

thm_2Elist_2ELAST__CONS (TMUqdPsE- HeS7ytwMhdonDtzB14QR1z7HqUw)

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Definition 1 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p \Rightarrow P \Rightarrow Q)$ of type ι .

Definition 2 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 3 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 4 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c_2Emin_2E_3D (2^{A-27a})))$

Definition 5 We define c_2Ebool_2EF to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 6 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t)))$

Definition 7 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2EF$

Let $ty_2Elist_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty_2Elist_2Elist A0) \quad (1)$$

Let $c_2Elist_2ENIL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Elist_2ENIL A_27a \in (ty_2Elist_2Elist A_27a) \quad (2)$$

Definition 8 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A$. **if** $(\exists x \in A.p (ap P x))$ **then** *(the* $(\lambda x.x \in A \wedge p)$ *of type* $\iota \Rightarrow \iota$.

Definition 9 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a.(ap$

Let $c_2Elist_2ECONS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Elist_2ECONS A_27a \in (((ty_2Elist_2Elist A_27a)^{(ty_2Elist_2Elist A_27a)})^{A_27a}) \quad (3)$$

Let $c_2Elist_2ELAST : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2ELAST\ A_27a \in (A_27a^{(ty_2Elist_2Elist\ A_27a)}) \quad (4)$$

Assume the following.

$$True \quad (5)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \quad (6)$$

Assume the following.

$$(\forall V0t \in 2. (False \Rightarrow (p\ V0t))) \quad (7)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0t \in 2. ((\forall V1x \in A_27a. (p\ V0t)) \Leftrightarrow (p\ V0t))) \quad (8)$$

Assume the following.

$$(\forall V0t \in 2. (((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \wedge True) \Leftrightarrow (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge (((p\ V0t) \wedge False) \Leftrightarrow False) \wedge (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)))))) \quad (9)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. ((V0x = V0x) \Leftrightarrow True)) \quad (10)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0t1 \in A_27a. (\forall V1t2 \in A_27a. (((ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27a)\ c_2Ebool_2ET)\ V0t1)\ V1t2) = V0t1) \wedge ((ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27a)\ c_2Ebool_2EF)\ V0t1)\ V1t2) = V1t2)))))) \quad (11)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0a1 \in (ty_2Elist_2Elist\ A_27a). (\forall V1a0 \in A_27a. (\neg ((ap\ (ap\ (c_2Elist_2ECONS\ A_27a)\ V1a0)\ V0a1) = (c_2Elist_2ENIL\ A_27a)))))) \quad (12)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0h \in A_27a. (\forall V1t \in (ty_2Elist_2Elist\ A_27a). ((ap\ (c_2Elist_2ELAST\ A_27a)\ (ap\ (ap\ (c_2Elist_2ECONS\ A_27a)\ V0h)\ V1t)) = (ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27a)\ (ap\ (ap\ (c_2Emin_2E_3D\ (ty_2Elist_2Elist\ A_27a)\ V1t)\ (c_2Elist_2ENIL\ A_27a)))\ V0h)\ (ap\ (c_2Elist_2ELAST\ A_27a)\ V1t)))))) \quad (13)$$

Theorem 1

$$\begin{aligned} \forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow & ((\forall V0x \in A_{.27a}.((ap\ (c_{.2Elist_{.2ELAST}} \\ A_{.27a})\ (ap\ (ap\ (c_{.2Elist_{.2ECONS}}\ A_{.27a})\ V0x)\ (c_{.2Elist_{.2ENIL}}\ A_{.27a}))) = \\ & V0x)) \wedge (\forall V1x \in A_{.27a}.(\forall V2y \in A_{.27a}.(\forall V3z \in (\\ & ty_{.2Elist_{.2Elist}}\ A_{.27a}).((ap\ (c_{.2Elist_{.2ELAST}}\ A_{.27a})\ (ap\ (ap \\ & (c_{.2Elist_{.2ECONS}}\ A_{.27a})\ V1x)\ (ap\ (ap\ (c_{.2Elist_{.2ECONS}}\ A_{.27a})\ V2y) \\ & V3z))) = (ap\ (c_{.2Elist_{.2ELAST}}\ A_{.27a})\ (ap\ (ap\ (c_{.2Elist_{.2ECONS}}\ A_{.27a}) \\ & V2y)\ V3z)))))) \end{aligned}$$