

thm_2Elist_2ELIST__REL__MAP1 (TMbe- QogFp8pDGVVJfngPEUwAJhV1nUpZHWZ)

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Definition 1 We define `c_2Emin_2E_3D` to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define `c_2Ebool_2ET` to be $(ap (ap (c_2Emin_2E_3D (2^2))) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define `c_2Ebool_2E_21` to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a}))))$

Definition 4 We define `c_2Ebool_2EF` to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 5 We define `c_2Emin_2E_3D_3D_3E` to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 6 We define `c_2Ebool_2E_7E` to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2EF))$

Definition 7 We define `c_2Ecombin_2Eo` to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda V0f \in (A_27b^{A_27c}).\lambda V1g$

Let `ty_2Elist_2Elist` : $\iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty_2Elist_2Elist A0) \quad (1)$$

Let `c_2Elist_2EMAP` : $\iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Elist_2EMAP A_27a A_27b \in (((ty_2Elist_2Elist A_27b)^{(ty_2Elist_2Elist A_27a)})^{(A_27b^{A_27a})}) \quad (2)$$

Let `ty_2Enum_2Enum` : ι be given. Assume the following.

$$nonempty ty_2Enum_2Enum \quad (3)$$

Let `c_2Elist_2EEL` : $\iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Elist_2EEL A_27a \in ((A_27a^{(ty_2Elist_2Elist A_27a)})^{ty_2Enum_2Enum}) \quad (4)$$

Definition 8 We define `c_2Ebool_2E_2F_5C` to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t))))$

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \quad (5)$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \quad (6)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \quad (7)$$

Definition 9 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap\ c_2Enum_2EABS_num$

Definition 10 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.\mathbf{if} (\exists x \in A.p (ap\ P\ x)) \mathbf{then} (the (\lambda x.x \in A \wedge$
of type $\iota \Rightarrow \iota$.

Definition 11 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ V0P (ap (c_2Emin_2E_40$

Definition 12 We define $c_2Eprim_rec_2E_3C$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Let $c_2Elist_2ELENGTH : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2ELENGTH\ A_27a \in (ty_2Enum_2Enum^{(ty_2Elist_2Elist\ A_27a)}) \quad (8)$$

Let $c_2Elist_2ELIST_REL : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Elist_2ELIST_REL\ A_27a\ A_27b \in (((2^{(ty_2Elist_2Elist\ A_27b)})^{(ty_2Elist_2Elist\ A_27a)})^{(2^{A_27b})^{A_27a}}) \quad (9)$$

Assume the following.

$$True \quad (10)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \quad (11)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (12)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow \neg(p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow \neg(p\ V0t)))))) \quad (13)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p\ V0t1) \Rightarrow ((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \quad (14)$$

Assume the following.

$$2.(((p \ V0x) \Leftrightarrow (p \ V1x_27)) \wedge ((p \ V1x_27) \Rightarrow ((p \ V2y) \Leftrightarrow (p \ V3y_27)))) \Rightarrow \\ (((p \ V0x) \Rightarrow (p \ V2y)) \Leftrightarrow ((p \ V1x_27) \Rightarrow (p \ V3y_27)))) \quad (15)$$

Assume the following.

$$\forall A_27a.\text{nonempty } A_27a \Rightarrow \forall A_27b.\text{nonempty } A_27b \Rightarrow \forall A_27c. \\ \text{nonempty } A_27c \Rightarrow (\forall V0f \in (A_27b^{A_27a}). (\forall V1g \in (A_27a^{A_27c}). \\ (\forall V2x \in A_27c. ((ap (ap (ap (c_2Ecombin_2Eo A_27c A_27b A_27a) \\ V0f) V1g) V2x) = (ap V0f (ap V1g V2x)))))) \quad (16)$$

Assume the following.

$$\forall A_27a.\text{nonempty } A_27a \Rightarrow \forall A_27b.\text{nonempty } A_27b \Rightarrow (\\ \forall V0l \in (ty_2Elist_2Elist A_27a). (\forall V1f \in (A_27b^{A_27a}). \\ ((ap (c_2Elist_2ELENGTH A_27b) (ap (ap (c_2Elist_2EMAP A_27a A_27b) \\ V1f) V0l)) = (ap (c_2Elist_2ELENGTH A_27a) V0l)))) \quad (17)$$

Assume the following.

$$\forall A_27a.\text{nonempty } A_27a \Rightarrow \forall A_27b.\text{nonempty } A_27b \Rightarrow (\\ \forall V0n \in ty_2Enum_2Enum. (\forall V1l \in (ty_2Elist_2Elist \\ A_27a). ((p (ap (ap c_2Eprim_rec_2E_3C V0n) (ap (c_2Elist_2ELENGTH \\ A_27a) V1l))) \Rightarrow (\forall V2f \in (A_27b^{A_27a}). ((ap (ap (c_2Elist_2EEL \\ A_27b) V0n) (ap (ap (c_2Elist_2EMAP A_27a A_27b) V2f) V1l)) = (ap \\ V2f (ap (ap (c_2Elist_2EEL A_27a) V0n) V1l)))))) \quad (18)$$

Assume the following.

$$\forall A_27a.\text{nonempty } A_27a \Rightarrow \forall A_27b.\text{nonempty } A_27b \Rightarrow (\\ \forall V0R \in ((2^{A_27b})^{A_27a}). (\forall V1l1 \in (ty_2Elist_2Elist \\ A_27a). (\forall V2l2 \in (ty_2Elist_2Elist A_27b). ((p (ap (ap (ap \\ (c_2Elist_2ELIST_REL A_27a A_27b) V0R) V1l1) V2l2)) \Leftrightarrow (((ap (c_2Elist_2ELENGTH \\ A_27a) V1l1) = (ap (c_2Elist_2ELENGTH A_27b) V2l2)) \wedge (\forall V3n \in \\ ty_2Enum_2Enum. ((p (ap (ap c_2Eprim_rec_2E_3C V3n) (ap (c_2Elist_2ELENGTH \\ A_27a) V1l1))) \Rightarrow (p (ap (ap V0R (ap (ap (c_2Elist_2EEL A_27a) V3n) \\ V1l1)) (ap (ap (c_2Elist_2EEL A_27b) V3n) V2l2)))))))) \quad (19)$$

Theorem 1

$$\forall A_27a.\text{nonempty } A_27a \Rightarrow \forall A_27b.\text{nonempty } A_27b \Rightarrow \forall A_27c. \\ \text{nonempty } A_27c \Rightarrow (\forall V0R \in ((2^{A_27b})^{A_27a}). (\forall V1f \in \\ (A_27a^{A_27c}). (\forall V2l1 \in (ty_2Elist_2Elist A_27c). (\forall V3l2 \in \\ (ty_2Elist_2Elist A_27b). ((p (ap (ap (ap (c_2Elist_2ELIST_REL \\ A_27a A_27b) V0R) (ap (ap (c_2Elist_2EMAP A_27c A_27a) V1f) V2l1)) \\ V3l2)) \Leftrightarrow (p (ap (ap (ap (c_2Elist_2ELIST_REL A_27c A_27b) (ap (ap \\ (c_2Ecombin_2Eo A_27c (2^{A_27b}) A_27a) V0R) V1f)) V2l1) V3l2)))))) \quad (19)$$