thm_2Elist_2ELENGTH__EQ__NUM (TMFjRQJ8hMobTeWr1NYFeKdxTnsY6sGKoci)

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Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V$

Definition 3 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

Definition 4 *We define* c_2Ebool_2EF *to be* $(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V0t \in 2.V0t))$.

Let $ty\_2Elist\_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Elist\_2Elist\ A0) \tag{1}$$

Let $c\_2Elist\_2ENIL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ENIL\ A\_27a \in (ty\_2Elist\_2Elist\ A\_27a) \tag{2}$$

Let $c\_2Enum\_2EZERO\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2EZERO\_\_REP \in omega \tag{3}$$

Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{4}$$

Let $c\_2Enum\_2EABS\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EABS\_\_num \in (ty\_2Enum\_2Enum^{omega}) \tag{5}$$

Definition 5 *We define* c_2Enum_2E0 *to be* $(ap\ c\_2Enum\_2EABS\_\_num\ c\_2Enum\_2EZERO\_\_REP)$.

Let $c\_2Elist\_2ECONS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ECONS\ A\_27a \in (((ty\_2Elist\_2Elist\ A\_27a)^{(ty\_2Elist\_2Elist\ A\_27a)})^{A\_27a}) \tag{6}$$

Let $c\_2Enum\_2EREP\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EREP\_\_num \in (omega^{ty\_2Enum\_2Enum}) \tag{7}$$

Let $c\_2Enum\_2ESUC\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2ESUC\_\_REP \in (omega^{omega}) \tag{8}$$

Definition 6 *We define* c_2Enum_2ESUC *to be* $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_\_num\ ($

Let $c\_2Elist\_2EAPPEND : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2EAPPEND\ A\_27a \in (((ty\_2Elist\_2Elist\ A\_27a)^{(ty\_2Elist\_2Elist\ A\_27a)})^{(ty\_2Elist\_2Elist\ A\_27a)}) \tag{9}$$

Definition 7 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 8 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in 2$

Definition 9 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^{A}.$**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge p$ *of type* $\iota \Rightarrow \iota$.

Definition 10 *We define* c_2Ebool_2E_3F *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ V0P\ (ap\ (\mathsf{c\_2Emin\_2E\_40}$

Let $c\_2Earithmetic\_2E\_2B : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{10}$$

Let $c\_2Elist\_2ELENGTH : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ELENGTH\ A\_27a \in (ty\_2Enum\_2Enum^{(ty\_2Elist\_2Elist\ A\_27a)}) \tag{11}$$

Assume the following.

$$True \tag{12}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A\_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \tag{13}$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \wedge True) \Leftrightarrow (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge ((((p\ V0t) \wedge False) \Leftrightarrow False) \wedge (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t))))))) \tag{14}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \tag{15}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0l \in (ty\_2Elist\_2Elist\ A\_27a).(((ap\ (c\_2Elist\_2ELENGTH\ A\_27a)\ V0l) = \mathsf{c\_2Enum\_2E0}) \Leftrightarrow (V0l = (c\_2Elist\_2ENIL\ A\_27a)))) \tag{16}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0l \in (ty\_2Elist\_2Elist\ A\_27a).(\forall V1n \in ty\_2Enum\_2Enum.(((ap\ (c\_2Elist\_2ELENGTH\ A\_27a)\ V0l) = (ap\ \mathsf{c\_2Enum\_2ESUC}\ V1n)) \Leftrightarrow (\exists V2h \in A\_27a.(\exists V3l\_27 \in (ty\_2Elist\_2Elist\ A\_27a).(((ap\ (c\_2Elist\_2ELENGTH\ A\_27a)\ V3l\_27) = V1n) \wedge (V0l = (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V2h)\ V3l\_27)))))))) \tag{17}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0l \in (ty\_2Elist\_2Elist\ A\_27a).(\forall V1n1 \in ty\_2Enum\_2Enum.(\forall V2n2 \in ty\_2Enum\_2Enum.(((ap\ (c\_2Elist\_2ELENGTH\ A\_27a)\ V0l) = (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V1n1)\ V2n2)) \Leftrightarrow (\exists V3l1 \in (ty\_2Elist\_2Elist\ A\_27a).(\exists V4l2 \in (ty\_2Elist\_2Elist\ A\_27a).(((ap\ (c\_2Elist\_2ELENGTH\ A\_27a)\ V3l1) = V1n1) \wedge (((ap\ (c\_2Elist\_2ELENGTH\ A\_27a)\ V4l2) = V2n2) \wedge (V0l = (ap\ (ap\ (c\_2Elist\_2EAPPEND\ A\_27a)\ V3l1)\ V4l2)))))))))) \tag{18}$$

Theorem 1

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow ((\forall V0l \in (ty\_2Elist\_2Elist\ A\_27a).(((ap\ (c\_2Elist\_2ELENGTH\ A\_27a)\ V0l) = \mathsf{c\_2Enum\_2E0}) \Leftrightarrow (V0l = (c\_2Elist\_2ENIL\ A\_27a)))) \wedge ((\forall V1l \in (ty\_2Elist\_2Elist\ A\_27a).(\forall V2n \in ty\_2Enum\_2Enum.(((ap\ (c\_2Elist\_2ELENGTH\ A\_27a)\ V1l) = (ap\ \mathsf{c\_2Enum\_2ESUC}\ V2n)) \Leftrightarrow (\exists V3h \in A\_27a.(\exists V4l\_27 \in (ty\_2Elist\_2Elist\ A\_27a).(((ap\ (c\_2Elist\_2ELENGTH\ A\_27a)\ V4l\_27) = V2n) \wedge (V1l = (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V3h)\ V4l\_27)))))))) \wedge (\forall V5l \in (ty\_2Elist\_2Elist\ A\_27a).(\forall V6n1 \in ty\_2Enum\_2Enum.(\forall V7n2 \in ty\_2Enum\_2Enum.(((ap\ (c\_2Elist\_2ELENGTH\ A\_27a)\ V5l) = (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V6n1)\ V7n2)) \Leftrightarrow (\exists V8l1 \in (ty\_2Elist\_2Elist\ A\_27a).(\exists V9l2 \in (ty\_2Elist\_2Elist\ A\_27a).(((ap\ (c\_2Elist\_2ELENGTH\ A\_27a)\ V8l1) = V6n1) \wedge (((ap\ (c\_2Elist\_2ELENGTH\ A\_27a)\ V9l2) = V7n2) \wedge (V5l = (ap\ (ap\ (c\_2Elist\_2EAPPEND\ A\_27a)\ V8l1)\ V9l2))))))))))))$$