



Let  $c\_2Elist\_2ECONS : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ECONS\ A\_27a \in (((ty\_2Elist\_2Elist\ A\_27a)^{(ty\_2Elist\_2Elist\ A\_27a)})_{A\_27a}) \quad (4)$$

Let  $c\_2Elist\_2ENIL : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ENIL\ A\_27a \in (ty\_2Elist\_2Elist\ A\_27a) \quad (5)$$

Assume the following.

$$True \quad (6)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \quad (7)$$

Assume the following.

$$(\forall V0t \in 2. (False \Rightarrow (p\ V0t))) \quad (8)$$

Assume the following.

$$(\forall V0t \in 2. ((p\ V0t) \vee (\neg(p\ V0t)))) \quad (9)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t \in 2. ((\forall V1x \in A\_27a. (p\ V0t)) \Leftrightarrow (p\ V0t))) \quad (10)$$

Assume the following.

$$(\forall V0t \in 2. (((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \wedge True) \Leftrightarrow (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge (((p\ V0t) \wedge False) \Leftrightarrow False) \wedge (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)))))) \quad (11)$$

Assume the following.

$$(\forall V0t \in 2. (((True \vee (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \vee True) \Leftrightarrow True) \wedge (((False \vee (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \vee False) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \vee (p\ V0t)) \Leftrightarrow (p\ V0t)))))) \quad (12)$$

Assume the following.

$$(\forall V0t \in 2. (((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow False) \Leftrightarrow (\neg(p\ V0t)))))) \quad (13)$$

Assume the following.

$$((\forall V0t \in 2. ((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t)) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True)))) \quad (14)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a. ((V0x = V0x) \Leftrightarrow True)) \quad (15)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a. (\forall V1y \in A\_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (16)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow \\ & (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow \neg(p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow \neg( \\ & p\ V0t)))))) \end{aligned} \quad (17)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t1 \in A\_27a. (\forall V1t2 \in \\ & A\_27a. (((ap\ (ap\ (ap\ (c\_2Ebool\_2ECOND\ A\_27a)\ c\_2Ebool\_2ET)\ V0t1) \\ & V1t2) = V0t1) \wedge ((ap\ (ap\ (ap\ (c\_2Ebool\_2ECOND\ A\_27a)\ c\_2Ebool\_2EF) \\ & V0t1)\ V1t2) = V1t2)))) \end{aligned} \quad (18)$$

Assume the following.

$$\begin{aligned} & (\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p\ V0t1) \Rightarrow \\ & ((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \end{aligned} \quad (19)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\ & \forall V0b \in 2. (\forall V1f \in (A\_27b^{A\_27a}). (\forall V2g \in (A\_27b^{A\_27a}). \\ & (\forall V3x \in A\_27a. ((ap\ (ap\ (ap\ (ap\ (c\_2Ebool\_2ECOND\ (A\_27b^{A\_27a}) \\ & V0b)\ V1f)\ V2g)\ V3x) = (ap\ (ap\ (ap\ (c\_2Ebool\_2ECOND\ A\_27b)\ V0b)\ (ap \\ & V1f\ V3x))\ (ap\ V2g\ V3x)))))) \end{aligned} \quad (20)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\ & \forall V0f \in (A\_27b^{A\_27a}). (\forall V1b \in 2. (\forall V2x \in A\_27a. \\ & (\forall V3y \in A\_27a. ((ap\ V0f\ (ap\ (ap\ (ap\ (c\_2Ebool\_2ECOND\ A\_27a) \\ & V1b)\ V2x)\ V3y) = (ap\ (ap\ (ap\ (c\_2Ebool\_2ECOND\ A\_27b)\ V1b)\ (ap\ V0f \\ & V2x))\ (ap\ V0f\ V3y)))))) \end{aligned} \quad (21)$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in 2. (\forall V1x\_27 \in 2. (\forall V2y \in 2. (\forall V3y\_27 \in \\ & 2. (((p\ V0x) \Leftrightarrow (p\ V1x\_27)) \wedge ((p\ V1x\_27) \Rightarrow ((p\ V2y) \Leftrightarrow (p\ V3y\_27)))))) \Rightarrow \\ & (((p\ V0x) \Rightarrow (p\ V2y)) \Leftrightarrow ((p\ V1x\_27) \Rightarrow (p\ V3y\_27)))))) \end{aligned} \quad (22)$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in 2. \\
& (\forall V2x \in A\_27a. (\forall V3x\_27 \in A\_27a. (\forall V4y \in A\_27a. \\
& (\forall V5y\_27 \in A\_27a. (((p\ V0P) \Leftrightarrow (p\ V1Q)) \wedge ((p\ V1Q) \Rightarrow (V2x = V3x\_27)) \wedge \\
& ((\neg(p\ V1Q)) \Rightarrow (V4y = V5y\_27)))))) \Rightarrow ((ap\ (ap\ (ap\ (c\_2Ebool\_2ECOND\ A\_27a) \\
& V0P)\ V2x)\ V4y) = (ap\ (ap\ (ap\ (c\_2Ebool\_2ECOND\ A\_27a)\ V1Q)\ V3x\_27) \\
& V5y\_27)))))))))
\end{aligned} \tag{23}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow ((\forall V0P \in (2^{A\_27a}). ((ap\ ( \\
& ap\ (c\_2Elist\_2EFILTER\ A\_27a)\ V0P)\ (c\_2Elist\_2ENIL\ A\_27a)) = (c\_2Elist\_2ENIL \\
& A\_27a))) \wedge (\forall V1P \in (2^{A\_27a}). (\forall V2h \in A\_27a. (\forall V3t \in \\
& (ty\_2Elist\_2Elist\ A\_27a). ((ap\ (ap\ (c\_2Elist\_2EFILTER\ A\_27a) \\
& V1P)\ (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V2h)\ V3t)) = (ap\ (ap\ (ap\ (c\_2Ebool\_2ECOND \\
& (ty\_2Elist\_2Elist\ A\_27a))\ (ap\ V1P\ V2h))\ (ap\ (ap\ (c\_2Elist\_2ECONS \\
& A\_27a)\ V2h)\ (ap\ (ap\ (c\_2Elist\_2EFILTER\ A\_27a)\ V1P)\ V3t))))\ (ap\ (ap \\
& (c\_2Elist\_2EFILTER\ A\_27a)\ V1P)\ V3t)))))))))
\end{aligned} \tag{24}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow ((\forall V0P \in (2^{A\_27a}). ((p\ (ap \\
& (ap\ (c\_2Elist\_2EEVERY\ A\_27a)\ V0P)\ (c\_2Elist\_2ENIL\ A\_27a))) \Leftrightarrow True)) \wedge \\
& (\forall V1P \in (2^{A\_27a}). (\forall V2h \in A\_27a. (\forall V3t \in (ty\_2Elist\_2Elist \\
& A\_27a). ((p\ (ap\ (ap\ (c\_2Elist\_2EEVERY\ A\_27a)\ V1P)\ (ap\ (ap\ (c\_2Elist\_2ECONS \\
& A\_27a)\ V2h)\ V3t))) \Leftrightarrow ((p\ (ap\ V1P\ V2h)) \wedge (p\ (ap\ (ap\ (c\_2Elist\_2EEVERY \\
& A\_27a)\ V1P)\ V3t)))))))))
\end{aligned} \tag{25}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in (2^{(ty\_2Elist\_2Elist\ A\_27a)}). \\
& (((p\ (ap\ V0P\ (c\_2Elist\_2ENIL\ A\_27a))) \wedge (\forall V1t \in (ty\_2Elist\_2Elist \\
& A\_27a). ((p\ (ap\ V0P\ V1t)) \Rightarrow (\forall V2h \in A\_27a. (p\ (ap\ V0P\ (ap\ (ap\ ( \\
& c\_2Elist\_2ECONS\ A\_27a)\ V2h)\ V1t)))))) \Rightarrow (\forall V3l \in (ty\_2Elist\_2Elist \\
& A\_27a). (p\ (ap\ V0P\ V3l))))))
\end{aligned} \tag{26}$$

### Theorem 1

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P1 \in (2^{A\_27a}). (\forall V1P2 \in \\
& (2^{A\_27a}). (\forall V2l \in (ty\_2Elist\_2Elist\ A\_27a). ((p\ (ap\ (ap \\
& (c\_2Elist\_2EEVERY\ A\_27a)\ V0P1)\ (ap\ (ap\ (c\_2Elist\_2EFILTER\ A\_27a) \\
& V1P2)\ V2l))) \Leftrightarrow (p\ (ap\ (ap\ (c\_2Elist\_2EEVERY\ A\_27a)\ (\lambda V3x \in A\_27a. \\
& (ap\ (ap\ c\_2Emin\_2E\_3D\_3D\_3E\ (ap\ V1P2\ V3x))\ (ap\ V0P1\ V3x))))\ V2l))))))
\end{aligned}$$