

thm_2Elist_2ELIST__TO__SET__APPEND
(TMa7kAeq3RHKk2L99fA3tQ7aj7wBoHc3vEW)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Let $ty_2Elist_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty_2Elist_2Elist A0) \quad (1)$$

Let $c_2Elist_2EAPPEND : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Elist_2EAPPEND A_27a \in (((ty_2Elist_2Elist A_27a)^{(ty_2Elist_2Elist A_27a)})^{(ty_2Elist_2Elist A_27a)}) \quad (2)$$

Let $c_2Elist_2ELIST_TO_SET : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Elist_2ELIST_TO_SET A_27a \in ((2^{A_27a})^{(ty_2Elist_2Elist A_27a)}) \quad (3)$$

Let $c_2Elist_2ECONS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Elist_2ECONS A_27a \in (((ty_2Elist_2Elist A_27a)^{(ty_2Elist_2Elist A_27a)})^{A_27a}) \quad (4)$$

Let $c_2Elist_2ENIL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Elist_2ENIL A_27a \in (ty_2Elist_2Elist A_27a) \quad (5)$$

Definition 3 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 4 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a}))$

Definition 5 We define c_2Ebool_2EF to be $(ap (c_2Ebool_2E_21 2)) (\lambda V0t \in 2.V0t)$.

Definition 6 We define $c_2Epred_set_2EEMPTY$ to be $\lambda A_27a : \iota. (\lambda V0x \in A_27a. c_2Ebool_2EF)$.

Definition 7 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2))$

Definition 8 We define c_2Ebool_2EIN to be $\lambda A_27a : \iota. (\lambda V0x \in A_27a. (\lambda V1f \in (2^{A_27a}). (ap V1f V0x)))$

Definition 9 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2))$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0. nonempty A0 \Rightarrow \forall A1. nonempty A1 \Rightarrow nonempty (ty_2Epair_2Eprod A0 A1) \quad (6)$$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty A_27a \Rightarrow \forall A_27b. nonempty A_27b \Rightarrow c_2Epair_2EABS_prod A_27a A_27b \in ((ty_2Epair_2Eprod A_27a A_27b)^{(2^{A_27b})^{A_27a}}) \quad (7)$$

Definition 10 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0x \in A_27a. \lambda V1y \in A_27b. (ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2))$

Let $c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty A_27a \Rightarrow \forall A_27b. nonempty A_27b \Rightarrow c_2Epred_set_2EGSPEC A_27a A_27b \in ((2^{A_27a})^{(ty_2Epair_2Eprod A_27a 2)^{A_27b}}) \quad (8)$$

Definition 11 We define $c_2Epred_set_2EINSERT$ to be $\lambda A_27a : \iota. \lambda V0x \in A_27a. \lambda V1s \in (2^{A_27a}). (ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2))$

Definition 12 We define $c_2Epred_set_2EUNION$ to be $\lambda A_27a : \iota. \lambda V0s \in (2^{A_27a}). \lambda V1t \in (2^{A_27a}). (ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2))$

Assume the following.

$$True \quad (9)$$

Assume the following.

$$\forall A_27a. nonempty A_27a \Rightarrow (\forall V0x \in A_27a. ((V0x = V0x) \Leftrightarrow True)) \quad (10)$$

Assume the following.

$$\forall A_27a. nonempty A_27a \Rightarrow ((\forall V0l \in (ty_2Elist_2Elist A_27a). ((ap (ap (c_2Elist_2EAPPEND A_27a) (c_2Elist_2ENIL A_27a)) V0l) = V0l)) \wedge (\forall V1l1 \in (ty_2Elist_2Elist A_27a). (\forall V2l2 \in (ty_2Elist_2Elist A_27a). (\forall V3h \in A_27a. ((ap (ap (c_2Elist_2EAPPEND A_27a) (ap (ap (c_2Elist_2ECONS A_27a) V3h) V1l1)) V2l2) = (ap (ap (c_2Elist_2ECONS A_27a) V3h) (ap (ap (c_2Elist_2EAPPEND A_27a) V1l1) V2l2)))))))))) \quad (11)$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \quad \forall V0h \in A_27b. (\forall V1t \in (ty_2Elist_2Elist\ A_27b). ((\\
& \quad (ap\ (c_2Elist_2ELIST_TO_SET\ A_27a)\ (c_2Elist_2ENIL\ A_27a)) = \\
& \quad (c_2Epred_set_2EEMPTY\ A_27a)) \wedge ((ap\ (c_2Elist_2ELIST_TO_SET \\
& A_27b)\ (ap\ (ap\ (c_2Elist_2ECONS\ A_27b)\ V0h)\ V1t)) = (ap\ (ap\ (c_2Epred_set_2EINSERT \\
& A_27b)\ V0h)\ (ap\ (c_2Elist_2ELIST_TO_SET\ A_27b)\ V1t)))))) \\
& \hspace{15em} (12)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in (2^{(ty_2Elist_2Elist\ A_27a)}). \\
& ((p\ (ap\ V0P\ (c_2Elist_2ENIL\ A_27a))) \wedge (\forall V1t \in (ty_2Elist_2Elist \\
& A_27a). ((p\ (ap\ V0P\ V1t)) \Rightarrow (\forall V2h \in A_27a. (p\ (ap\ V0P\ (ap\ (ap\ (\\
& c_2Elist_2ECONS\ A_27a)\ V2h)\ V1t)))))) \Rightarrow (\forall V3l \in (ty_2Elist_2Elist \\
& A_27a). (p\ (ap\ V0P\ V3l)))))) \\
& \hspace{15em} (13)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow ((\forall V0s \in (2^{A_27a}). ((ap\ (\\
& ap\ (c_2Epred_set_2EUNION\ A_27a)\ (c_2Epred_set_2EEMPTY\ A_27a)) \\
& V0s) = V0s)) \wedge (\forall V1s \in (2^{A_27a}). ((ap\ (ap\ (c_2Epred_set_2EUNION \\
& A_27a)\ V1s)\ (c_2Epred_set_2EEMPTY\ A_27a)) = V1s))) \\
& \hspace{15em} (14)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1s \in \\
& (2^{A_27a}). (\forall V2t \in (2^{A_27a}). ((ap\ (ap\ (c_2Epred_set_2EUNION \\
& A_27a)\ (ap\ (ap\ (c_2Epred_set_2EINSERT\ A_27a)\ V0x)\ V1s))\ V2t) = \\
& (ap\ (ap\ (c_2Epred_set_2EINSERT\ A_27a)\ V0x)\ (ap\ (ap\ (c_2Epred_set_2EUNION \\
& A_27a)\ V1s)\ V2t)))))) \\
& \hspace{15em} (15)
\end{aligned}$$

Theorem 1

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0l1 \in (ty_2Elist_2Elist \\
& A_27a). (\forall V1l2 \in (ty_2Elist_2Elist\ A_27a). ((ap\ (c_2Elist_2ELIST_TO_SET \\
& A_27a)\ (ap\ (ap\ (c_2Elist_2EAPPEND\ A_27a)\ V0l1)\ V1l2)) = (ap\ (ap\ (\\
& c_2Epred_set_2EUNION\ A_27a)\ (ap\ (c_2Elist_2ELIST_TO_SET \\
& A_27a)\ V0l1))\ (ap\ (c_2Elist_2ELIST_TO_SET\ A_27a)\ V1l2))))))
\end{aligned}$$