thm_2Elist_2EGENLIST__PLUS__APPEND (TMVBohi7hK1QmUg69dzaWpZWKY3xdcVE1vF)

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Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1$

Definition 3 *We define* c_2Ebool_2EBOUNDED *to be* $(\lambda V0v \in 2.\mathsf{c\_2Ebool\_2ET})$.

Let $ty\_2Elist\_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Elist\_2Elist\ A0) \tag{1}$$

Let $c\_2Elist\_2EAPPEND : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2EAPPEND\ A\_27a \in (((ty\_2Elist\_2Elist\ A\_27a)^{(ty\_2Elist\_2Elist\ A\_27a)})^{(ty\_2Elist\_2Elist\ A\_27a)}) \tag{2}$$

Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{3}$$

Let $c\_2Earithmetic\_2E\_2B : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{4}$$

Let $c\_2Elist\_2EGENLIST : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2EGENLIST\ A\_27a \in (((ty\_2Elist\_2Elist\ A\_27a)^{ty\_2Enum\_2Enum})^{(A\_27a^{ty\_2Enum\_2Enum})}) \tag{5}$$

Definition 4 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

Definition 5 *We define* c_2Emarker_2ECong *to be* $\lambda V0x \in 2.V0x$.

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ V1n) = (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V1n)\ V0m)))) \tag{6}$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ V1n) = (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V1n)\ V0m)))) \tag{7}$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.(\forall V2p \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V1n)\ V2p)) = (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ V1n))\ V2p))))) \tag{8}$$

Assume the following.

$$True \tag{9}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \tag{10}$$

Assume the following.

$$(\forall V0v \in 2.((p\ (ap\ \mathsf{c\_2Ebool\_2EBOUNDED}\ V0v)) \Leftrightarrow True)) \tag{11}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0f \in (A\_27a^{ty\_2Enum\_2Enum}).(\forall V1a \in ty\_2Enum\_2Enum.(\forall V2b \in ty\_2Enum\_2Enum.((ap\ (ap\ (c\_2Elist\_2EGENLIST\ A\_27a)\ V0f)\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V1a)\ V2b)) = (ap\ (ap\ (c\_2Elist\_2EAPPEND\ A\_27a)\ (ap\ (ap\ (c\_2Elist\_2EGENLIST\ A\_27a)\ V0f)\ V2b))\ (ap\ (ap\ (c\_2Elist\_2EGENLIST\ A\_27a)\ (\lambda V3t \in ty\_2Enum\_2Enum.(ap\ V0f\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V3t)\ V2b))))\ V1a)))))) \tag{12}$$

Theorem 1

$$(\forall V0a \in ty\_2Enum\_2Enum.(\forall V1n1 \in ty\_2Enum\_2Enum.(\forall V2n2 \in ty\_2Enum\_2Enum.((ap\ (ap\ (c\_2Elist\_2EAPPEND\ ty\_2Enum\_2Enum)\ (ap\ (ap\ (c\_2Elist\_2EGENLIST\ ty\_2Enum\_2Enum)\ (ap\ c\_2Earithmetic\_2E\_2B\ V0a))\ V1n1))\ (ap\ (ap\ (c\_2Elist\_2EGENLIST\ ty\_2Enum\_2Enum)\ (ap\ c\_2Earithmetic\_2E\_2B\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V1n1)\ V0a)))\ V2n2)) = (ap\ (ap\ (c\_2Elist\_2EGENLIST\ ty\_2Enum\_2Enum)\ (ap\ c\_2Earithmetic\_2E\_2B\ V0a))\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V1n1)\ V2n2))))))$$