

thm_2Elist_2EMAP__MAP__o
(TMQ1jS9amDiZhZLZcxLa3ANyjiiSoJBdTe9)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a}))$

Definition 4 We define $c_2Ecombin_2Eo$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda V0f \in (A_27b^{A_27c}).\lambda V1g$

Let $ty_2Elist_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty_2Elist_2Elist A0) \quad (1)$$

Let $c_2Elist_2EMAP : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Elist_2EMAP A_27a A_27b \in (((ty_2Elist_2Elist A_27b)^{(ty_2Elist_2Elist A_27a)})^{(A_27b^{A_27a})}) \quad (2)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow \forall A_27c. \\ & nonempty A_27c \Rightarrow (\forall V0f \in (A_27c^{A_27b}).(\forall V1g \in (A_27b^{A_27a}). \\ & ((ap (c_2Elist_2EMAP A_27a A_27c) (ap (ap (c_2Ecombin_2Eo A_27a \\ & A_27c A_27b) V0f) V1g))) = (ap (ap (c_2Ecombin_2Eo (ty_2Elist_2Elist \\ & A_27a) (ty_2Elist_2Elist A_27c) (ty_2Elist_2Elist A_27b)) (ap \\ & (c_2Elist_2EMAP A_27b A_27c) V0f)) (ap (c_2Elist_2EMAP A_27a A_27b) \\ & V1g)))))) \end{aligned} \quad (3)$$

Theorem 1

$$\begin{aligned} & \forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow \forall A_27c. \\ & nonempty A_27c \Rightarrow (\forall V0f \in (A_27c^{A_27b}).(\forall V1g \in (A_27b^{A_27a}). \\ & (\forall V2l \in (ty_2Elist_2Elist A_27a).((ap (ap (c_2Elist_2EMAP \\ & A_27b A_27c) V0f) (ap (ap (c_2Elist_2EMAP A_27a A_27b) V1g) V2l))) = \\ & (ap (ap (c_2Elist_2EMAP A_27a A_27c) (ap (ap (c_2Ecombin_2Eo A_27a \\ & A_27c A_27b) V0f) V1g)) V2l)))))) \end{aligned}$$