

thm_2Elist_2ELIST__REL__LENGTH
(TMFGmr9jG95DhZ1xR3n57jRG2Aj96tWzQbP)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a}))$

Definition 4 We define c_2Ebool_2EF to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 5 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 6 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2EF$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \tag{1}$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \tag{2}$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \tag{3}$$

Definition 7 We define c_2Enum_2E0 to be $(ap c_2Enum_2EABS_num c_2Enum_2EZERO_REP)$.

Let $ty_2Elist_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty (ty_2Elist_2Elist\ A0) \tag{4}$$

Let $c_2Elist_2ELENGTH : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2ELENGTH\ A_27a \in (ty_2Enum_2Enum^{(ty_2Elist_2Elist\ A_27a)}) \tag{5}$$

Let $c_2Elist_2ECONS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2ECONS\ A_27a \in (((ty_2Elist_2Elist\ A_27a)^{(ty_2Elist_2Elist\ A_27a)})^{A_27a}) \quad (6)$$

Let $c_2Elist_2ELIST_REL : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Elist_2ELIST_REL\ A_27a\ A_27b \in (((2^{(ty_2Elist_2Elist\ A_27b)})^{(ty_2Elist_2Elist\ A_27a)})^{(2^{A_27b})^{A_27a}}) \quad (7)$$

Let $c_2Elist_2ENIL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2ENIL\ A_27a \in (ty_2Elist_2Elist\ A_27a) \quad (8)$$

Definition 8 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in 2.$

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \quad (9)$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \quad (10)$$

Definition 9 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap\ c_2Enum_2EABS_num\ ($

Assume the following.

$$True \quad (11)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Rightarrow True) \Leftrightarrow \\ & True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((\\ & (p\ V0t) \Rightarrow False) \Leftrightarrow (\neg (p\ V0t)))))) \end{aligned} \quad (12)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \quad (13)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (14)$$

Assume the following.

$$\begin{aligned} & (\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p\ V0t1) \Rightarrow \\ & ((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \end{aligned} \quad (15)$$

Assume the following.

$$2.(((p \ V0x) \Leftrightarrow (p \ V1x_27)) \wedge ((p \ V1x_27) \Rightarrow ((p \ V2y) \Leftrightarrow (p \ V3y_27)))) \Rightarrow \quad (16)$$

$$(((p \ V0x) \Rightarrow (p \ V2y)) \Leftrightarrow ((p \ V1x_27) \Rightarrow (p \ V3y_27))))$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (((ap \ (c_2Elist_2ELENGTH \ A_27a) \ (c_2Elist_2ENIL \ A_27a)) = c_2Enum_2E0) \wedge (\forall V0h \in \ A_27a. (\forall V1t \in \ (ty_2Elist_2Elist \ A_27a). ((ap \ (c_2Elist_2ELENGTH \ A_27a) \ (ap \ (ap \ (c_2Elist_2ECONS \ A_27a) \ V0h) \ V1t)) = (ap \ c_2Enum_2ESUC \ (ap \ (c_2Elist_2ELENGTH \ A_27a) \ V1t))))))) \quad (17)$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow \forall A_27b.nonempty \ A_27b \Rightarrow (\forall V0R \in \ ((2^{A_27b})^{A_27a}). (\forall V1LIST_REL_27 \in \ ((2^{(ty_2Elist_2Elist \ A_27b)})^{(ty_2Elist_2Elist \ A_27a)}). ((p \ (ap \ (ap \ V1LIST_REL_27 \ (c_2Elist_2ENIL \ A_27a)) \ (c_2Elist_2ENIL \ A_27b))) \wedge (\forall V2h1 \in \ A_27a. (\forall V3h2 \in \ A_27b. (\forall V4t1 \in \ (ty_2Elist_2Elist \ A_27a). (\forall V5t2 \in \ (ty_2Elist_2Elist \ A_27b). (((p \ (ap \ (ap \ V0R \ V2h1) \ V3h2)) \wedge ((p \ (ap \ (ap \ (ap \ (c_2Elist_2ELIST_REL \ A_27a \ A_27b) \ V0R) \ V4t1) \ V5t2))) \wedge (p \ (ap \ (ap \ V1LIST_REL_27 \ V4t1) \ V5t2)))))) \Rightarrow (p \ (ap \ (ap \ V1LIST_REL_27 \ (ap \ (ap \ (c_2Elist_2ECONS \ A_27a) \ V2h1) \ V4t1)) \ (ap \ (ap \ (c_2Elist_2ECONS \ A_27b) \ V3h2) \ V5t2)))))) \Rightarrow (\forall V6a0 \in \ (ty_2Elist_2Elist \ A_27a). (\forall V7a1 \in \ (ty_2Elist_2Elist \ A_27b). ((p \ (ap \ (ap \ (ap \ (c_2Elist_2ELIST_REL \ A_27a \ A_27b) \ V0R) \ V6a0) \ V7a1)) \Rightarrow (p \ (ap \ (ap \ V1LIST_REL_27 \ V6a0) \ V7a1)))))) \quad (18)$$

Assume the following.

$$(\forall V0m \in \ ty_2Enum_2Enum. (\forall V1n \in \ ty_2Enum_2Enum. ((ap \ c_2Enum_2ESUC \ V0m) = (ap \ c_2Enum_2ESUC \ V1n)) \Leftrightarrow (V0m = V1n))) \quad (19)$$

Theorem 1

$$\forall A_27a.nonempty \ A_27a \Rightarrow \forall A_27b.nonempty \ A_27b \Rightarrow (\forall V0R \in \ ((2^{A_27b})^{A_27a}). (\forall V1x \in \ (ty_2Elist_2Elist \ A_27a). (\forall V2y \in \ (ty_2Elist_2Elist \ A_27b). ((p \ (ap \ (ap \ (ap \ (c_2Elist_2ELIST_REL \ A_27a \ A_27b) \ V0R) \ V1x) \ V2y)) \Rightarrow ((ap \ (c_2Elist_2ELENGTH \ A_27a) \ V1x) = (ap \ (c_2Elist_2ELENGTH \ A_27b) \ V2y))))))$$