

thm_2Elist_2EEVERY__CONG
(TMZR23hY6W2ZJeW5t9X2vPKWkcY6ZPvTM5X)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_2T$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a}))$

Definition 4 We define $c_2Ebool_2E_2F$ to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 5 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 6 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_2F$

Let $ty_2Elist_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty_2Elist_2Elist A0) \quad (1)$$

Let $c_2Elist_2EEVERY : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Elist_2EEVERY A_27a \in ((2^{(ty_2Elist_2Elist A_27a)})^{(2^{A_27a})}) \quad (2)$$

Definition 7 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t$

Let $c_2Elist_2ECONS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Elist_2ECONS A_27a \in (((ty_2Elist_2Elist A_27a)^{(ty_2Elist_2Elist A_27a)})^{A_27a}) \quad (3)$$

Let $c_2Elist_2ENIL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Elist_2ENIL A_27a \in (ty_2Elist_2Elist A_27a) \quad (4)$$

Let $c_2Elist_2ELIST_TO_SET : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Elist_2ELIST_TO_SET A_27a \in ((2^{A_27a})^{(ty_2Elist_2Elist A_27a)}) \quad (5)$$

Definition 8 We define c_2Ebool_2EIN to be $\lambda A_27a : \iota. (\lambda V0x \in A_27a. (\lambda V1f \in (2^{A_27a}). (ap\ V1f\ V0x)))$

Definition 9 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in 2.$

Assume the following.

$$True \tag{6}$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \wedge True) \Leftrightarrow \\ & (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge (((p\ V0t) \wedge False) \Leftrightarrow False) \wedge \\ & (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)))))) \end{aligned} \tag{7}$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \vee (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \vee True) \Leftrightarrow True) \wedge \\ & (((False \vee (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \vee False) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \vee \\ & (p\ V0t)) \Leftrightarrow (p\ V0t)))))) \end{aligned} \tag{8}$$

Assume the following.

$$\forall A_27a. nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. ((V0x = V0x) \Leftrightarrow True)) \tag{9}$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow \\ & (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow \neg(p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow \neg(\\ & p\ V0t)))))) \end{aligned} \tag{10}$$

Assume the following.

$$\begin{aligned} & \forall A_27a. nonempty\ A_27a \Rightarrow ((\forall V0P \in (2^{A_27a}). ((p\ (ap \\ & (ap\ (c_2Elist_2EEVERY\ A_27a)\ V0P)\ (c_2Elist_2ENIL\ A_27a))) \Leftrightarrow True)) \wedge \\ & (\forall V1P \in (2^{A_27a}). (\forall V2h \in A_27a. (\forall V3t \in (ty_2Elist_2Elist \\ & A_27a). ((p\ (ap\ (ap\ (c_2Elist_2EEVERY\ A_27a)\ V1P)\ (ap\ (ap\ (c_2Elist_2ECONS \\ & A_27a)\ V2h)\ V3t))) \Leftrightarrow ((p\ (ap\ V1P\ V2h)) \wedge (p\ (ap\ (ap\ (c_2Elist_2EEVERY \\ & A_27a)\ V1P)\ V3t))))))) \end{aligned} \tag{11}$$

Assume the following.

$$\begin{aligned} & \forall A_27a. nonempty\ A_27a \Rightarrow (\forall V0P \in (2^{(ty_2Elist_2Elist\ A_27a)}). \\ & (((p\ (ap\ V0P\ (c_2Elist_2ENIL\ A_27a))) \wedge (\forall V1t \in (ty_2Elist_2Elist \\ & A_27a). ((p\ (ap\ V0P\ V1t)) \Rightarrow (\forall V2h \in A_27a. (p\ (ap\ V0P\ (ap\ (ap\ (\\ & c_2Elist_2ECONS\ A_27a)\ V2h)\ V1t)))))) \Rightarrow (\forall V3l \in (ty_2Elist_2Elist \\ & A_27a). (p\ (ap\ V0P\ V3l)))))) \end{aligned} \tag{12}$$

Assume the following.

$$\begin{aligned}
& \forall A_{.27a}. \text{nonempty } A_{.27a} \Rightarrow ((\forall V0x \in A_{.27a}. ((p (ap (ap \\
& (c_2Ebool_2EIN A_{.27a}) V0x) (ap (c_2Elist_2ELIST_TO_SET A_{.27a}) \\
& (c_2Elist_2ENIL A_{.27a})))) \Leftrightarrow \text{False})) \wedge (\forall V1x \in A_{.27a}. (\forall V2h \in \\
& A_{.27a}. (\forall V3t \in (ty_2Elist_2Elist A_{.27a}). ((p (ap (ap (c_2Ebool_2EIN \\
& A_{.27a}) V1x) (ap (c_2Elist_2ELIST_TO_SET A_{.27a}) (ap (ap (c_2Elist_2ECONS \\
& A_{.27a}) V2h) V3t)))) \Leftrightarrow ((V1x = V2h) \vee (p (ap (ap (c_2Ebool_2EIN A_{.27a}) \\
& V1x) (ap (c_2Elist_2ELIST_TO_SET A_{.27a}) V3t))))))))))
\end{aligned} \tag{13}$$

Theorem 1

$$\begin{aligned}
& \forall A_{.27a}. \text{nonempty } A_{.27a} \Rightarrow (\forall V0l1 \in (ty_2Elist_2Elist \\
& A_{.27a}). (\forall V1l2 \in (ty_2Elist_2Elist A_{.27a}). (\forall V2P \in \\
& (2^{A_{.27a}}). (\forall V3P_{.27} \in (2^{A_{.27a}}). (((V0l1 = V1l2) \wedge (\forall V4x \in \\
& A_{.27a}. ((p (ap (ap (c_2Ebool_2EIN A_{.27a}) V4x) (ap (c_2Elist_2ELIST_TO_SET \\
& A_{.27a}) V1l2)))) \Rightarrow ((p (ap V2P V4x)) \Leftrightarrow (p (ap V3P_{.27} V4x)))))) \Rightarrow ((p (ap \\
& (ap (c_2Elist_2EEVERY A_{.27a}) V2P) V0l1)) \Leftrightarrow (p (ap (ap (c_2Elist_2EEVERY \\
& A_{.27a}) V3P_{.27} V1l2))))))))))
\end{aligned}$$