

thm\_2Elist\_2ELENGTH\_SNOC  
(TMWQ8aRS2x85L2J6YiS7dKb6nfEg1PVYbMn)

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**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Ebool\_2ET$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2))) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x)$

Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{1}$$

Let  $c\_2Enum\_2EREP\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EREP\_num \in (\omega^{ty\_2Enum\_2Enum}) \tag{2}$$

Let  $c\_2Enum\_2ESUC\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2ESUC\_REP \in (\omega^{\omega}) \tag{3}$$

Let  $c\_2Enum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum^{\omega}) \tag{4}$$

**Definition 3** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap (ap (c\_2Emin\_2E\_3D (2^{A\_27a}))$

**Definition 4** We define  $c\_2Enum\_2ESUC$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.(ap c\_2Enum\_2EABS\_num$

Let  $c\_2Enum\_2EZERO\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2EZERO\_REP \in \omega \tag{5}$$

**Definition 5** We define  $c\_2Enum\_2E0$  to be  $(ap c\_2Enum\_2EABS\_num c\_2Enum\_2EZERO\_REP)$ .

Let  $ty\_2Elist\_2Elist : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Elist\_2Elist\ A0) \tag{6}$$

Let  $c\_2Elist\_2ELENGTH : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ELENGTH\ A\_27a \in (ty\_2Enum\_2Enum^{(ty\_2Elist\_2Elist\ A\_27a)}) \tag{7}$$

**Definition 6** We define  $c\_2Emin\_2E\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p P \Rightarrow p Q)$  of type  $\iota$ .

Let  $c\_2Elist\_2ECONS : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_2Elist\_2ECONS A\_27a \in (((ty\_2Elist\_2Elist A\_27a)^{(ty\_2Elist\_2Elist A\_27a)})_{A\_27a}) \quad (8)$$

Let  $c\_2Elist\_2ENIL : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_2Elist\_2ENIL A\_27a \in (ty\_2Elist\_2Elist A\_27a) \quad (9)$$

Let  $c\_2Elist\_2ESNOC : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_2Elist\_2ESNOC A\_27a \in (((ty\_2Elist\_2Elist A\_27a)^{(ty\_2Elist\_2Elist A\_27a)})_{A\_27a}) \quad (10)$$

**Definition 7** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in 2.))$

Assume the following.

$$True \quad (11)$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A\_27a.(p V0t)) \Leftrightarrow (p V0t))) \quad (12)$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \quad (13)$$

Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty A\_27a \Rightarrow & (((ap (c\_2Elist\_2ELENGTH A\_27a) \\ & (c\_2Elist\_2ENIL A\_27a)) = c\_2Enum\_2E0) \wedge (\forall V0h \in A\_27a. \\ & \forall V1t \in (ty\_2Elist\_2Elist A\_27a).((ap (c\_2Elist\_2ELENGTH \\ A\_27a) (ap (ap (c\_2Elist\_2ECONS A\_27a) V0h) V1t)) = (ap c\_2Enum\_2ESUC \\ & (ap (c\_2Elist\_2ELENGTH A\_27a) V1t)))))) \end{aligned} \quad (14)$$

Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty A\_27a \Rightarrow & (\forall V0P \in (2^{(ty\_2Elist\_2Elist A\_27a)}). \\ & (((p (ap V0P (c\_2Elist\_2ENIL A\_27a))) \wedge (\forall V1t \in (ty\_2Elist\_2Elist \\ & A\_27a).((p (ap V0P V1t)) \Rightarrow (\forall V2h \in A\_27a.(p (ap V0P (ap (ap ( \\ c\_2Elist\_2ECONS A\_27a) V2h) V1t)))))) \Rightarrow (\forall V3l \in (ty\_2Elist\_2Elist \\ & A\_27a).(p (ap V0P V3l)))))) \end{aligned} \quad (15)$$

Assume the following.

$$\begin{aligned}
\forall A\_27a.nonempty\ A\_27a \Rightarrow & ((\forall V0x \in A\_27a. ((ap\ (ap\ (c\_2Elist\_2ESNOC \\
& A\_27a)\ V0x)\ (c\_2Elist\_2ENIL\ A\_27a)) = (ap\ (ap\ (c\_2Elist\_2ECONS \\
& A\_27a)\ V0x)\ (c\_2Elist\_2ENIL\ A\_27a)))) \wedge (\forall V1x \in A\_27a. (\forall V2x\_27 \in \\
& A\_27a. (\forall V3l \in (ty\_2Elist\_2Elist\ A\_27a). ((ap\ (ap\ (c\_2Elist\_2ESNOC \\
& A\_27a)\ V1x)\ (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V2x\_27)\ V3l)) = (ap\ ( \\
& ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V2x\_27)\ (ap\ (ap\ (c\_2Elist\_2ESNOC\ A\_27a) \\
& V1x)\ V3l)))))))))
\end{aligned}
\tag{16}$$

**Theorem 1**

$$\begin{aligned}
\forall A\_27a.nonempty\ A\_27a \Rightarrow & (\forall V0x \in A\_27a. (\forall V1l \in \\
& (ty\_2Elist\_2Elist\ A\_27a). ((ap\ (c\_2Elist\_2ELENGTH\ A\_27a)\ (ap \\
& (ap\ (c\_2Elist\_2ESNOC\ A\_27a)\ V0x)\ V1l)) = (ap\ c\_2Enum\_2ESUC\ (ap\ ( \\
& c\_2Elist\_2ELENGTH\ A\_27a)\ V1l))))))
\end{aligned}$$