

thm_2Elist_2EFILTER__NEQ__NIL (TMWsj- fanDBYgnZYu2aHQUPh9Gb1doq9dEjM)

October 26, 2020

Definition 1 We define `c_2Emin_2E_3D` to be $\lambda A. \lambda x \in A. \lambda y \in A. \text{inj_o } (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define `c_2Ebool_2E_2T` to be $(\text{ap } (\text{ap } (\text{c_2Emin_2E_3D } (2^2)) (\lambda V 0x \in 2. V 0x)) (\lambda V 1x \in 2. V 1x))$

Definition 3 We define `c_2Ebool_2E_21` to be $\lambda A. 27a : \iota. (\lambda V 0P \in (2^{A-27a}). (\text{ap } (\text{ap } (\text{c_2Emin_2E_3D } (2^{A-27a}))))$

Definition 4 We define `c_2Ebool_2E_2F` to be $(\text{ap } (\text{c_2Ebool_2E_21 } 2) (\lambda V 0t \in 2. V 0t))$.

Definition 5 We define `c_2Emin_2E_3D_3D_3E` to be $\lambda P \in 2. \lambda Q \in 2. \text{inj_o } (p P \Rightarrow p Q)$ of type ι .

Definition 6 We define `c_2Ebool_2E_5C_2F` to be $(\lambda V 0t1 \in 2. (\lambda V 1t2 \in 2. (\text{ap } (\text{c_2Ebool_2E_21 } 2) (\lambda V 2t \in 2. V 2t))))$

Let `ty_2Elist_2Elist` : $\iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A 0. \text{nonempty } A 0 \Rightarrow \text{nonempty } (\text{ty_2Elist_2Elist } A 0) \quad (1)$$

Let `c_2Elist_2EELIST__TO__SET` : $\iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A. 27a. \text{nonempty } A. 27a \Rightarrow \text{c_2Elist_2EELIST_TO_SET } A. 27a \in ((2^{A-27a}) (\text{ty_2Elist_2Elist } A. 27a)) \quad (2)$$

Definition 7 We define `c_2Ebool_2E_2IN` to be $\lambda A. 27a : \iota. (\lambda V 0x \in A. 27a. (\lambda V 1f \in (2^{A-27a}). (\text{ap } V 1f V 0x))))$

Definition 8 We define `c_2Ebool_2E_2F_5C` to be $(\lambda V 0t1 \in 2. (\lambda V 1t2 \in 2. (\text{ap } (\text{c_2Ebool_2E_21 } 2) (\lambda V 2t \in 2. V 2t))))$

Definition 9 We define `c_2Emin_2E_40` to be $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. p (\text{ap } P x)) \text{ then } (\text{the } (\lambda x. x \in A \wedge p (\text{ap } P x)))$ of type $\iota \Rightarrow \iota$.

Definition 10 We define `c_2Ebool_2E_3F` to be $\lambda A. 27a : \iota. (\lambda V 0P \in (2^{A-27a}). (\text{ap } V 0P (\text{ap } (\text{c_2Emin_2E_40 } (2^{A-27a}))))$

Let `c_2Elist_2EEXISTS` : $\iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A. 27a. \text{nonempty } A. 27a \Rightarrow \text{c_2Elist_2EEXISTS } A. 27a \in ((2^{(\text{ty_2Elist_2Elist } A. 27a)}) (2^{A-27a})) \quad (3)$$

Definition 11 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E$

Let $c_2Elist_2EEVERY : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Elist_2EEVERY A_27a \in ((2^{(ty_2Elist_2Elist A_27a)})^{(2^{A_27a})}) \quad (4)$$

Let $c_2Elist_2ENIL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Elist_2ENIL A_27a \in (ty_2Elist_2Elist A_27a) \quad (5)$$

Let $c_2Elist_2EFILTER : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Elist_2EFILTER A_27a \in (((ty_2Elist_2Elist A_27a)^{(ty_2Elist_2Elist A_27a)})^{(2^{A_27a})}) \quad (6)$$

Assume the following.

$$True \quad (7)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A_27a.(p V0t)) \Leftrightarrow (p V0t))) \quad (8)$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \quad (9)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \quad (10)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0P \in (2^{A_27a}).((\neg(\forall V1x \in A_27a.(p (ap V0P V1x)))) \Leftrightarrow (\exists V2x \in A_27a.(\neg(p (ap V0P V2x)))))) \quad (11)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0P \in (2^{A_27a}).((\neg(\exists V1x \in A_27a.(p (ap V0P V1x)))) \Leftrightarrow (\forall V2x \in A_27a.(\neg(p (ap V0P V2x)))))) \quad (12)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p V0A) \wedge (p V1B))) \Leftrightarrow ((\neg(p V0A)) \vee (\neg(p V1B)))))) \wedge (((\neg((p V0A) \vee (p V1B))) \Leftrightarrow ((\neg(p V0A)) \wedge (\neg(p V1B)))))) \quad (13)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in (2^{A_27a}). (\forall V1l \in \\ & (ty_2Elist_2Elist\ A_27a). ((p\ (ap\ (ap\ (c_2Elist_2EEXISTS\ A_27a) \\ & V0P)\ V1l)) \Leftrightarrow (\exists V2e \in A_27a. ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a) \\ & V2e)\ (ap\ (c_2Elist_2ELIST_TO_SET\ A_27a)\ V1l))) \wedge (p\ (ap\ V0P\ V2e))))))))) \end{aligned} \quad (14)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in (2^{A_27a}). (\forall V1l \in \\ & (ty_2Elist_2Elist\ A_27a). ((p\ (ap\ (ap\ (c_2Elist_2EVERY\ A_27a) \\ & V0P)\ V1l)) \Leftrightarrow (\neg (p\ (ap\ (ap\ (c_2Elist_2EEXISTS\ A_27a)\ (\lambda V2x \in A_27a. \\ & (ap\ c_2Ebool_2E_7E\ (ap\ V0P\ V2x))))\ V1l))))))))) \end{aligned} \quad (15)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in (2^{A_27a}). (\forall V1l \in \\ & (ty_2Elist_2Elist\ A_27a). (((ap\ (ap\ (c_2Elist_2EFILTER\ A_27a) \\ & V0P)\ V1l) = (c_2Elist_2ENIL\ A_27a)) \Leftrightarrow (p\ (ap\ (ap\ (c_2Elist_2EVERY \\ & A_27a)\ (\lambda V2x \in A_27a. (ap\ c_2Ebool_2E_7E\ (ap\ V0P\ V2x))))\ V1l)))))) \end{aligned} \quad (16)$$

Theorem 1

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in (2^{A_27a}). (\forall V1l \in \\ & (ty_2Elist_2Elist\ A_27a). ((\neg (ap\ (ap\ (c_2Elist_2EFILTER\ A_27a) \\ & V0P)\ V1l) = (c_2Elist_2ENIL\ A_27a))) \Leftrightarrow (\exists V2x \in A_27a. ((p\ (\\ & ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V2x)\ (ap\ (c_2Elist_2ELIST_TO_SET \\ & A_27a)\ V1l))) \wedge (p\ (ap\ V0P\ V2x))))))))) \end{aligned}$$