

# thm\_2Elist\_2EFront\_\_CONS (TM- Lxm5qKG14urCKhnX34pAhjBBX29cx8gGe)

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**Definition 1** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p P \Rightarrow p Q)$  of type  $\iota$ .

**Definition 2** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 3** We define  $c\_2Ebool\_2ET$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

**Definition 4** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c\_2Emin\_2E\_3D (2^{A-27a}))$

**Definition 5** We define  $c\_2Ebool\_2EF$  to be  $(ap (c\_2Ebool\_2E\_21 2) (\lambda V0t \in 2.V0t))$ .

**Definition 6** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in 2.V2t)))$

**Definition 7** We define  $c\_2Ebool\_2E\_7E$  to be  $(\lambda V0t \in 2.(ap (ap c\_2Emin\_2E\_3D\_3D\_3E V0t) c\_2Ebool\_2EF$

Let  $ty\_2Elist\_2Elist : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty\_2Elist\_2Elist A0) \quad (1)$$

Let  $c\_2Elist\_2ENIL : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_2Elist\_2ENIL A\_27a \in (ty\_2Elist\_2Elist A\_27a) \quad (2)$$

**Definition 8** We define  $c\_2Emin\_2E\_40$  to be  $\lambda A.\lambda P \in 2^A$ . **if**  $(\exists x \in A.p (ap P x))$  **then** *(the*  $(\lambda x.x \in A \wedge p$  of type  $\iota \Rightarrow \iota$ .

**Definition 9** We define  $c\_2Ebool\_2ECOND$  to be  $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.(ap$

Let  $c\_2Elist\_2ECONS : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_2Elist\_2ECONS A\_27a \in (((ty\_2Elist\_2Elist A\_27a)^{(ty\_2Elist\_2Elist A\_27a)})^{A\_27a}) \quad (3)$$

Let  $c\_2Elist\_2EFRONT : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2EFRONT\ A\_27a \in ((ty\_2Elist\_2Elist\ A\_27a)^{(ty\_2Elist\_2Elist\ A\_27a)}) \quad (4)$$

Assume the following.

$$True \quad (5)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \quad (6)$$

Assume the following.

$$(\forall V0t \in 2. (False \Rightarrow (p\ V0t))) \quad (7)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t \in 2. ((\forall V1x \in A\_27a. (p\ V0t)) \Leftrightarrow (p\ V0t))) \quad (8)$$

Assume the following.

$$(\forall V0t \in 2. (((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \wedge True) \Leftrightarrow (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge (((p\ V0t) \wedge False) \Leftrightarrow False) \wedge (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)))))) \quad (9)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a. ((V0x = V0x) \Leftrightarrow True)) \quad (10)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t1 \in A\_27a. (\forall V1t2 \in A\_27a. (((ap\ (ap\ (ap\ (c\_2Ebool\_2ECOND\ A\_27a)\ c\_2Ebool\_2ET)\ V0t1)\ V1t2) = V0t1) \wedge ((ap\ (ap\ (ap\ (c\_2Ebool\_2ECOND\ A\_27a)\ c\_2Ebool\_2EF)\ V0t1)\ V1t2) = V1t2)))))) \quad (11)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0a1 \in (ty\_2Elist\_2Elist\ A\_27a). (\forall V1a0 \in A\_27a. (\neg ((ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V1a0)\ V0a1) = (c\_2Elist\_2ENIL\ A\_27a)))))) \quad (12)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0h \in A\_27a. (\forall V1t \in (ty\_2Elist\_2Elist\ A\_27a). (((ap\ (c\_2Elist\_2EFRONT\ A\_27a)\ (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V0h)\ V1t)) = (ap\ (ap\ (ap\ (c\_2Ebool\_2ECOND\ (ty\_2Elist\_2Elist\ A\_27a))\ (ap\ (ap\ (c\_2Emin\_2E\_3D\ (ty\_2Elist\_2Elist\ A\_27a))\ V1t)\ (c\_2Elist\_2ENIL\ A\_27a))))\ (c\_2Elist\_2ENIL\ A\_27a))\ (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V0h)\ (ap\ (c\_2Elist\_2EFRONT\ A\_27a)\ V1t)))))) \quad (13)$$

**Theorem 1**

$$\begin{aligned} \forall A_{27a}. \text{nonempty } A_{27a} \Rightarrow & ((\forall V0x \in A_{27a}. ((\text{ap } (c\_2Elist\_2EFront \\ A_{27a}) (\text{ap } (\text{ap } (c\_2Elist\_2ECONS } & A_{27a}) V0x) (c\_2Elist\_2ENIL } \\ A_{27a}))) = & (c\_2Elist\_2ENIL } \\ A_{27a}))) \wedge (\forall V1x \in A_{27a}. (\forall V2y \in A_{27a}. & \\ (\forall V3z \in (ty\_2Elist\_2Elist } & A_{27a}). ((\text{ap } (c\_2Elist\_2EFront \\ A_{27a}) (\text{ap } (\text{ap } (c\_2Elist\_2ECONS } & A_{27a}) V1x) (\text{ap } (\text{ap } (c\_2Elist\_2ECONS } \\ A_{27a}) V2y) V3z))) = & (\text{ap } (\text{ap } (c\_2Elist\_2ECONS } \\ A_{27a}) V1x) (\text{ap } (c\_2Elist\_2EFront & \\ A_{27a}) (\text{ap } (\text{ap } (c\_2Elist\_2ECONS } & A_{27a}) V2y) V3z)))))) \end{aligned}$$