

thm_2Elist_2EMAP__CONG

(TMZqVAanQFRJxMAC5LfF877JrLs4b3bf2E5)

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Definition 1 We define `c_2Emin_2E_3D` to be $\lambda A. \lambda x \in A. \lambda y \in A. \text{inj_o } (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define `c_2Ebool_2ET` to be $(\text{ap } (\text{ap } (\text{c_2Emin_2E_3D } (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define `c_2Ebool_2E_21` to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (\text{ap } (\text{ap } (\text{c_2Emin_2E_3D } (2^{A_27a}))$

Definition 4 We define `c_2Ebool_2EF` to be $(\text{ap } (\text{c_2Ebool_2E_21 } 2) (\lambda V0t \in 2.V0t))$.

Definition 5 We define `c_2Emin_2E_3D_3D_3E` to be $\lambda P \in 2. \lambda Q \in 2. \text{inj_o } (p P \Rightarrow p Q)$ of type ι .

Definition 6 We define `c_2Ebool_2E_7E` to be $(\lambda V0t \in 2. (\text{ap } (\text{ap } (\text{c_2Emin_2E_3D_3D_3E } V0t) \text{ c_2Ebool_2EF$

Let `ty_2Elist_2Elist` : $\iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0. \text{nonempty } A0 \Rightarrow \text{nonempty } (\text{ty_2Elist_2Elist } A0) \quad (1)$$

Let `c_2Elist_2EMAP` : $\iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a. \text{nonempty } A_27a \Rightarrow \forall A_27b. \text{nonempty } A_27b \Rightarrow \text{c_2Elist_2EMAP} \\ & A_27a A_27b \in (((\text{ty_2Elist_2Elist } A_27b)^{(\text{ty_2Elist_2Elist } A_27a)})^{(A_27b^{A_27a})}) \end{aligned} \quad (2)$$

Definition 7 We define `c_2Ebool_2E_5C_2F` to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (\text{ap } (\text{c_2Ebool_2E_21 } 2) (\lambda V2t \in 2.V2t))$

Let `c_2Elist_2ECONS` : $\iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. \text{nonempty } A_27a \Rightarrow \text{c_2Elist_2ECONS } A_27a \in (((\text{ty_2Elist_2Elist } A_27a)^{(\text{ty_2Elist_2Elist } A_27a)})^{A_27a}) \quad (3)$$

Let `c_2Elist_2ENIL` : $\iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. \text{nonempty } A_27a \Rightarrow \text{c_2Elist_2ENIL } A_27a \in (\text{ty_2Elist_2Elist } A_27a) \quad (4)$$

Let `c_2Elist_2ELIST__TO__SET` : $\iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. \text{nonempty } A_27a \Rightarrow \text{c_2Elist_2ELIST_TO_SET } A_27a \in ((2^{A_27a})^{(\text{ty_2Elist_2Elist } A_27a)}) \quad (5)$$

Definition 8 We define c_2Ebool_2EIN to be $\lambda A_27a : \iota. (\lambda V0x \in A_27a. (\lambda V1f \in (2^{A_27a}). (ap\ V1f\ V0x)))$

Definition 9 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in 2.$

Assume the following.

$$True \tag{6}$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \wedge True) \Leftrightarrow \\ & (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge (((p\ V0t) \wedge False) \Leftrightarrow False) \wedge \\ & (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)))))) \end{aligned} \tag{7}$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \vee (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \vee True) \Leftrightarrow True) \wedge \\ & (((False \vee (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \vee False) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \vee \\ & (p\ V0t)) \Leftrightarrow (p\ V0t)))))) \end{aligned} \tag{8}$$

Assume the following.

$$\forall A_27a. nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. ((V0x = V0x) \Leftrightarrow True)) \tag{9}$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow \\ & (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow \neg(p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow \neg(\\ & p\ V0t)))))) \end{aligned} \tag{10}$$

Assume the following.

$$\begin{aligned} & \forall A_27a. nonempty\ A_27a \Rightarrow \forall A_27b. nonempty\ A_27b \Rightarrow (\\ & (\forall V0f \in (A_27b^{A_27a}). ((ap\ (ap\ (c_2Elist_2EMAP\ A_27a\ A_27b)\ V0f)\ (c_2Elist_2ENIL\ A_27a)) = (c_2Elist_2ENIL\ A_27b))) \wedge (\forall V1f \in \\ & (A_27b^{A_27a}). (\forall V2h \in A_27a. (\forall V3t \in (ty_2Elist_2Elist\ A_27a). ((ap\ (ap\ (c_2Elist_2EMAP\ A_27a\ A_27b)\ V1f)\ (ap\ (ap\ (c_2Elist_2ECONS\ A_27a)\ V2h)\ V3t)) = (ap\ (ap\ (c_2Elist_2ECONS\ A_27b)\ (ap\ V1f\ V2h)) \\ & (ap\ (ap\ (c_2Elist_2EMAP\ A_27a\ A_27b)\ V1f)\ V3t)))))) \end{aligned} \tag{11}$$

Assume the following.

$$\begin{aligned} & \forall A_27a. nonempty\ A_27a \Rightarrow (\forall V0P \in (2^{(ty_2Elist_2Elist\ A_27a)}). \\ & (((p\ (ap\ V0P\ (c_2Elist_2ENIL\ A_27a))) \wedge (\forall V1t \in (ty_2Elist_2Elist\ A_27a). ((p\ (ap\ V0P\ V1t)) \Rightarrow (\forall V2h \in A_27a. (p\ (ap\ V0P\ (ap\ (ap\ (\\ & c_2Elist_2ECONS\ A_27a)\ V2h)\ V1t)))))) \Rightarrow (\forall V3l \in (ty_2Elist_2Elist\ A_27a). (p\ (ap\ V0P\ V3l)))))) \end{aligned} \tag{12}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow ((\forall V0x \in A_27a. ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V0x)\ (ap\ (c_2Elist_2ELIST_TO_SET\ A_27a)\ (c_2Elist_2ENIL\ A_27a)))) \Leftrightarrow False)) \wedge (\forall V1x \in A_27a. (\forall V2h \in \\
& A_27a. (\forall V3t \in (ty_2Elist_2Elist\ A_27a). ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V1x)\ (ap\ (c_2Elist_2ELIST_TO_SET\ A_27a)\ (ap\ (ap\ (c_2Elist_2ECONS\ A_27a)\ V2h)\ V3t)))) \Leftrightarrow ((V1x = V2h) \vee (p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V1x)\ (ap\ (c_2Elist_2ELIST_TO_SET\ A_27a)\ V3t))))))))))
\end{aligned} \tag{13}$$

Theorem 1

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \forall V0l1 \in (ty_2Elist_2Elist\ A_27a). (\forall V1l2 \in (ty_2Elist_2Elist\ A_27a). (\forall V2f \in (A_27b^{A_27a}). (\forall V3f_27 \in (A_27b^{A_27a}). \\
& (((V0l1 = V1l2) \wedge (\forall V4x \in A_27a. ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V4x)\ (ap\ (c_2Elist_2ELIST_TO_SET\ A_27a)\ V1l2)))) \Rightarrow ((ap\ V2f\ V4x) = (ap\ V3f_27\ V4x)))))) \Rightarrow ((ap\ (ap\ (c_2Elist_2EMAP\ A_27a\ A_27b)\ V2f)\ V0l1) = (ap\ (ap\ (c_2Elist_2EMAP\ A_27a\ A_27b)\ V3f_27)\ V1l2))))))
\end{aligned}$$