thm_2Elist_2ELENGTH_REVERSE (TMVDp-pHhPC1Q6PSz88s7baFGQXjkDwua3fD)

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Let $c\_2Enum\_2EZERO\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2EZERO\_\_REP \in omega \tag{1}$$

Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{2}$$

Let $c\_2Enum\_2EABS\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EABS\_\_num \in (ty\_2Enum\_2Enum^{omega}) \tag{3}$$

Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$
of type $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Enum_2E0 *to be* $(ap\ c\_2Enum\_2EABS\_\_num\ c\_2Enum\_2EZERO\_\_REP)$.

Definition 3 *We define* c_2Earithmetic_2EZERO *to be* c_2Enum_2E0.

Let $c\_2Enum\_2EREP\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EREP\_\_num \in (omega^{ty\_2Enum\_2Enum}) \tag{4}$$

Let $c\_2Enum\_2ESUC\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2ESUC\_\_REP \in (omega^{omega}) \tag{5}$$

Definition 4 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ ($c_2Emin_2E_3D$\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1$

Definition 5 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ ($c_2Emin_2E_3D$\ (2^{A\_27}$

Definition 6 *We define* c_2Enum_2ESUC *to be* $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_\_num\ ($

Let $c\_2Earithmetic\_2E\_2B : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{6}$$

Definition 7 *We define* c_2Earithmetic_2EBIT1 *to be* $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic\_$

Definition 8 *We define* c_2Earithmetic_2ENUMERAL *to be* $\lambda V0x \in ty\_2Enum\_2Enum.V0x.$

Definition 9 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$
of type ι.

Let $ty\_2Elist\_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Elist\_2Elist\ A0) \qquad (7)$$

Let $c\_2Elist\_2ELENGTH : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ELENGTH\ A\_27a \in (ty\_2Enum\_2Enum^{(ty\_2Elist\_2Elist\ A\_27a)})$$
$$(8)$$

Let $c\_2Elist\_2EAPPEND : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2EAPPEND\ A\_27a \in (((ty\_2Elist\_2Elist$$
$$A\_27a)^{(ty\_2Elist\_2Elist\ A\_27a)})^{(ty\_2Elist\_2Elist\ A\_27a)})$$
$$(9)$$

Let $c\_2Elist\_2ECONS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ECONS\ A\_27a \in (((ty\_2Elist\_2Elist$$
$$A\_27a)^{(ty\_2Elist\_2Elist\ A\_27a)})^{A\_27a})$$
$$(10)$$

Let $c\_2Elist\_2ENIL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ENIL\ A\_27a \in (ty\_2Elist\_2Elist$$
$$A\_27a)$$
$$(11)$$

Let $c\_2Elist\_2EREVERSE : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2EREVERSE\ A\_27a \in ((ty\_2Elist\_2Elist$$
$$A\_27a)^{(ty\_2Elist\_2Elist\ A\_27a)})$$
$$(12)$$

Definition 10 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c\_2Ebool\_2E\_21\ 2)\ (\lambda V2t \in$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m)$$
$$\mathsf{c\_2Enum\_2E0}) = V0m)) \qquad (13)$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.($$
$$(ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ V1n) = (ap\ (ap\ c\_2Earithmetic\_2E\_2B$$
$$V1n)\ V0m))))$$
$$(14)$$

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Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.((ap \ \mathsf{c\_2Enum\_2ESUC} \ V0m) = (ap \ (ap$$
$$c\_2Earithmetic\_2E\_2B \ V0m) \ (ap \ \mathsf{c\_2Earithmetic\_2ENUMERAL} \ (ap \ \mathsf{c\_2Earithmetic\_2EBIT1}$$
$$\mathsf{c\_2Earithmetic\_2EZERO})))))$$

$$(15)$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.($$
$$\forall V2p \in ty\_2Enum\_2Enum.(((ap \ (ap \ c\_2Earithmetic\_2E\_2B \ V0m)$$
$$V2p) = (ap \ (ap \ c\_2Earithmetic\_2E\_2B \ V1n) \ V2p)) \Leftrightarrow (V0m = V1n)))))$$

$$(16)$$

Assume the following.

$$True \qquad\qquad (17)$$

Assume the following.

$$\forall A\_27a.nonempty \ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow$$
$$True)) \qquad\qquad (18)$$

Assume the following.

$$\forall A\_27a.nonempty \ A\_27a \Rightarrow (((ap \ (c\_2Elist\_2ELENGTH \ A\_27a)$$
$$(c\_2Elist\_2ENIL \ A\_27a)) = \mathsf{c\_2Enum\_2E0}) \wedge (\forall V0h \in A\_27a.($$
$$\forall V1t \in (ty\_2Elist\_2Elist \ A\_27a).((ap \ (c\_2Elist\_2ELENGTH$$
$$A\_27a) \ (ap \ (ap \ (c\_2Elist\_2ECONS \ A\_27a) \ V0h) \ V1t)) = (ap \ \mathsf{c\_2Enum\_2ESUC}$$
$$(ap \ (c\_2Elist\_2ELENGTH \ A\_27a) \ V1t)))))))$$

$$(19)$$

Assume the following.

$$\forall A\_27a.nonempty \ A\_27a \Rightarrow (\forall V0P \in (2^{(ty\_2Elist\_2Elist \ A\_27a)}).$$
$$(((p \ (ap \ V0P \ (c\_2Elist\_2ENIL \ A\_27a))) \wedge (\forall V1t \in (ty\_2Elist\_2Elist$$
$$A\_27a).((p \ (ap \ V0P \ V1t)) \Rightarrow (\forall V2h \in A\_27a.(p \ (ap \ V0P \ (ap \ (ap \ ($$
$$c\_2Elist\_2ECONS \ A\_27a) \ V2h) \ V1t))))))))) \Rightarrow (\forall V3l \in (ty\_2Elist\_2Elist$$
$$A\_27a).(p \ (ap \ V0P \ V3l)))))$$

$$(20)$$

Assume the following.

$$\forall A\_27a.nonempty \ A\_27a \Rightarrow (\forall V0l1 \in (ty\_2Elist\_2Elist$$
$$A\_27a).(\forall V1l2 \in (ty\_2Elist\_2Elist \ A\_27a).((ap \ (c\_2Elist\_2ELENGTH$$
$$A\_27a) \ (ap \ (ap \ (c\_2Elist\_2EAPPEND \ A\_27a) \ V0l1) \ V1l2)) = (ap \ (ap \ c\_2Earithmetic\_2E\_2B$$
$$(ap \ (c\_2Elist\_2ELENGTH \ A\_27a) \ V0l1)) \ (ap \ (c\_2Elist\_2ELENGTH \ A\_27a)$$
$$V1l2)))))$$

$$(21)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (((ap\ (c\_2Elist\_2EREVERSE\ A\_27a)$$
$$(c\_2Elist\_2ENIL\ A\_27a)) = (c\_2Elist\_2ENIL\ A\_27a)) \wedge (\forall V0h \in$$
$$A\_27a.(\forall V1t \in (ty\_2Elist\_2Elist\ A\_27a).((ap\ (c\_2Elist\_2EREVERSE$$
$$A\_27a)\ (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V0h)\ V1t)) = (ap\ (ap\ (c\_2Elist\_2EAPPEND$$
$$A\_27a)\ (ap\ (c\_2Elist\_2EREVERSE\ A\_27a)\ V1t))\ (ap\ (ap\ (c\_2Elist\_2ECONS$$
$$A\_27a)\ V0h)\ (c\_2Elist\_2ENIL\ A\_27a)))))))$$

$$(22)$$

Theorem 1

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0l \in (ty\_2Elist\_2Elist$$
$$A\_27a).((ap\ (c\_2Elist\_2ELENGTH\ A\_27a)\ (ap\ (c\_2Elist\_2EREVERSE$$
$$A\_27a)\ V0l)) = (ap\ (c\_2Elist\_2ELENGTH\ A\_27a)\ V0l)))$$