thm_2Elist_2EGENLIST__NUMERALS (TMcept9rf9moxhjMkdmrqSRmgrrB3vh5iET)

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Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{1}$$

Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (c\_2Emin\_2E\_3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1$

Definition 3 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (c\_2Emin\_2E\_3D\ (2^{A\_27$

Definition 4 *We define* c_2Earithmetic_2ENUMERAL *to be* $\lambda V0x \in ty\_2Enum\_2Enum.V0x$.

Definition 5 *We define* c_2Ebool_2EF *to be* $(ap\ (c\_2Ebool\_2E\_21\ 2)\ (\lambda V0t \in 2.V0t))$.

Let $ty\_2Elist\_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Elist\_2Elist\ A0) \tag{2}$$

Let $c\_2Elist\_2ECONS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ECONS\ A\_27a \in (((ty\_2Elist\_2Elist\ A\_27a)^{(ty\_2Elist\_2Elist\ A\_27a)})^{A\_27a}) \tag{3}$$

Let $c\_2Enum\_2EREP\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EREP\_\_num \in (omega^{ty\_2Enum\_2Enum}) \tag{4}$$

Let $c\_2Enum\_2ESUC\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2ESUC\_\_REP \in (omega^{omega}) \tag{5}$$

Let $c\_2Enum\_2EABS\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EABS\_\_num \in (ty\_2Enum\_2Enum^{omega}) \tag{6}$$

Definition 6 *We define* c_2Enum_2ESUC *to be* $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_\_num\ ($

Let $c\_2Enum\_2EZERO\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2EZERO\_\_REP \in omega \tag{7}$$

Definition 7 *We define* c_2Enum_2E0 *to be* $(ap\ c\_2Enum\_2EABS\_\_num\ c\_2Enum\_2EZERO\_\_REP)$.

Definition 8 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 9 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in 2$

Let $c\_2Elist\_2ENIL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ENIL\ A\_27a \in (ty\_2Elist\_2Elist\ A\_27a) \tag{8}$$

Let $c\_2Elist\_2EGENLIST\_\_AUX : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2EGENLIST\_\_AUX\ A\_27a \in ((((ty\_2Elist\_2Elist\ A\_27a)^{(ty\_2Elist\_2Elist\ A\_27a)})^{ty\_2Enum\_2Enum})^{(A\_27a^{ty\_2Enum\_2Enum})}) \tag{9}$$

Let $c\_2Elist\_2EGENLIST : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2EGENLIST\ A\_27a \in (((ty\_2Elist\_2Elist\ A\_27a)^{ty\_2Enum\_2Enum})^{(A\_27a^{ty\_2Enum\_2Enum})}) \tag{10}$$

Assume the following.

$$True \tag{11}$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \wedge True) \Leftrightarrow (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge ((((p\ V0t) \wedge False) \Leftrightarrow False) \wedge (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t))))))) \tag{12}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \tag{13}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow ((\forall V0f \in (A\_27a^{ty\_2Enum\_2Enum}).(\forall V1l \in (ty\_2Elist\_2Elist\ A\_27a).((ap\ (ap\ (ap\ (c\_2Elist\_2EGENLIST\_\_AUX\ A\_27a)\ V0f)\ \mathsf{c\_2Enum\_2E0})\ V1l) = V1l))) \wedge (\forall V2f \in (A\_27a^{ty\_2Enum\_2Enum}).(\forall V3n \in ty\_2Enum\_2Enum.(\forall V4l \in (ty\_2Elist\_2Elist\ A\_27a).((ap\ (ap\ (ap\ (c\_2Elist\_2EGENLIST\_\_AUX\ A\_27a)\ V2f)\ (ap\ \mathsf{c\_2Enum\_2ESUC}\ V3n))\ V4l) = (ap\ (ap\ (ap\ (c\_2Elist\_2EGENLIST\_\_AUX\ A\_27a)\ V2f)\ V3n)\ (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ (ap\ V2f\ V3n))\ V4l))))))) \tag{14}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0f \in (A\_27a^{ty\_2Enum\_2Enum}). (\forall V1n \in ty\_2Enum\_2Enum.((ap\ (ap\ (c\_2Elist\_2EGENLIST\ A\_27a)\ V0f)\ V1n) = (ap\ (ap\ (ap\ (c\_2Elist\_2EGENLIST\_\_AUX\ A\_27a)\ V0f)\ V1n)\ (c\_2Elist\_2ENIL\ A\_27a))))) \tag{15}$$

Theorem 1

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0f \in (A\_27a^{ty\_2Enum\_2Enum}). (\forall V1n \in ty\_2Enum\_2Enum.(((ap\ (ap\ (c\_2Elist\_2EGENLIST\ A\_27a)\ V0f)\ \mathsf{c\_2Enum\_2E0}) = (c\_2Elist\_2ENIL\ A\_27a)) \wedge ((ap\ (ap\ (c\_2Elist\_2EGENLIST\ A\_27a)\ V0f)\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V1n)) = (ap\ (ap\ (ap\ (c\_2Elist\_2EGENLIST\_\_AUX\ A\_27a)\ V0f)\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V1n))\ (c\_2Elist\_2ENIL\ A\_27a))))))$$