

thm\_2Elist\_2EEVERY\_\_SNOC  
(TMaWCWZY8hNHPaCEbeQy5JAuW1BujBZE6zq)

October 26, 2020

**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Ebool\_2ET$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

**Definition 3** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap (ap (c\_2Emin\_2E\_3D (2^{A\_27a}))$

**Definition 4** We define  $c\_2Ebool\_2EF$  to be  $(ap (c\_2Ebool\_2E\_21 2) (\lambda V0t \in 2.V0t))$ .

Let  $ty\_2Elist\_2Elist : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty\_2Elist\_2Elist A0) \quad (1)$$

Let  $c\_2Elist\_2EEVERY : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_2Elist\_2EEVERY A\_27a \in ((2^{(ty\_2Elist\_2Elist A\_27a)})^{(2^{A\_27a})}) \quad (2)$$

**Definition 5** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p P \Rightarrow p Q)$  of type  $\iota$ .

Let  $c\_2Elist\_2ECONS : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_2Elist\_2ECONS A\_27a \in (((ty\_2Elist\_2Elist A\_27a)^{(ty\_2Elist\_2Elist A\_27a)})^{A\_27a}) \quad (3)$$

Let  $c\_2Elist\_2ENIL : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_2Elist\_2ENIL A\_27a \in (ty\_2Elist\_2Elist A\_27a) \quad (4)$$

Let  $c\_2Elist\_2ESNOC : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_2Elist\_2ESNOC A\_27a \in (((ty\_2Elist\_2Elist A\_27a)^{(ty\_2Elist\_2Elist A\_27a)})^{A\_27a}) \quad (5)$$

**Definition 6** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in 2))$   
Assume the following.

$$True \tag{6}$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A\_27a.(p V0t)) \Leftrightarrow (p V0t))) \tag{7}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p V0t1) \wedge ((p V1t2) \wedge (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \wedge (p V2t3)))))) \tag{8}$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \tag{9}$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \tag{10}$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow ((\forall V0P \in (2^{A\_27a}).((p (ap (ap (c\_2Elist\_2EVERY A\_27a) V0P) (c\_2Elist\_2ENIL A\_27a))) \Leftrightarrow True)) \wedge (\forall V1P \in (2^{A\_27a}).(\forall V2h \in A\_27a.(\forall V3t \in (ty\_2Elist\_2Elist A\_27a).((p (ap (ap (c\_2Elist\_2EVERY A\_27a) V1P) (ap (ap (c\_2Elist\_2ECONS A\_27a) V2h) V3t))) \Leftrightarrow ((p (ap V1P V2h)) \wedge (p (ap (ap (c\_2Elist\_2EVERY A\_27a) V1P) V3t)))))))))) \tag{11}$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0P \in (2^{(ty\_2Elist\_2Elist A\_27a)}).(((p (ap V0P (c\_2Elist\_2ENIL A\_27a))) \wedge (\forall V1t \in (ty\_2Elist\_2Elist A\_27a).((p (ap V0P V1t)) \Rightarrow (\forall V2h \in A\_27a.(p (ap V0P (ap (ap (c\_2Elist\_2ECONS A\_27a) V2h) V1t)))))) \Rightarrow (\forall V3l \in (ty\_2Elist\_2Elist A\_27a).(p (ap V0P V3l)))))) \tag{12}$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow ((\forall V0x \in A\_27a.((ap (ap (c\_2Elist\_2ESNOC A\_27a) V0x) (c\_2Elist\_2ENIL A\_27a)) = (ap (ap (c\_2Elist\_2ECONS A\_27a) V0x) (c\_2Elist\_2ENIL A\_27a)))) \wedge (\forall V1x \in A\_27a.(\forall V2x.27 \in A\_27a.(\forall V3l \in (ty\_2Elist\_2Elist A\_27a).((ap (ap (c\_2Elist\_2ESNOC A\_27a) V1x) (ap (ap (c\_2Elist\_2ECONS A\_27a) V2x.27) V3l)) = (ap (ap (c\_2Elist\_2ECONS A\_27a) V2x.27) (ap (ap (c\_2Elist\_2ESNOC A\_27a) V1x) V3l)))))) \tag{13}$$

**Theorem 1**

$$\forall A_{27a}. \text{nonempty } A_{27a} \Rightarrow (\forall V0P \in (2^{A_{27a}}). (\forall V1x \in A_{27a}. (\forall V2l \in (\text{ty\_2Elist\_2Elist } A_{27a}). ((p (ap (ap (c\_2Elist\_2EVERY A_{27a}) V0P) (ap (ap (c\_2Elist\_2ESNOC A_{27a}) V1x) V2l))) \Leftrightarrow ((p (ap (ap (c\_2Elist\_2EVERY A_{27a}) V0P) V2l)) \wedge (p (ap V0P V1x))))))))))$$