thm_2Elist_2ELUPDATE__LENGTH (TMQ7Lfq1Zaje93VQocAWMwphbYcrWTigYQK)

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Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V$

Let $ty\_2Elist\_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Elist\_2Elist\ A0) \quad (1)$$

Let $c\_2Elist\_2EAPPEND : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2EAPPEND\ A\_27a \in (((ty\_2Elist\_2Elist\ A\_27a)^{(ty\_2Elist\_2Elist\ A\_27a)})^{(ty\_2Elist\_2Elist\ A\_27a)}) \quad (2)$$

Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \quad (3)$$

Let $c\_2Elist\_2ELENGTH : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ELENGTH\ A\_27a \in (ty\_2Enum\_2Enum^{(ty\_2Elist\_2Elist\ A\_27a)}) \quad (4)$$

Definition 3 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Let $c\_2Enum\_2EREP\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EREP\_\_num \in (omega^{ty\_2Enum\_2Enum}) \quad (5)$$

Let $c\_2Enum\_2ESUC\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2ESUC\_\_REP \in (omega^{omega}) \quad (6)$$

Let $c\_2Enum\_2EABS\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EABS\_\_num \in (ty\_2Enum\_2Enum^{omega}) \quad (7)$$

Definition 4 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a:\iota.(\lambda V0P\in(2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

Definition 5 *We define* c_2Enum_2ESUC *to be* $\lambda V0m\in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_\_num\ ($

Let $c\_2Elist\_2ECONS:\iota\Rightarrow\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow c\_2Elist\_2ECONS\ A\_27a\in(((ty\_2Elist\_2Elist\ A\_27a)^{(ty\_2Elist\_2Elist\ A\_27a)})^{A\_27a}) \tag{8}$$

Let $c\_2Enum\_2EZERO\_\_REP:\iota$ be given. Assume the following.

$$c\_2Enum\_2EZERO\_\_REP\in omega \tag{9}$$

Definition 6 *We define* c_2Enum_2E0 *to be* $(ap\ c\_2Enum\_2EABS\_\_num\ c\_2Enum\_2EZERO\_\_REP)$.

Let $c\_2Elist\_2ENIL:\iota\Rightarrow\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow c\_2Elist\_2ENIL\ A\_27a\in(ty\_2Elist\_2Elist\ A\_27a) \tag{10}$$

Let $c\_2Elist\_2ELUPDATE:\iota\Rightarrow\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow c\_2Elist\_2ELUPDATE\ A\_27a\in((((ty\_2Elist\_2Elist\ A\_27a)^{(ty\_2Elist\_2Elist\ A\_27a)})^{ty\_2Enum\_2Enum})^{A\_27a}) \tag{11}$$

Definition 7 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1\in 2.(\lambda V1t2\in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t\in 2$

Assume the following.

$$True \tag{12}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0t\in 2.((\forall V1x\in A\_27a.(p\ V0t))\Leftrightarrow(p\ V0t))) \tag{13}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0x\in A\_27a.((V0x=V0x)\Leftrightarrow True)) \tag{14}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow((\forall V0l\in(ty\_2Elist\_2Elist\ A\_27a).((ap\ (ap\ (c\_2Elist\_2EAPPEND\ A\_27a)\ (c\_2Elist\_2ENIL\ A\_27a))\ V0l)=V0l))\wedge(\forall V1l1\in(ty\_2Elist\_2Elist\ A\_27a).(\forall V2l2\in(ty\_2Elist\_2Elist\ A\_27a).(\forall V3h\in A\_27a.((ap\ (ap\ (c\_2Elist\_2EAPPEND\ A\_27a)\ (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V3h)\ V1l1))\ V2l2)=(ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V3h)\ (ap\ (ap\ (c\_2Elist\_2EAPPEND\ A\_27a)\ V1l1)\ V2l2))))))) \tag{15}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (((ap\ (c\_2Elist\_2ELENGTH\ A\_27a)\ (c\_2Elist\_2ENIL\ A\_27a)) = \mathsf{c\_2Enum\_2E0}) \wedge (\forall V0h \in A\_27a.(\forall V1t \in (ty\_2Elist\_2Elist\ A\_27a).((ap\ (c\_2Elist\_2ELENGTH\ A\_27a)\ (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V0h)\ V1t)) = (ap\ \mathsf{c\_2Enum\_2ESUC}\ (ap\ (c\_2Elist\_2ELENGTH\ A\_27a)\ V1t))))))) \tag{16}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in (2^{(ty\_2Elist\_2Elist\ A\_27a)}).(((p\ (ap\ V0P\ (c\_2Elist\_2ENIL\ A\_27a))) \wedge (\forall V1t \in (ty\_2Elist\_2Elist\ A\_27a).((p\ (ap\ V0P\ V1t)) \Rightarrow (\forall V2h \in A\_27a.(p\ (ap\ V0P\ (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V2h)\ V1t))))))) \Rightarrow (\forall V3l \in (ty\_2Elist\_2Elist\ A\_27a).(p\ (ap\ V0P\ V3l))))) \tag{17}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow ((\forall V0e \in A\_27a.(\forall V1n \in ty\_2Enum\_2Enum.((ap\ (ap\ (ap\ (c\_2Elist\_2ELUPDATE\ A\_27a)\ V0e)\ V1n)\ (c\_2Elist\_2ENIL\ A\_27a)) = (c\_2Elist\_2ENIL\ A\_27a)))) \wedge ((\forall V2e \in A\_27a.(\forall V3x \in A\_27a.(\forall V4l \in (ty\_2Elist\_2Elist\ A\_27a).((ap\ (ap\ (ap\ (c\_2Elist\_2ELUPDATE\ A\_27a)\ V2e)\ \mathsf{c\_2Enum\_2E0})\ (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V3x)\ V4l)) = (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V2e)\ V4l))))) \wedge (\forall V5e \in A\_27a.(\forall V6n \in ty\_2Enum\_2Enum.(\forall V7x \in A\_27a.(\forall V8l \in (ty\_2Elist\_2Elist\ A\_27a).((ap\ (ap\ (ap\ (c\_2Elist\_2ELUPDATE\ A\_27a)\ V5e)\ (ap\ \mathsf{c\_2Enum\_2ESUC}\ V6n))\ (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V7x)\ V8l)) = (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V7x)\ (ap\ (ap\ (ap\ (c\_2Elist\_2ELUPDATE\ A\_27a)\ V5e)\ V6n)\ V8l)))))))))) \tag{18}$$

Theorem 1

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0xs \in (ty\_2Elist\_2Elist\ A\_27a).(\forall V1x \in A\_27a.(\forall V2y \in A\_27a.(\forall V3ys \in (ty\_2Elist\_2Elist\ A\_27a).((ap\ (ap\ (ap\ (c\_2Elist\_2ELUPDATE\ A\_27a)\ V1x)\ (ap\ (c\_2Elist\_2ELENGTH\ A\_27a)\ V0xs))\ (ap\ (ap\ (c\_2Elist\_2EAPPEND\ A\_27a)\ V0xs)\ (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V2y)\ V3ys))) = (ap\ (ap\ (c\_2Elist\_2EAPPEND\ A\_27a)\ V0xs)\ (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V1x)\ V3ys)))))))$$