap (c_2Elist_2EEVERY A_27a) V1P) (ap (ap (c_2Elist_2ECONS \\ & A_27a) V2h) V3t))) \Leftrightarrow ((p (ap V1P V2h)) \wedge (p (ap (ap (c_2Elist_2EEVERY \\ & A_27a) V1P) V3t)))))))))) \end{aligned} \tag{12}$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty A_27a \Rightarrow & (\forall V0P \in (2^{(ty_2Elist_2Elist A_27a)}). \\ & (((p (ap V0P (c_2Elist_2ENIL A_27a))) \wedge (\forall V1t \in (ty_2Elist_2Elist \\ & A_27a).((p (ap V0P V1t)) \Rightarrow (\forall V2h \in A_27a.(p (ap V0P (ap (ap (\\ & c_2Elist_2ECONS A_27a) V2h) V1t)))))) \Rightarrow (\forall V3l \in (ty_2Elist_2Elist \\ & A_27a).(p (ap V0P V3l)))))) \end{aligned} \tag{13}$$

Theorem 1

$$\begin{aligned} & \forall A_{27a}. \text{nonempty } A_{27a} \Rightarrow (\forall V0P \in (2^{A_{27a}}). (\forall V1l1 \in \\ & \quad (ty_2Elist_2Elist \ A_{27a}). (\forall V2l2 \in (ty_2Elist_2Elist \ A_{27a}). \\ & ((p \ (ap \ (ap \ (c_2Elist_2EVERY \ A_{27a}) \ V0P) \ (ap \ (ap \ (c_2Elist_2EAPPEND \\ & \quad A_{27a}) \ V1l1) \ V2l2))) \Leftrightarrow ((p \ (ap \ (ap \ (c_2Elist_2EVERY \ A_{27a}) \ V0P) \\ & \quad V1l1)) \wedge (p \ (ap \ (ap \ (c_2Elist_2EVERY \ A_{27a}) \ V0P) \ V2l2)))))) \end{aligned}$$