thm_2Elist_2ELENGTH__o__REVERSE (TMEo4P68ktPDPqGxKdQ5ScQifaNVkx1SAjA)

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Definition 1 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 2 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 3 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V$

Definition 4 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

Definition 5 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in 2$

Definition 6 *We define* c_2Ecombin_2Eo *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda V0f \in (A\_27b^{A\_27c}).\lambda V1g$

Let $ty\_2Elist\_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Elist\_2Elist\ A0) \tag{1}$$

Let $c\_2Elist\_2EREVERSE : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2EREVERSE\ A\_27a \in ((ty\_2Elist\_2Elist\ A\_27a)^{(ty\_2Elist\_2Elist\ A\_27a)}) \tag{2}$$

Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{3}$$

Let $c\_2Elist\_2ELENGTH : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ELENGTH\ A\_27a \in (ty\_2Enum\_2Enum^{(ty\_2Elist\_2Elist\ A\_27a)}) \tag{4}$$

Assume the following.

$$True \tag{5}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A\_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \tag{6}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \tag{7}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0f \in (A\_27b^{A\_27a}).(\forall V1g \in (A\_27b^{A\_27a}).((V0f = V1g) \Leftrightarrow (\forall V2x \in A\_27a.((ap\ V0f\ V2x) = (ap\ V1g\ V2x)))))) \tag{8}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow \forall A\_27c. nonempty\ A\_27c \Rightarrow (\forall V0f \in (A\_27b^{A\_27a}).(\forall V1g \in (A\_27a^{A\_27c}). (\forall V2x \in A\_27c.((ap\ (ap\ (ap\ (\mathsf{c\_2Ecombin\_2Eo}\ A\_27c\ A\_27b\ A\_27a)\ V0f)\ V1g)\ V2x) = (ap\ V0f\ (ap\ V1g\ V2x)))))) \tag{9}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0l \in (ty\_2Elist\_2Elist\ A\_27a).((ap\ (c\_2Elist\_2ELENGTH\ A\_27a)\ (ap\ (c\_2Elist\_2EREVERSE\ A\_27a)\ V0l)) = (ap\ (c\_2Elist\_2ELENGTH\ A\_27a)\ V0l))) \tag{10}$$

Theorem 1

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow \forall A\_27c. nonempty\ A\_27c \Rightarrow (\forall V0f \in ((ty\_2Elist\_2Elist\ A\_27c)^{A\_27b}). (((ap\ (ap\ (\mathsf{c\_2Ecombin\_2Eo}\ (ty\_2Elist\_2Elist\ A\_27a)\ ty\_2Enum\_2Enum\ (ty\_2Elist\_2Elist\ A\_27a))\ (c\_2Elist\_2ELENGTH\ A\_27a))\ (c\_2Elist\_2EREVERSE\ A\_27a)) = (c\_2Elist\_2ELENGTH\ A\_27a)) \wedge ((ap\ (ap\ (\mathsf{c\_2Ecombin\_2Eo}\ A\_27b\ ty\_2Enum\_2Enum\ (ty\_2Elist\_2Elist\ A\_27c))\ (c\_2Elist\_2ELENGTH\ A\_27c))\ (ap\ (ap\ (\mathsf{c\_2Ecombin\_2Eo}\ A\_27b\ (ty\_2Elist\_2Elist\ A\_27c)\ (ty\_2Elist\_2Elist\ A\_27c))\ (c\_2Elist\_2EREVERSE\ A\_27c))\ V0f)) = (ap\ (ap\ (\mathsf{c\_2Ecombin\_2Eo}\ A\_27b\ ty\_2Enum\_2Enum\ (ty\_2Elist\_2Elist\ A\_27c))\ (c\_2Elist\_2ELENGTH\ A\_27c))\ V0f))))$$