

thm_2Elist_2EMAP__EQ__NIL (TMF- pWX3eD2gNMZCKHh6QqmnBKahFKVSP7Xf)

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Definition 1 We define `c_2Emin_2E_3D` to be $\lambda A. \lambda x \in A. \lambda y \in A. \text{inj_o } (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define `c_2Ebool_2E_2T` to be $(\text{ap } (\text{ap } (\text{c_2Emin_2E_3D } (2^2)) (\lambda V 0x \in 2.V 0x)) (\lambda V 1x \in 2.V 1x))$

Definition 3 We define `c_2Ebool_2E_21` to be $\lambda A. 27a : \iota. (\lambda V 0P \in (2^{A-27a}). (\text{ap } (\text{ap } (\text{c_2Emin_2E_3D } (2^{A-27a}))))$

Definition 4 We define `c_2Ebool_2E_2F` to be $(\text{ap } (\text{c_2Ebool_2E_21 } 2) (\lambda V 0t \in 2.V 0t))$.

Let `ty_2Elist_2Elist` : $\iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A 0. \text{nonempty } A 0 \Rightarrow \text{nonempty } (\text{ty_2Elist_2Elist } A 0) \quad (1)$$

Let `c_2Elist_2EMAP` : $\iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A. 27a. \text{nonempty } A. 27a \Rightarrow \forall A. 27b. \text{nonempty } A. 27b \Rightarrow \text{c_2Elist_2EMAP } A. 27a \ A. 27b \in (((\text{ty_2Elist_2Elist } A. 27b)^{(\text{ty_2Elist_2Elist } A. 27a)})^{(A. 27b^{A-27a})}) \quad (2)$$

Definition 5 We define `c_2Emin_2E_3D_3D_3E` to be $\lambda P \in 2. \lambda Q \in 2. \text{inj_o } (p P \Rightarrow p Q)$ of type ι .

Definition 6 We define `c_2Ebool_2E_2F_5C` to be $(\lambda V 0t1 \in 2. (\lambda V 1t2 \in 2. (\text{ap } (\text{c_2Ebool_2E_21 } 2) (\lambda V 2t \in 2.V 2t))))$

Let `c_2Elist_2ENIL` : $\iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A. 27a. \text{nonempty } A. 27a \Rightarrow \text{c_2Elist_2ENIL } A. 27a \in (\text{ty_2Elist_2Elist } A. 27a) \quad (3)$$

Let `c_2Elist_2ECONS` : $\iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A. 27a. \text{nonempty } A. 27a \Rightarrow \text{c_2Elist_2ECONS } A. 27a \in (((\text{ty_2Elist_2Elist } A. 27a)^{(\text{ty_2Elist_2Elist } A. 27a)})^{A. 27a}) \quad (4)$$

Definition 7 We define `c_2Ebool_2E_7E` to be $(\lambda V 0t \in 2. (\text{ap } (\text{ap } (\text{c_2Emin_2E_3D_3D_3E } V 0t) \text{c_2Ebool_2E_2F})))$

Assume the following.

$$True \quad (5)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p \ V0t1) \Rightarrow (p \ V1t2)) \Rightarrow (((p \ V1t2) \Rightarrow (p \ V0t1)) \Rightarrow ((p \ V0t1) \Leftrightarrow (p \ V1t2)))))) \quad (6)$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p \ V0t))) \quad (7)$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A_27a.(p \ V0t)) \Leftrightarrow (p \ V0t))) \quad (8)$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \wedge True) \Leftrightarrow (p \ V0t)) \wedge (((False \wedge (p \ V0t)) \Leftrightarrow False) \wedge (((p \ V0t) \wedge False) \Leftrightarrow False) \wedge (((p \ V0t) \wedge (p \ V0t)) \Leftrightarrow (p \ V0t)))))) \quad (9)$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p \ V0t))) \Leftrightarrow (p \ V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \quad (10)$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \quad (11)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Leftrightarrow True) \Leftrightarrow (p \ V0t)) \wedge (((False \Leftrightarrow (p \ V0t)) \Leftrightarrow (\neg(p \ V0t))) \wedge (((p \ V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p \ V0t)))))) \quad (12)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty \ A_27a \Rightarrow \forall A_27b.nonempty \ A_27b \Rightarrow (\\ & (\forall V0f \in (A_27b^{A_27a}).((ap \ (ap \ (c_2Elist_2EMAP \ A_27a \ A_27b) \\ & V0f) \ (c_2Elist_2ENIL \ A_27a)) = (c_2Elist_2ENIL \ A_27b))) \wedge (\forall V1f \in \\ & (A_27b^{A_27a}).(\forall V2h \in A_27a.(\forall V3t \in (ty_2Elist_2Elist \\ & A_27a).((ap \ (ap \ (c_2Elist_2EMAP \ A_27a \ A_27b) \ V1f) \ (ap \ (ap \ (c_2Elist_2ECONS \\ & A_27a) \ V2h) \ V3t)) = (ap \ (ap \ (c_2Elist_2ECONS \ A_27b) \ (ap \ V1f \ V2h)) \\ & (ap \ (ap \ (c_2Elist_2EMAP \ A_27a \ A_27b) \ V1f) \ V3t)))))) \end{aligned} \quad (13)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0P \in (2^{(ty_2Elist_2Elist\ A.27a)}), \\ & (((p\ (ap\ V0P\ (c_2Elist_2ENIL\ A.27a))) \wedge (\forall V1t \in (ty_2Elist_2Elist \\ & A.27a).(p\ (ap\ V0P\ V1t)) \Rightarrow (\forall V2h \in A.27a.(p\ (ap\ V0P\ (ap\ (ap\ (\\ & c_2Elist_2ECONS\ A.27a\ V2h)\ V1t)))))) \Rightarrow (\forall V3l \in (ty_2Elist_2Elist \\ & A.27a).(p\ (ap\ V0P\ V3l)))))) \end{aligned} \quad (14)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0a1 \in (ty_2Elist_2Elist \\ & A.27a).(\forall V1a0 \in A.27a. (\neg((c_2Elist_2ENIL\ A.27a) = (ap\ (\\ & ap\ (c_2Elist_2ECONS\ A.27a\ V1a0)\ V0a1)))))) \end{aligned} \quad (15)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0a1 \in (ty_2Elist_2Elist \\ & A.27a).(\forall V1a0 \in A.27a. (\neg((ap\ (ap\ (c_2Elist_2ECONS\ A.27a) \\ & V1a0)\ V0a1) = (c_2Elist_2ENIL\ A.27a)))))) \end{aligned} \quad (16)$$

Theorem 1

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\ & \forall V0l \in (ty_2Elist_2Elist\ A.27a).(\forall V1f \in (A.27b^{A.27a}). \\ & (((ap\ (ap\ (c_2Elist_2EMAP\ A.27a\ A.27b)\ V1f)\ V0l) = (c_2Elist_2ENIL \\ & A.27b)) \Leftrightarrow (V0l = (c_2Elist_2ENIL\ A.27a))) \wedge (((c_2Elist_2ENIL\ A.27b) = \\ & (ap\ (ap\ (c_2Elist_2EMAP\ A.27a\ A.27b)\ V1f)\ V0l)) \Leftrightarrow (V0l = (c_2Elist_2ENIL \\ & A.27a)))))) \end{aligned}$$