

thm\_2Elist\_2EEXISTS\_SNOC  
(TMJVZrZjmCFLvm3corgkhtWCAjcb6acmjBXN)

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**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Ebool\_2E\_2T$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

**Definition 3** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p P \Rightarrow p Q)$  of type  $\iota$ .

**Definition 4** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap (ap (c\_2Emin\_2E\_3D (2^{A\_27a}))$

**Definition 5** We define  $c\_2Ebool\_2E\_5C\_2E\_2F$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in 2.V2t))$

**Definition 6** We define  $c\_2Ebool\_2E\_2F$  to be  $(ap (c\_2Ebool\_2E\_21 2) (\lambda V0t \in 2.V0t))$ .

Let  $ty\_2Elist\_2Elist : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty\_2Elist\_2Elist A0) \quad (1)$$

Let  $c\_2Elist\_2EEXISTS : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_2Elist\_2EEXISTS A\_27a \in ((2^{(ty\_2Elist\_2Elist A\_27a)})^{(2^{A\_27a})}) \quad (2)$$

Let  $c\_2Elist\_2ECONS : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_2Elist\_2ECONS A\_27a \in (((ty\_2Elist\_2Elist A\_27a)^{(ty\_2Elist\_2Elist A\_27a)})^{A\_27a}) \quad (3)$$

Let  $c\_2Elist\_2ENIL : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_2Elist\_2ENIL A\_27a \in (ty\_2Elist\_2Elist A\_27a) \quad (4)$$

Let  $c\_2Elist\_2ESNOC : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_2Elist\_2ESNOC A\_27a \in (((ty\_2Elist\_2Elist A\_27a)^{(ty\_2Elist\_2Elist A\_27a)})^{A\_27a}) \quad (5)$$

**Definition 7** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in 2))$   
Assume the following.

$$True \tag{6}$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \vee (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \vee True) \Leftrightarrow True) \wedge \\ & (((False \vee (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee False) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee \\ & (p V0t)) \Leftrightarrow (p V0t)))))) \end{aligned} \tag{7}$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \tag{8}$$

Assume the following.

$$\begin{aligned} & (\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.(((p V0A) \vee ( \\ & (p V1B) \vee (p V2C)) \Leftrightarrow (((p V0A) \vee (p V1B)) \vee (p V2C)))))) \end{aligned} \tag{9}$$

Assume the following.

$$\begin{aligned} & (\forall V0A \in 2.(\forall V1B \in 2.(((p V0A) \vee (p V1B)) \Leftrightarrow ((p V1B) \vee \\ & (p V0A)))))) \end{aligned} \tag{10}$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty A\_27a \Rightarrow ((\forall V0P \in (2^{A\_27a}).((p (ap \\ & (ap (c\_2Elist\_2EEXISTs A\_27a) V0P) (c\_2Elist\_2ENIL A\_27a))) \Leftrightarrow \\ & False)) \wedge (\forall V1P \in (2^{A\_27a}).(\forall V2h \in A\_27a.(\forall V3t \in \\ & (ty\_2Elist\_2Elist A\_27a).((p (ap (ap (c\_2Elist\_2EEXISTs A\_27a) \\ & V1P) (ap (ap (c\_2Elist\_2ECONS A\_27a) V2h) V3t))) \Leftrightarrow ((p (ap V1P V2h)) \vee \\ & (p (ap (ap (c\_2Elist\_2EEXISTs A\_27a) V1P) V3t))))))))) \end{aligned} \tag{11}$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0P \in (2^{(ty\_2Elist\_2Elist A\_27a)}). \\ & (((p (ap V0P (c\_2Elist\_2ENIL A\_27a))) \wedge (\forall V1t \in (ty\_2Elist\_2Elist \\ & A\_27a).((p (ap V0P V1t)) \Rightarrow (\forall V2h \in A\_27a.(p (ap V0P (ap (ap ( \\ & c\_2Elist\_2ECONS A\_27a) V2h) V1t)))))) \Rightarrow (\forall V3l \in (ty\_2Elist\_2Elist \\ & A\_27a).(p (ap V0P V3l)))))) \end{aligned} \tag{12}$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty A\_27a \Rightarrow ((\forall V0x \in A\_27a.((ap (ap (c\_2Elist\_2ESNOC \\ & A\_27a) V0x) (c\_2Elist\_2ENIL A\_27a)) = (ap (ap (c\_2Elist\_2ECONS \\ & A\_27a) V0x) (c\_2Elist\_2ENIL A\_27a)))) \wedge (\forall V1x \in A\_27a.(\forall V2x\_27 \in \\ & A\_27a.(\forall V3l \in (ty\_2Elist\_2Elist A\_27a).((ap (ap (c\_2Elist\_2ESNOC \\ & A\_27a) V1x) (ap (ap (c\_2Elist\_2ECONS A\_27a) V2x\_27) V3l)) = (ap ( \\ & ap (c\_2Elist\_2ECONS A\_27a) V2x\_27) (ap (ap (c\_2Elist\_2ESNOC A\_27a) \\ & V1x) V3l))))))))) \end{aligned} \tag{13}$$

**Theorem 1**

$\forall A_{27a}. \text{nonempty } A_{27a} \Rightarrow (\forall V0P \in (2^{A_{27a}}). (\forall V1x \in$   
 $A_{27a}. (\forall V2l \in (\text{ty\_2Elist\_2Elist } A_{27a}). ((p (ap (ap (c\_2Elist\_2EEXISTS$   
 $A_{27a} V0P) (ap (ap (c\_2Elist\_2ESNOC } A_{27a} V1x) V2l))) \Leftrightarrow ((p (ap$   
 $V0P V1x)) \vee (p (ap (ap (c\_2Elist\_2EEXISTS } A_{27a} V0P) V2l))))))$