thm_2Elist_2EEVERY__GENLIST (TMErykbkc8UvS9TrREggjrdqnCZbiHvetEb)

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Let $ty\_2Elist\_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0. nonempty\ A0 \Rightarrow nonempty\ (ty\_2Elist\_2Elist\ A0) \qquad (1)$$

Let $c\_2Elist\_2ECONS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a. nonempty\ A\_27a \Rightarrow c\_2Elist\_2ECONS\ A\_27a \in (((ty\_2Elist\_2Elist$$
$$A\_27a)^{(ty\_2Elist\_2Elist\ A\_27a)})^{A\_27a})$$
$$\qquad (2)$$

Definition 1 *We define* $\mathsf{c\_2Emin\_2E\_3D}$ *to be* $\lambda A. \lambda x \in A. \lambda y \in A. inj\_\_o\ (x = y)$
of type $\iota \Rightarrow \iota$.

Definition 2 *We define* $\mathsf{c\_2Ebool\_2ET}$ *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2. V0x))\ (\lambda V1x \in 2. V1x$

Let $c\_2Elist\_2EEVERY : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a. nonempty\ A\_27a \Rightarrow c\_2Elist\_2EEVERY\ A\_27a \in ((2^{(ty\_2Elist\_2Elist\ A\_27a)})^{(2^{A\_27a})})$$
$$\qquad (3)$$

Let $c\_2Elist\_2ESNOC : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a. nonempty\ A\_27a \Rightarrow c\_2Elist\_2ESNOC\ A\_27a \in (((ty\_2Elist\_2Elist$$
$$A\_27a)^{(ty\_2Elist\_2Elist\ A\_27a)})^{A\_27a})$$
$$\qquad (4)$$

Let $c\_2Elist\_2ENIL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a. nonempty\ A\_27a \Rightarrow c\_2Elist\_2ENIL\ A\_27a \in (ty\_2Elist\_2Elist$$
$$A\_27a)$$
$$\qquad (5)$$

Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \qquad (6)$$

Let $c\_2Elist\_2EGENLIST : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a. nonempty\ A\_27a \Rightarrow c\_2Elist\_2EGENLIST\ A\_27a \in (((ty\_2Elist\_2Elist$$
$$A\_27a)^{ty\_2Enum\_2Enum})^{(A\_27a^{ty\_2Enum\_2Enum})})$$
$$\qquad (7)$$

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Let $c\_2Enum\_2EZERO\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2EZERO\_\_REP \in omega \qquad (8)$$

Let $c\_2Enum\_2EABS\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EABS\_\_num \in (ty\_2Enum\_2Enum^{omega}) \qquad (9)$$

Definition 3 *We define* c_2Enum_2E0 *to be* $(ap\ c\_2Enum\_2EABS\_\_num\ c\_2Enum\_2EZERO\_\_REP)$.

Let $c\_2Enum\_2EREP\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EREP\_\_num \in (omega^{ty\_2Enum\_2Enum}) \qquad (10)$$

Let $c\_2Enum\_2ESUC\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2ESUC\_\_REP \in (omega^{omega}) \qquad (11)$$

Definition 4 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ ($c_2Emin_2E_3D$\ (2^{A\_27a}$

Definition 5 *We define* c_2Enum_2ESUC *to be* $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_\_num\ ($

Definition 6 *We define* c_2Ebool_2EF *to be* $(ap\ ($c_2Ebool_2E_21$\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 7 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P{\Rightarrow}p\ Q)$
of type ι.

Definition 8 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap\ $c_2Emin_2E_3D_3D_3E$\ V0t)\ $c_2Ebool_2EF

Definition 9 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ ($c_2Ebool_2E_21$\ 2)\ (\lambda V2t \in 2$

Definition 10 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.$**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A{\wedge}$
of type $\iota{\Rightarrow}\iota$.

Definition 11 *We define* c_2Ebool_2E_3F *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ V0P\ (ap\ ($c_2Emin_2E_40

Definition 12 *We define* c_2Eprim__rec_2E_3C *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enu$

Definition 13 *We define* c_2Ebool_2E_5C_2F *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ ($c_2Ebool_2E_21$\ 2)\ (\lambda V2t \in$

Assume the following.

$$True \qquad (12)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1){\Rightarrow}(p\ V1t2)){\Rightarrow}(((p \\ V1t2){\Rightarrow}(p\ V0t1)){\Rightarrow}((p\ V0t1){\Leftrightarrow}(p\ V1t2)))))) \qquad (13)$$

Assume the following.

$$(\forall V0t \in 2.(False{\Rightarrow}(p\ V0t))) \qquad (14)$$

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Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in \\ A\_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \tag{15}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Rightarrow True) \Leftrightarrow \\ True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((\\ (p\ V0t) \Rightarrow False) \Leftrightarrow (\neg (p\ V0t)))))))) \tag{16}$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge \\ ((\neg False) \Leftrightarrow True))) \tag{17}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(V0x = V0x)) \tag{18}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in \\ A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \tag{19}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Leftrightarrow True) \Leftrightarrow \\ (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg(p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg(\\ p\ V0t))))))) \tag{20}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.(((p\ V0A) \vee (\\ (p\ V1B) \wedge (p\ V2C))) \Leftrightarrow (((p\ V0A) \vee (p\ V1B)) \wedge ((p\ V0A) \vee (p\ V2C))))))) \tag{21}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p\ V0t1) \Rightarrow \\ ((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \tag{22}$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1x\_27 \in 2.(\forall V2y \in 2.(\forall V3y\_27 \in \\ 2.((((p\ V0x) \Leftrightarrow (p\ V1x\_27)) \wedge ((p\ V1x\_27) \Rightarrow ((p\ V2y) \Leftrightarrow (p\ V3y\_27)))) \Rightarrow \\ (((p\ V0x) \Rightarrow (p\ V2y)) \Leftrightarrow ((p\ V1x\_27) \Rightarrow (p\ V3y\_27))))))))) \tag{23}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow ((\forall V0P \in (2^{A\_27a}).((p\ (ap \\ (ap\ (c\_2Elist\_2EEVERY\ A\_27a)\ V0P)\ (c\_2Elist\_2ENIL\ A\_27a))) \Leftrightarrow True)) \wedge \\ (\forall V1P \in (2^{A\_27a}).(\forall V2h \in A\_27a.(\forall V3t \in (ty\_2Elist\_2Elist \\ A\_27a).((p\ (ap\ (ap\ (c\_2Elist\_2EEVERY\ A\_27a)\ V1P)\ (ap\ (ap\ (c\_2Elist\_2ECONS \\ A\_27a)\ V2h)\ V3t))) \Leftrightarrow ((p\ (ap\ V1P\ V2h)) \wedge (p\ (ap\ (ap\ (c\_2Elist\_2EEVERY \\ A\_27a)\ V1P)\ V3t))))))))) \tag{24}$$

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Assume the following.

$$\forall A\_27a.nonempty~A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}).(\forall V1x \in$$
$$A\_27a.(\forall V2l \in (ty\_2Elist\_2Elist~A\_27a).((p~(ap~(ap~(c\_2Elist\_2EEVERY$$
$$A\_27a)~V0P)~(ap~(ap~(c\_2Elist\_2ESNOC~A\_27a)~V1x)~V2l))) \Leftrightarrow ((p~(ap$$
$$(ap~(c\_2Elist\_2EEVERY~A\_27a)~V0P)~V2l)) \wedge (p~(ap~V0P~V1x)))))))$$
$$\tag{25}$$

Assume the following.

$$\forall A\_27a.nonempty~A\_27a \Rightarrow ((\forall V0f \in (A\_27a^{ty\_2Enum\_2Enum}).$$
$$((ap~(ap~(c\_2Elist\_2EGENLIST~A\_27a)~V0f)~\mathsf{c\_2Enum\_2E0}) = (c\_2Elist\_2ENIL$$
$$A\_27a))) \wedge (\forall V1f \in (A\_27a^{ty\_2Enum\_2Enum}).(\forall V2n \in$$
$$ty\_2Enum\_2Enum.((ap~(ap~(c\_2Elist\_2EGENLIST~A\_27a)~V1f)~(ap~\mathsf{c\_2Enum\_2ESUC}$$
$$V2n)) = (ap~(ap~(c\_2Elist\_2ESNOC~A\_27a)~(ap~V1f~V2n))~(ap~(ap~(c\_2Elist\_2EGENLIST$$
$$A\_27a)~V1f)~V2n))))))$$
$$\tag{26}$$

Assume the following.

$$(\forall V0P \in (2^{ty\_2Enum\_2Enum}).(((p~(ap~V0P~\mathsf{c\_2Enum\_2E0})) \wedge$$
$$(\forall V1n \in ty\_2Enum\_2Enum.((p~(ap~V0P~V1n)) \Rightarrow (p~(ap~V0P~(ap~\mathsf{c\_2Enum\_2ESUC}$$
$$V1n)))))) \Rightarrow (\forall V2n \in ty\_2Enum\_2Enum.(p~(ap~V0P~V2n)))))$$
$$\tag{27}$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.(\neg(p~(ap~(ap~\mathsf{c\_2Eprim\_\_rec\_2E\_3C}$$
$$V0n)~\mathsf{c\_2Enum\_2E0}))))$$
$$\tag{28}$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.($$
$$(p~(ap~(ap~\mathsf{c\_2Eprim\_\_rec\_2E\_3C}~V0m)~(ap~\mathsf{c\_2Enum\_2ESUC}~V1n))) \Leftrightarrow ($$
$$(V0m = V1n) \vee (p~(ap~(ap~\mathsf{c\_2Eprim\_\_rec\_2E\_3C}~V0m)~V1n))))))$$
$$\tag{29}$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p~V0t))) \Leftrightarrow (p~V0t)))$$
$$\tag{30}$$

Assume the following.

$$(\forall V0A \in 2.((p~V0A) \Rightarrow ((\neg(p~V0A)) \Rightarrow False)))$$
$$\tag{31}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p~V0A) \vee (p~V1B))) \Rightarrow False) \Leftrightarrow$$
$$(((p~V0A) \Rightarrow False) \Rightarrow ((\neg(p~V1B)) \Rightarrow False)))))$$
$$\tag{32}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p~V0A)) \vee (p~V1B))) \Rightarrow False) \Leftrightarrow$$
$$((p~V0A) \Rightarrow ((\neg(p~V1B)) \Rightarrow False)))))$$
$$\tag{33}$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p\ V0A))\Rightarrow False)\Rightarrow(((p\ V0A)\Rightarrow False)\Rightarrow False))) \tag{34}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p)\Leftrightarrow($$
$$(p\ V1q)\Leftrightarrow(p\ V2r)))\Leftrightarrow(((p\ V0p)\vee((p\ V1q)\vee(p\ V2r)))\wedge(((p\ V0p)\vee((\neg($$
$$p\ V2r))\vee(\neg(p\ V1q))))\wedge(((p\ V1q)\vee((\neg(p\ V2r))\vee(\neg(p\ V0p))))\wedge((p\ V2r)\vee$$
$$((\neg(p\ V1q))\vee(\neg(p\ V0p)))))))))))) \tag{35}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p)\Leftrightarrow($$
$$(p\ V1q)\wedge(p\ V2r)))\Leftrightarrow(((p\ V0p)\vee((\neg(p\ V1q))\vee(\neg(p\ V2r))))\wedge(((p\ V1q)\vee$$
$$(\neg(p\ V0p)))\wedge((p\ V2r)\vee(\neg(p\ V0p)))))))))) \tag{36}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p)\Leftrightarrow($$
$$(p\ V1q)\vee(p\ V2r)))\Leftrightarrow(((p\ V0p)\vee(\neg(p\ V1q)))\wedge(((p\ V0p)\vee(\neg(p\ V2r)))\wedge$$
$$((p\ V1q)\vee((p\ V2r)\vee(\neg(p\ V0p)))))))))) \tag{37}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p)\Leftrightarrow($$
$$(p\ V1q)\Rightarrow(p\ V2r)))\Leftrightarrow(((p\ V0p)\vee(p\ V1q))\wedge(((p\ V0p)\vee(\neg(p\ V2r)))\wedge((\tag{38}$$
$$\neg(p\ V1q))\vee((p\ V2r)\vee(\neg(p\ V0p)))))))))$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((p\ V0p)\Leftrightarrow(\neg(p\ V1q)))\Leftrightarrow(((p\ V0p)\vee$$
$$(p\ V1q))\wedge((\neg(p\ V1q))\vee(\neg(p\ V0p))))))) \tag{39}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p\ V0p)\Rightarrow(p\ V1q)))\Rightarrow(p\ V0p)))) \tag{40}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p\ V0p)\Rightarrow(p\ V1q)))\Rightarrow(\neg(p\ V1q))))) \tag{41}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p\ V0p)\vee(p\ V1q)))\Rightarrow(\neg(p\ V0p))))) \tag{42}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p\ V0p)\vee(p\ V1q)))\Rightarrow(\neg(p\ V1q))))) \tag{43}$$

Assume the following.

$$(\forall V0p \in 2.((\neg(\neg(p\ V0p)))\Rightarrow(p\ V0p))) \tag{44}$$

Theorem 1

$$\forall A\_27a.nonempty \ A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}).(\forall V1f \in$$
$$(A\_27a^{ty\_2Enum\_2Enum}).(\forall V2n \in ty\_2Enum\_2Enum.((p \ (ap$$
$$(ap \ (c\_2Elist\_2EEVERY \ A\_27a) \ V0P) \ (ap \ (ap \ (c\_2Elist\_2EGENLIST$$
$$A\_27a) \ V1f) \ V2n))) \Leftrightarrow (\forall V3i \in ty\_2Enum\_2Enum.((p \ (ap \ (ap \ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}$$
$$V3i) \ V2n)) \Rightarrow (p \ (ap \ V0P \ (ap \ V1f \ V3i)))))))))$$