thm_2Elist_2ELENGTH__EQ__NUM__compute (TMLQDAwpXvVXgFtQMqg-WcaG5uLcq3sfVAXW)

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Let $c\_2Enum\_2EZERO\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2EZERO\_\_REP \in omega \tag{1}$$

Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{2}$$

Let $c\_2Enum\_2EABS\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EABS\_\_num \in (ty\_2Enum\_2Enum^{omega}) \tag{3}$$

Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Enum_2E0 *to be* $(ap\ c\_2Enum\_2EABS\_\_num\ c\_2Enum\_2EZERO\_\_REP)$.

Let $c\_2Enum\_2EREP\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EREP\_\_num \in (omega^{ty\_2Enum\_2Enum}) \tag{4}$$

Let $c\_2Enum\_2ESUC\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2ESUC\_\_REP \in (omega^{omega}) \tag{5}$$

Definition 3 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V$

Definition 4 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

Definition 5 *We define* c_2Enum_2ESUC *to be* $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_\_num\ ($

Let $c\_2Earithmetic\_2E\_2B : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{6}$$

Definition 6 *We define* c_2Earithmetic_2EBIT2 *to be* $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic\_$

Definition 7 *We define* c_2Earithmetic_2EZERO *to be* c_2Enum_2E0.

Let $c\_2Earithmetic\_2E\_2D : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2D \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{7}$$

Definition 8 *We define* c_2Earithmetic_2EBIT1 *to be* $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic\_$

Definition 9 *We define* c_2Earithmetic_2ENUMERAL *to be* $\lambda V0x \in ty\_2Enum\_2Enum.V0x.$

Definition 10 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Let $ty\_2Elist\_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Elist\_2Elist\ A0) \tag{8}$$

Let $c\_2Elist\_2EAPPEND : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2EAPPEND\ A\_27a \in (((ty\_2Elist\_2Elist\ A\_27a)^{(ty\_2Elist\_2Elist\ A\_27a)})^{(ty\_2Elist\_2Elist\ A\_27a)}) \tag{9}$$

Let $c\_2Elist\_2ECONS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ECONS\ A\_27a \in (((ty\_2Elist\_2Elist\ A\_27a)^{(ty\_2Elist\_2Elist\ A\_27a)})^{A\_27a}) \tag{10}$$

Definition 11 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.$**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge$ *of type* $\iota \Rightarrow \iota$.

Definition 12 *We define* c_2Ebool_2E_3F *to be* $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ V0P\ (ap\ ($c_2Emin_2E_40

Let $c\_2Elist\_2ENIL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ENIL\ A\_27a \in (ty\_2Elist\_2Elist\ A\_27a) \tag{11}$$

Let $c\_2Elist\_2ELENGTH : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ELENGTH\ A\_27a \in (ty\_2Enum\_2Enum^{(ty\_2Elist\_2Elist\ A\_27a)}) \tag{12}$$

Definition 13 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ ($c_2Ebool_2E_21 $2)\ (\lambda V2t \in$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0f \in ((A\_27a^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}). (\forall V1g \in (A\_27a^{ty\_2Enum\_2Enum}).((\forall V2n \in ty\_2Enum\_2Enum. ((ap\ V1g\ (ap\ \mathsf{c\_2Enum\_2ESUC}\ V2n)) = (ap\ (ap\ V0f\ V2n)\ (ap\ \mathsf{c\_2Enum\_2ESUC}\ V2n))))\Leftrightarrow((\forall V3n \in ty\_2Enum\_2Enum.((ap\ V1g\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V3n))) = (ap\ (ap\ V0f\ (ap\ (ap\ c\_2Earithmetic\_2E\_2D\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V3n)))\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO}))))\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V3n)))))\wedge (\forall V4n \in ty\_2Enum\_2Enum.((ap\ V1g\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V4n))) = (ap\ (ap\ V0f\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V4n)))\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V4n)))))))))) \tag{13}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1)\Rightarrow(p\ V1t2))\Rightarrow(((p\ V1t2)\Rightarrow(p\ V0t1))\Rightarrow((p\ V0t1)\Leftrightarrow(p\ V1t2)))))) \tag{14}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p\ V0t1)\wedge ((p\ V1t2)\wedge(p\ V2t3)))\Leftrightarrow(((p\ V0t1)\wedge(p\ V1t2))\wedge(p\ V2t3)))))) \tag{15}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y)\Leftrightarrow(V1y = V0x)))) \tag{16}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow ((\forall V0l \in (ty\_2Elist\_2Elist\ A\_27a).(((ap\ (c\_2Elist\_2ELENGTH\ A\_27a)\ V0l) = \mathsf{c\_2Enum\_2E0})\Leftrightarrow(V0l = (c\_2Elist\_2ENIL\ A\_27a))))\wedge((\forall V1l \in (ty\_2Elist\_2Elist\ A\_27a).(\forall V2n \in ty\_2Enum\_2Enum.(((ap\ (c\_2Elist\_2ELENGTH\ A\_27a)\ V1l) = (ap\ \mathsf{c\_2Enum\_2ESUC}\ V2n))\Leftrightarrow(\exists V3h \in A\_27a.(\exists V4l\_27 \in (ty\_2Elist\_2Elist\ A\_27a).(((ap\ (c\_2Elist\_2ELENGTH\ A\_27a)\ V4l\_27) = V2n)\wedge(V1l = (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V3h)\ V4l\_27))))))))\wedge (\forall V5l \in (ty\_2Elist\_2Elist\ A\_27a).(\forall V6n1 \in ty\_2Enum\_2Enum. (\forall V7n2 \in ty\_2Enum\_2Enum.(((ap\ (c\_2Elist\_2ELENGTH\ A\_27a)\ V5l) = (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V6n1)\ V7n2))\Leftrightarrow(\exists V8l1 \in (ty\_2Elist\_2Elist\ A\_27a).(\exists V9l2 \in (ty\_2Elist\_2Elist\ A\_27a). (((ap\ (c\_2Elist\_2ELENGTH\ A\_27a)\ V8l1) = V6n1)\wedge(((ap\ (c\_2Elist\_2ELENGTH\ A\_27a)\ V9l2) = V7n2)\wedge(V5l = (ap\ (ap\ (c\_2Elist\_2EAPPEND\ A\_27a)\ V8l1)\ V9l2)))))))))))) \tag{17}$$

Theorem 1

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow ((\forall V0l \in (ty\_2Elist\_2Elist\ A\_27a).(((ap\ (c\_2Elist\_2ELENGTH\ A\_27a)\ V0l) = \mathsf{c\_2Enum\_2E0}) \Leftrightarrow (V0l = (c\_2Elist\_2ENIL\ A\_27a)))) \wedge ((\forall V1l \in (ty\_2Elist\_2Elist\ A\_27a).(\forall V2n \in ty\_2Enum\_2Enum.(((ap\ (c\_2Elist\_2ELENGTH\ A\_27a)\ V1l) = (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V2n))) \Leftrightarrow (\exists V3h \in A\_27a.(\exists V4l\_27 \in (ty\_2Elist\_2Elist\ A\_27a).(((ap\ (c\_2Elist\_2ELENGTH\ A\_27a)\ V4l\_27) = (ap\ (ap\ c\_2Earithmetic\_2E\_2D\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V2n)))\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO})))) \wedge (V1l = (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V3h)\ V4l\_27)))))))) \wedge ((\forall V5l \in (ty\_2Elist\_2Elist\ A\_27a).(\forall V6n \in ty\_2Enum\_2Enum.(((ap\ (c\_2Elist\_2ELENGTH\ A\_27a)\ V5l) = (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V6n))) \Leftrightarrow (\exists V7h \in A\_27a.(\exists V8l\_27 \in (ty\_2Elist\_2Elist\ A\_27a).(((ap\ (c\_2Elist\_2ELENGTH\ A\_27a)\ V8l\_27) = (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V6n))) \wedge (V5l = (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V7h)\ V8l\_27)))))))) \wedge (\forall V9l \in (ty\_2Elist\_2Elist\ A\_27a).(\forall V10n1 \in ty\_2Enum\_2Enum.(\forall V11n2 \in ty\_2Enum\_2Enum.(((ap\ (c\_2Elist\_2ELENGTH\ A\_27a)\ V9l) = (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V10n1)\ V11n2)) \Leftrightarrow (\exists V12l1 \in (ty\_2Elist\_2Elist\ A\_27a).(\exists V13l2 \in (ty\_2Elist\_2Elist\ A\_27a).(((ap\ (c\_2Elist\_2ELENGTH\ A\_27a)\ V12l1) = V10n1) \wedge (((ap\ (c\_2Elist\_2ELENGTH\ A\_27a)\ V13l2) = V11n2) \wedge (V9l = (ap\ (ap\ (c\_2Elist\_2EAPPEND\ A\_27a)\ V12l1)\ V13l2))))))))))))$$