

# thm\_2Elist\_2EEVERY\_\_EL (TM- LVtwyVsTHx2VojQMDdor3hF7NeQYVcf4x)

October 26, 2020

**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Ebool\_2ET$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

**Definition 3** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap (ap (c\_2Emin\_2E\_3D (2^{A\_27a})) (\lambda V1x \in 2.V1x)) (\lambda V2x \in 2.V2x)))$

**Definition 4** We define  $c\_2Ebool\_2EF$  to be  $(ap (c\_2Ebool\_2E\_21 2) (\lambda V0t \in 2.V0t))$ .

Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{1}$$

Let  $ty\_2Elist\_2Elist : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Elist\_2Elist\ A0) \tag{2}$$

Let  $c\_2Elist\_2ELENGTH : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ELENGTH\ A\_27a \in (ty\_2Enum\_2Enum^{(ty\_2Elist\_2Elist\ A\_27a)}) \tag{3}$$

Let  $c\_2Elist\_2EEVERY : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2EEVERY\ A\_27a \in ((2^{(ty\_2Elist\_2Elist\ A\_27a)})^{(2^{A\_27a})}) \tag{4}$$

Let  $c\_2Elist\_2ETL : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ETL\ A\_27a \in ((ty\_2Elist\_2Elist\ A\_27a)^{(ty\_2Elist\_2Elist\ A\_27a)}) \tag{5}$$

Let  $c\_2Elist\_2EHD : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2EHD\ A\_27a \in (A\_27a^{(ty\_2Elist\_2Elist\ A\_27a)}) \tag{6}$$

Let  $c\_2Elist\_2EEL : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2EEL\ A\_27a \in ((A\_27a^{(ty\_2Elist\_2Elist\ A\_27a)})^{ty\_2Enum\_2Enum}) \quad (7)$$

Let  $c\_2Elist\_2ECONS : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ECONS\ A\_27a \in (((ty\_2Elist\_2Elist\ A\_27a)^{(ty\_2Elist\_2Elist\ A\_27a)})^{A\_27a}) \quad (8)$$

Let  $c\_2Elist\_2ENIL : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ENIL\ A\_27a \in (ty\_2Elist\_2Elist\ A\_27a) \quad (9)$$

**Definition 5** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o\ (p\ P \Rightarrow p\ Q)$  of type  $\iota$ .

**Definition 6** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c\_2Ebool\_2E\_21\ 2)\ (\lambda V2t \in 2.$

**Definition 7** We define  $c\_2Ebool\_2E\_7E$  to be  $(\lambda V0t \in 2.(ap\ (ap\ c\_2Emin\_2E\_3D\_3D\_3E\ V0t)\ c\_2Ebool\_2E\_7E$

Let  $c\_2Enum\_2EZERO\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2EZERO\_REP \in \omega \quad (10)$$

Let  $c\_2Enum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum^{\omega}) \quad (11)$$

**Definition 8** We define  $c\_2Enum\_2E0$  to be  $(ap\ c\_2Enum\_2EABS\_num\ c\_2Enum\_2EZERO\_REP)$ .

Let  $c\_2Enum\_2EREP\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EREP\_num \in (\omega^{ty\_2Enum\_2Enum}) \quad (12)$$

Let  $c\_2Enum\_2ESUC\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2ESUC\_REP \in (\omega^{\omega}) \quad (13)$$

**Definition 9** We define  $c\_2Enum\_2ESUC$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_num\ (ap\ c\_2Enum\_2EREP\_num\ m))$

**Definition 10** We define  $c\_2Emin\_2E\_40$  to be  $\lambda A.\lambda P \in 2^A.if\ (\exists x \in A.p\ (ap\ P\ x))\ then\ (the\ (\lambda x.x \in A)\ P)$  of type  $\iota \Rightarrow \iota$ .

**Definition 11** We define  $c\_2Ebool\_2E\_3F$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ V0P\ (ap\ (c\_2Emin\_2E\_40\ A\_27a)\ P)))$

**Definition 12** We define  $c\_2Eprim\_rec\_2E\_3C$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum.(ap\ c\_2Emin\_2E\_40\ (ap\ (ap\ c\_2Ebool\_2E\_3F\ V0m)\ V1n))$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. (p (ap (ap (ap c\_2Eprim\_rec\_2E\_3C (ap c\_2Enum\_2ESUC V0m)) (ap c\_2Enum\_2ESUC V1n))) \Leftrightarrow (p (ap (ap c\_2Eprim\_rec\_2E\_3C V0m) V1n)))))) \quad (14)$$

Assume the following.

$$True \quad (15)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \quad (16)$$

Assume the following.

$$(\forall V0t \in 2. (False \Rightarrow (p V0t))) \quad (17)$$

Assume the following.

$$\forall A\_27a. nonempty A\_27a \Rightarrow (\forall V0t \in 2. ((\forall V1x \in A\_27a. (p V0t)) \Leftrightarrow (p V0t))) \quad (18)$$

Assume the following.

$$(\forall V0t \in 2. (((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge ((p V0t) \Rightarrow False) \Leftrightarrow (\neg (p V0t)))))) \quad (19)$$

Assume the following.

$$(\forall V0t \in 2. (((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg (p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg (p V0t)))))) \quad (20)$$

Assume the following.

$$\forall A\_27a. nonempty A\_27a \Rightarrow (\forall V0h \in A\_27a. (\forall V1t \in (ty\_2Elist\_2Elist A\_27a). ((ap (c\_2Elist\_2EHD A\_27a) (ap (ap (c\_2Elist\_2ECONS A\_27a) V0h) V1t)) = V0h))) \quad (21)$$

Assume the following.

$$\forall A\_27a. nonempty A\_27a \Rightarrow (\forall V0h \in A\_27a. (\forall V1t \in (ty\_2Elist\_2Elist A\_27a). ((ap (c\_2Elist\_2ETL A\_27a) (ap (ap (c\_2Elist\_2ECONS A\_27a) V0h) V1t)) = V1t))) \quad (22)$$

Assume the following.

$$\forall A\_27a. nonempty A\_27a \Rightarrow (((ap (c\_2Elist\_2ELENGTH A\_27a) (c\_2Elist\_2ENIL A\_27a)) = c\_2Enum\_2E0) \wedge (\forall V0h \in A\_27a. (\forall V1t \in (ty\_2Elist\_2Elist A\_27a). ((ap (c\_2Elist\_2ELENGTH A\_27a) (ap (ap (c\_2Elist\_2ECONS A\_27a) V0h) V1t)) = (ap c\_2Enum\_2ESUC (ap (c\_2Elist\_2ELENGTH A\_27a) V1t)))))) \quad (23)$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow ((\forall V0P \in (2^{A.27a}).((p\ (ap \\
& (ap\ (c.2Elist.2EVERY\ A.27a)\ V0P)\ (c.2Elist.2ENIL\ A.27a))) \Leftrightarrow True)) \wedge \\
& (\forall V1P \in (2^{A.27a}).(\forall V2h \in A.27a.(\forall V3t \in (ty.2Elist.2Elist \\
& A.27a).((p\ (ap\ (ap\ (c.2Elist.2EVERY\ A.27a)\ V1P)\ (ap\ (ap\ (c.2Elist.2ECONS \\
& A.27a)\ V2h)\ V3t))) \Leftrightarrow ((p\ (ap\ V1P\ V2h)) \wedge (p\ (ap\ (ap\ (c.2Elist.2EVERY \\
& A.27a)\ V1P)\ V3t)))))))))
\end{aligned} \tag{24}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow ((\forall V0l \in (ty.2Elist.2Elist \\
& A.27a).((ap\ (ap\ (c.2Elist.2EEL\ A.27a)\ c.2Enum.2E0)\ V0l) = (ap\ ( \\
& c.2Elist.2EHD\ A.27a)\ V0l))) \wedge (\forall V1l \in (ty.2Elist.2Elist \\
& A.27a).(\forall V2n \in ty.2Enum.2Enum.((ap\ (ap\ (c.2Elist.2EEL \\
& A.27a)\ (ap\ c.2Enum.2ESUC\ V2n))\ V1l) = (ap\ (ap\ (c.2Elist.2EEL\ A.27a) \\
& V2n)\ (ap\ (c.2Elist.2ETL\ A.27a)\ V1l))))))
\end{aligned} \tag{25}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0P \in (2^{(ty.2Elist.2Elist\ A.27a)}). \\
& (((p\ (ap\ V0P\ (c.2Elist.2ENIL\ A.27a))) \wedge (\forall V1t \in (ty.2Elist.2Elist \\
& A.27a).((p\ (ap\ V0P\ V1t)) \Rightarrow (\forall V2h \in A.27a.(p\ (ap\ V0P\ (ap\ (ap\ ( \\
& c.2Elist.2ECONS\ A.27a)\ V2h)\ V1t)))))) \Rightarrow (\forall V3l \in (ty.2Elist.2Elist \\
& A.27a).(p\ (ap\ V0P\ V3l))))))
\end{aligned} \tag{26}$$

Assume the following.

$$\begin{aligned}
& (\forall V0P \in (2^{ty.2Enum.2Enum}).(((p\ (ap\ V0P\ c.2Enum.2E0)) \wedge \\
& (\forall V1n \in ty.2Enum.2Enum.((p\ (ap\ V0P\ V1n)) \Rightarrow (p\ (ap\ V0P\ (ap\ c.2Enum.2ESUC \\
& V1n)))))) \Rightarrow (\forall V2n \in ty.2Enum.2Enum.(p\ (ap\ V0P\ V2n))))))
\end{aligned} \tag{27}$$

Assume the following.

$$(\forall V0n \in ty.2Enum.2Enum.(\neg(p\ (ap\ (ap\ c.2Eprim\_rec.2E.3C\ V0n)\ c.2Enum.2E0)))) \tag{28}$$

Assume the following.

$$(\forall V0n \in ty.2Enum.2Enum.(p\ (ap\ (ap\ c.2Eprim\_rec.2E.3C\ c.2Enum.2E0) \\
(ap\ c.2Enum.2ESUC\ V0n)))) \tag{29}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty.2Enum.2Enum.(\forall V1n \in ty.2Enum.2Enum.( \\
& (p\ (ap\ (ap\ c.2Eprim\_rec.2E.3C\ V0m)\ V1n)) \Rightarrow (p\ (ap\ (ap\ c.2Eprim\_rec.2E.3C \\
& (ap\ c.2Enum.2ESUC\ V0m))\ (ap\ c.2Enum.2ESUC\ V1n))))))
\end{aligned} \tag{30}$$

**Theorem 1**

$$\begin{aligned} & \forall A_{27a}. \text{nonempty } A_{27a} \Rightarrow (\forall V0l \in (ty\_2Elist\_2Elist \\ & A_{27a}). (\forall V1P \in (2^{A_{27a}}). ((p (ap (ap (c\_2Elist\_2EVERY \\ & A_{27a}) V1P) V0l)) \Leftrightarrow (\forall V2n \in ty\_2Enum\_2Enum. ((p (ap (ap c\_2Eprim\_rec\_2E\_3C \\ & V2n) (ap (c\_2Elist\_2ELENGTH A_{27a}) V0l))) \Rightarrow (p (ap V1P (ap (ap (c\_2Elist\_2EEL \\ & A_{27a}) V2n) V0l)))))))))) \end{aligned}$$