

# thm\_2Elist\_2EFILTER\_\_REVERSE (TMJS- bCPEMZ9WzBMh3Etk4fP3V3Y63wSewmW)

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**Definition 1** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p P \Rightarrow p Q)$  of type  $\iota$ .

**Definition 2** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 3** We define  $c\_2Ebool\_2E\_2T$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

**Definition 4** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap (ap (c\_2Emin\_2E\_3D (2^{A\_27a}))$

**Definition 5** We define  $c\_2Ebool\_2E\_5C\_2F$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in 2.V2t))$

**Definition 6** We define  $c\_2Ebool\_2E\_2F$  to be  $(ap (c\_2Ebool\_2E\_21 2) (\lambda V0t \in 2.V0t))$ .

**Definition 7** We define  $c\_2Ebool\_2E\_7E$  to be  $(\lambda V0t \in 2.(ap (ap c\_2Emin\_2E\_3D\_3D\_3E V0t) c\_2Ebool\_2E\_2F$

**Definition 8** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in 2.V2t))$

**Definition 9** We define  $c\_2Emin\_2E\_40$  to be  $\lambda A.\lambda P \in 2^A.\mathbf{if} (\exists x \in A.p (ap P x)) \mathbf{then} (the (\lambda x.x \in A \wedge p$  of type  $\iota \Rightarrow \iota$ .

**Definition 10** We define  $c\_2Ebool\_2ECOND$  to be  $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.($

Let  $ty\_2Elist\_2Elist : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty\_2Elist\_2Elist A0) \quad (1)$$

Let  $c\_2Elist\_2EFILTER : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_2Elist\_2EFILTER A\_27a \in (((ty\_2Elist\_2Elist A\_27a)(ty\_2Elist\_2Elist A\_27a))(2^{A\_27a})) \quad (2)$$

Let  $c\_2Elist\_2EAPPEND : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_2Elist\_2EAPPEND A\_27a \in (((ty\_2Elist\_2Elist A\_27a)(ty\_2Elist\_2Elist A\_27a))(ty\_2Elist\_2Elist A\_27a)) \quad (3)$$

Let  $c\_2Elist\_2ECONS : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ECONS\ A\_27a \in (((ty\_2Elist\_2Elist\ A\_27a)^{(ty\_2Elist\_2Elist\ A\_27a)})^{A\_27a}) \quad (4)$$

Let  $c\_2Elist\_2ENIL : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ENIL\ A\_27a \in (ty\_2Elist\_2Elist\ A\_27a) \quad (5)$$

Let  $c\_2Elist\_2EREVERSE : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2EREVERSE\ A\_27a \in ((ty\_2Elist\_2Elist\ A\_27a)^{(ty\_2Elist\_2Elist\ A\_27a)}) \quad (6)$$

Assume the following.

$$True \quad (7)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \quad (8)$$

Assume the following.

$$(\forall V0t \in 2. (False \Rightarrow (p\ V0t))) \quad (9)$$

Assume the following.

$$(\forall V0t \in 2. ((p\ V0t) \vee (\neg(p\ V0t)))) \quad (10)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t \in 2. ((\forall V1x \in A\_27a. (p\ V0t)) \Leftrightarrow (p\ V0t))) \quad (11)$$

Assume the following.

$$(\forall V0t \in 2. (((True \vee (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \vee True) \Leftrightarrow True) \wedge (((False \vee (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \vee False) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \vee (p\ V0t)) \Leftrightarrow (p\ V0t)))))) \quad (12)$$

Assume the following.

$$(\forall V0t \in 2. (((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((p\ V0t) \Rightarrow False) \Leftrightarrow (\neg(p\ V0t)))))) \quad (13)$$

Assume the following.

$$((\forall V0t \in 2. ((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t)) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True)))) \quad (14)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a. ((V0x = V0x) \Leftrightarrow True)) \quad (15)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a. (\forall V1y \in A\_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (16)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow \\ & (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg(p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg( \\ & p\ V0t)))))) \end{aligned} \quad (17)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t1 \in A\_27a. (\forall V1t2 \in \\ & A\_27a. (((ap\ (ap\ (ap\ (c\_2Ebool\_2ECOND\ A\_27a)\ c\_2Ebool\_2ET)\ V0t1)\ V1t2) = V0t1) \wedge ((ap\ (ap\ (ap\ (c\_2Ebool\_2ECOND\ A\_27a)\ c\_2Ebool\_2EF) \\ & V0t1)\ V1t2) = V1t2)))) \end{aligned} \quad (18)$$

Assume the following.

$$\begin{aligned} & (\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p\ V0t1) \Rightarrow \\ & ((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \end{aligned} \quad (19)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\ & \forall V0b \in 2. (\forall V1f \in (A\_27b^{A\_27a}). (\forall V2g \in (A\_27b^{A\_27a}). \\ & (\forall V3x \in A\_27a. ((ap\ (ap\ (ap\ (ap\ (c\_2Ebool\_2ECOND\ (A\_27b^{A\_27a})\ V0b)\ V1f)\ V2g)\ V3x) = (ap\ (ap\ (ap\ (c\_2Ebool\_2ECOND\ A\_27b)\ V0b)\ (ap \\ & V1f\ V3x))\ (ap\ V2g\ V3x)))))) \end{aligned} \quad (20)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\ & \forall V0f \in (A\_27b^{A\_27a}). (\forall V1b \in 2. (\forall V2x \in A\_27a. \\ & (\forall V3y \in A\_27a. ((ap\ V0f\ (ap\ (ap\ (ap\ (c\_2Ebool\_2ECOND\ A\_27a)\ V1b)\ V2x)\ V3y) = (ap\ (ap\ (ap\ (c\_2Ebool\_2ECOND\ A\_27b)\ V1b)\ (ap\ V0f \\ & V2x))\ (ap\ V0f\ V3y)))))) \end{aligned} \quad (21)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in 2. \\ & (\forall V2x \in A\_27a. (\forall V3x\_27 \in A\_27a. (\forall V4y \in A\_27a. \\ & (\forall V5y\_27 \in A\_27a. (((p\ V0P) \Leftrightarrow (p\ V1Q)) \wedge (((p\ V1Q) \Rightarrow (V2x = V3x\_27)) \wedge \\ & ((\neg(p\ V1Q)) \Rightarrow (V4y = V5y\_27)))))) \Rightarrow ((ap\ (ap\ (ap\ (c\_2Ebool\_2ECOND\ A\_27a)\ V0P)\ V2x)\ V4y) = (ap\ (ap\ (ap\ (c\_2Ebool\_2ECOND\ A\_27a)\ V1Q)\ V3x\_27) \\ & V5y\_27)))))) \end{aligned} \quad (22)$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow ((\forall V0P \in (2^{A.27a}).((ap\ ( \\
& ap\ (c.2Elist.2EFILTER\ A.27a)\ V0P)\ (c.2Elist.2ENIL\ A.27a)) = (c.2Elist.2ENIL \\
& A.27a))) \wedge (\forall V1P \in (2^{A.27a}).(\forall V2h \in A.27a.(\forall V3t \in \\
& (ty.2Elist.2Elist\ A.27a).((ap\ (ap\ (c.2Elist.2EFILTER\ A.27a) \\
V1P)\ (ap\ (ap\ (c.2Elist.2ECONS\ A.27a)\ V2h)\ V3t)) = (ap\ (ap\ (ap\ (c.2Ebool.2ECOND \\
& (ty.2Elist.2Elist\ A.27a))\ (ap\ V1P\ V2h))\ (ap\ (ap\ (c.2Elist.2ECONS \\
& A.27a)\ V2h)\ (ap\ (ap\ (c.2Elist.2EFILTER\ A.27a)\ V1P)\ V3t))))))\ (ap\ (ap \\
& (c.2Elist.2EFILTER\ A.27a)\ V1P)\ V3t))))))
\end{aligned} \tag{23}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0P \in (ty.2Elist.2Elist\ A.27a). \\
& (((p\ (ap\ V0P\ (c.2Elist.2ENIL\ A.27a))) \wedge (\forall V1t \in (ty.2Elist.2Elist \\
& A.27a).(p\ (ap\ V0P\ V1t)) \Rightarrow (\forall V2h \in A.27a.(p\ (ap\ V0P\ (ap\ (ap\ ( \\
& c.2Elist.2ECONS\ A.27a)\ V2h)\ V1t)))))) \Rightarrow (\forall V3l \in (ty.2Elist.2Elist \\
& A.27a).(p\ (ap\ V0P\ V3l))))))
\end{aligned} \tag{24}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0l \in (ty.2Elist.2Elist \\
& A.27a).((ap\ (ap\ (c.2Elist.2EAPPEND\ A.27a)\ V0l)\ (c.2Elist.2ENIL \\
& A.27a)) = V0l))
\end{aligned} \tag{25}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0P \in (2^{A.27a}).(\forall V1L \in \\
& (ty.2Elist.2Elist\ A.27a).(\forall V2M \in (ty.2Elist.2Elist\ A.27a). \\
& ((ap\ (ap\ (c.2Elist.2EFILTER\ A.27a)\ V0P)\ (ap\ (ap\ (c.2Elist.2EAPPEND \\
& A.27a)\ V1L)\ V2M)) = (ap\ (ap\ (c.2Elist.2EAPPEND\ A.27a)\ (ap\ (ap\ (c.2Elist.2EFILTER \\
& A.27a)\ V0P)\ V1L))\ (ap\ (ap\ (c.2Elist.2EFILTER\ A.27a)\ V0P)\ V2M))))))
\end{aligned} \tag{26}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow (((ap\ (c.2Elist.2EREVERSE\ A.27a) \\
& (c.2Elist.2ENIL\ A.27a)) = (c.2Elist.2ENIL\ A.27a)) \wedge (\forall V0h \in \\
& A.27a.(\forall V1t \in (ty.2Elist.2Elist\ A.27a).((ap\ (c.2Elist.2EREVERSE \\
& A.27a)\ (ap\ (ap\ (c.2Elist.2ECONS\ A.27a)\ V0h)\ V1t)) = (ap\ (ap\ (c.2Elist.2EAPPEND \\
& A.27a)\ (ap\ (c.2Elist.2EREVERSE\ A.27a)\ V1t))\ (ap\ (ap\ (c.2Elist.2ECONS \\
& A.27a)\ V0h)\ (c.2Elist.2ENIL\ A.27a))))))
\end{aligned} \tag{27}$$

**Theorem 1**

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0l \in (ty.2Elist.2Elist \\
& A.27a).(\forall V1P \in (2^{A.27a}).((ap\ (ap\ (c.2Elist.2EFILTER\ A.27a) \\
V1P)\ (ap\ (c.2Elist.2EREVERSE\ A.27a)\ V0l)) = (ap\ (c.2Elist.2EREVERSE \\
& A.27a)\ (ap\ (ap\ (c.2Elist.2EFILTER\ A.27a)\ V1P)\ V0l))))))
\end{aligned}$$