

thm_2Elist_2EFLAT__EQ__NIL
(TMVrdZ8djUVqefBKK5QjVuPej3HRRRF33qEH)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_2T$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a}))$

Definition 4 We define $c_2Ebool_2E_2F$ to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 5 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 6 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_2F$

Let $ty_2Elist_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty_2Elist_2Elist A0) \quad (1)$$

Let $c_2Elist_2EFLAT : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Elist_2EFLAT A_27a \in ((ty_2Elist_2Elist A_27a)^{(ty_2Elist_2Elist (ty_2Elist_2Elist A_27a))}) \quad (2)$$

Let $c_2Elist_2EEVERY : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Elist_2EEVERY A_27a \in ((2^{(ty_2Elist_2Elist A_27a)})^{(2^{A_27a})}) \quad (3)$$

Let $c_2Elist_2ECONS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Elist_2ECONS A_27a \in (((ty_2Elist_2Elist A_27a)^{(ty_2Elist_2Elist A_27a)})^{A_27a}) \quad (4)$$

Let $c_2Elist_2EAPPEND : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Elist_2EAPPEND A_27a \in (((ty_2Elist_2Elist A_27a)^{(ty_2Elist_2Elist A_27a)})^{(ty_2Elist_2Elist A_27a)}) \quad (5)$$

Let $c_2Elist_2ENIL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2ENIL\ A_27a \in (ty_2Elist_2Elist\ A_27a) \quad (6)$$

Definition 7 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in 2.$

Assume the following.

$$True \quad (7)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \wedge True) \Leftrightarrow \\ & (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge (((p\ V0t) \wedge False) \Leftrightarrow False) \wedge \\ & (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)))))) \end{aligned} \quad (8)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Rightarrow True) \Leftrightarrow \\ & True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((\\ & (p\ V0t) \Rightarrow False) \Leftrightarrow (\neg(p\ V0t)))))) \end{aligned} \quad (9)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \quad (10)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (11)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow \\ & (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg(p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg(\\ & p\ V0t)))))) \end{aligned} \quad (12)$$

Assume the following.

$$\begin{aligned} & (\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p\ V0t1) \Rightarrow \\ & (p\ V1t2) \Rightarrow (p\ V2t3)) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \end{aligned} \quad (13)$$

Assume the following.

$$\begin{aligned} & (\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Leftrightarrow (p\ V1t2)) \Leftrightarrow (((p \\ & V0t1) \Rightarrow (p\ V1t2)) \wedge ((p\ V1t2) \Rightarrow (p\ V0t1)))))) \end{aligned} \quad (14)$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in 2.(\forall V1x_27 \in 2.(\forall V2y \in 2.(\forall V3y_27 \in \\ & 2.(((p\ V0x) \Leftrightarrow (p\ V1x_27)) \wedge ((p\ V1x_27) \Rightarrow ((p\ V2y) \Leftrightarrow (p\ V3y_27)))) \Rightarrow \\ & (((p\ V0x) \Rightarrow (p\ V2y)) \Leftrightarrow ((p\ V1x_27) \Rightarrow (p\ V3y_27)))))) \end{aligned} \quad (15)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow & (((ap\ (c_2Elist_2EFLAT\ A_27a)\ (\\ & c_2Elist_2ENIL\ (ty_2Elist_2Elist\ A_27a))) = (c_2Elist_2ENIL \\ & A_27a)) \wedge (\forall V0h \in (ty_2Elist_2Elist\ A_27a). (\forall V1t \in \\ & (ty_2Elist_2Elist\ (ty_2Elist_2Elist\ A_27a)). ((ap\ (c_2Elist_2EFLAT \\ & A_27a)\ (ap\ (ap\ (c_2Elist_2ECONS\ (ty_2Elist_2Elist\ A_27a)\ V0h) \\ & V1t)) = (ap\ (ap\ (c_2Elist_2EAPPEND\ A_27a)\ V0h)\ (ap\ (c_2Elist_2EFLAT \\ & A_27a)\ V1t)))))) \end{aligned} \quad (16)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow & ((\forall V0P \in (2^{A_27a}). ((p\ (ap \\ & (ap\ (c_2Elist_2EEVERY\ A_27a)\ V0P)\ (c_2Elist_2ENIL\ A_27a))) \Leftrightarrow True)) \wedge \\ & (\forall V1P \in (2^{A_27a}). (\forall V2h \in A_27a. (\forall V3t \in (ty_2Elist_2Elist \\ & A_27a). ((p\ (ap\ (ap\ (c_2Elist_2EEVERY\ A_27a)\ V1P)\ (ap\ (ap\ (c_2Elist_2ECONS \\ & A_27a)\ V2h)\ V3t))) \Leftrightarrow ((p\ (ap\ V1P\ V2h)) \wedge (p\ (ap\ (ap\ (c_2Elist_2EEVERY \\ & A_27a)\ V1P)\ V3t)))))) \end{aligned} \quad (17)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow & (\forall V0P \in (2^{(ty_2Elist_2Elist\ A_27a)}). \\ & (((p\ (ap\ V0P\ (c_2Elist_2ENIL\ A_27a))) \wedge (\forall V1t \in (ty_2Elist_2Elist \\ & A_27a). ((p\ (ap\ V0P\ V1t)) \Rightarrow (\forall V2h \in A_27a. (p\ (ap\ V0P\ (ap\ (ap\ (\\ & c_2Elist_2ECONS\ A_27a)\ V2h)\ V1t)))))) \Rightarrow (\forall V3l \in (ty_2Elist_2Elist \\ & A_27a). (p\ (ap\ V0P\ V3l)))) \end{aligned} \quad (18)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow & ((\forall V0l1 \in (ty_2Elist_2Elist \\ & A_27a). (\forall V1l2 \in (ty_2Elist_2Elist\ A_27a). (((c_2Elist_2ENIL \\ & A_27a) = (ap\ (ap\ (c_2Elist_2EAPPEND\ A_27a)\ V0l1)\ V1l2)) \Leftrightarrow ((V0l1 = \\ & (c_2Elist_2ENIL\ A_27a)) \wedge (V1l2 = (c_2Elist_2ENIL\ A_27a)))))) \wedge \\ & (\forall V2l1 \in (ty_2Elist_2Elist\ A_27a). (\forall V3l2 \in (ty_2Elist_2Elist \\ & A_27a). (((ap\ (ap\ (c_2Elist_2EAPPEND\ A_27a)\ V2l1)\ V3l2) = (c_2Elist_2ENIL \\ & A_27a)) \Leftrightarrow ((V2l1 = (c_2Elist_2ENIL\ A_27a)) \wedge (V3l2 = (c_2Elist_2ENIL \\ & A_27a)))))) \end{aligned} \quad (19)$$

Theorem 1

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow & (\forall V0ls \in (ty_2Elist_2Elist \\ & (ty_2Elist_2Elist\ A_27a)). (((ap\ (c_2Elist_2EFLAT\ A_27a)\ V0ls) = \\ & (c_2Elist_2ENIL\ A_27a)) \Leftrightarrow (p\ (ap\ (ap\ (c_2Elist_2EEVERY\ (ty_2Elist_2Elist \\ & A_27a))\ (ap\ (c_2Emin_2E_3D\ (ty_2Elist_2Elist\ A_27a))\ (c_2Elist_2ENIL \\ & A_27a)))\ V0ls)))) \end{aligned}$$