

thm\_2Elist\_2EMAP\_\_o  
(TMYDR2NNQTCwH8qZwXdVtt4jb73d9V2RZBq)

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**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Ebool\_2ET$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2))) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x)$

**Definition 3** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c\_2Emin\_2E\_3D (2^{A-27a})) (V0P)) (V1P)))$

**Definition 4** We define  $c\_2Ecombin\_2Eo$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda V0f \in (A\_27b^{A-27c}).\lambda V1g \in (A\_27c^{A-27a}).\lambda V2h \in (A\_27a^{A-27b}).(ap (ap (ap (c\_2Ecombin\_2Eo (A\_27a) (A\_27b) (A\_27c)) (V0f) (V1g)) (V2h)))$

Let  $ty\_2Elist\_2Elist : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Elist\_2Elist\ A0) \quad (1)$$

Let  $c\_2Elist\_2EMAP : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Elist\_2EMAP\ A\_27a\ A\_27b \in (((ty\_2Elist\_2Elist\ A\_27b)^{(ty\_2Elist\_2Elist\ A\_27a)})^{(A\_27b^{A-27a})})^{(A\_27a)} \quad (2)$$

Let  $c\_2Elist\_2ECONS : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ECONS\ A\_27a \in (((ty\_2Elist\_2Elist\ A\_27a)^{(ty\_2Elist\_2Elist\ A\_27a)})^{(A\_27a)})^{(A\_27a)} \quad (3)$$

Let  $c\_2Elist\_2ENIL : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ENIL\ A\_27a \in (ty\_2Elist\_2Elist\ A\_27a)^{(A\_27a)} \quad (4)$$

**Definition 5** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p \Rightarrow q)$  of type  $\iota$ .

**Definition 6** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21\ 2)) (\lambda V2t \in 2.V2t)))$

Assume the following.

$$True \quad (5)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \quad (6)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t \in 2. ((\forall V1x \in A\_27a.(p\ V0t) \Leftrightarrow (p\ V0t))) \quad (7)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a. ((V0x = V0x) \Leftrightarrow True)) \quad (8)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow \forall A\_27c. \\ & nonempty\ A\_27c \Rightarrow (\forall V0f \in (A\_27b^{A\_27a}). (\forall V1g \in (A\_27a^{A\_27c}). \\ & (\forall V2x \in A\_27c. ((ap\ (ap\ (ap\ (c\_2Ecombin\_2Eo\ A\_27c\ A\_27b\ A\_27a) \\ & V0f)\ V1g)\ V2x) = (ap\ V0f\ (ap\ V1g\ V2x)))))) \quad (9) \end{aligned}$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\ & (\forall V0f \in (A\_27b^{A\_27a}). ((ap\ (ap\ (c\_2Elist\_2EMAP\ A\_27a\ A\_27b) \\ & V0f)\ (c\_2Elist\_2ENIL\ A\_27a)) = (c\_2Elist\_2ENIL\ A\_27b))) \wedge (\forall V1f \in \\ & (A\_27b^{A\_27a}). (\forall V2h \in A\_27a. (\forall V3t \in (ty\_2Elist\_2Elist \\ & A\_27a). ((ap\ (ap\ (c\_2Elist\_2EMAP\ A\_27a\ A\_27b)\ V1f)\ (ap\ (ap\ (c\_2Elist\_2ECONS \\ & A\_27a)\ V2h)\ V3t)) = (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27b)\ (ap\ V1f\ V2h)) \\ & (ap\ (ap\ (c\_2Elist\_2EMAP\ A\_27a\ A\_27b)\ V1f)\ V3t)))))) \quad (10) \end{aligned}$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in (2^{(ty\_2Elist\_2Elist\ A\_27a)}). \\ & (((p\ (ap\ V0P\ (c\_2Elist\_2ENIL\ A\_27a))) \wedge (\forall V1t \in (ty\_2Elist\_2Elist \\ & A\_27a). ((p\ (ap\ V0P\ V1t)) \Rightarrow (\forall V2h \in A\_27a. (p\ (ap\ V0P\ (ap\ (ap\ ( \\ & c\_2Elist\_2ECONS\ A\_27a)\ V2h)\ V1t)))))) \Rightarrow (\forall V3l \in (ty\_2Elist\_2Elist \\ & A\_27a). (p\ (ap\ V0P\ V3l)))))) \quad (11) \end{aligned}$$

### Theorem 1

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow \forall A\_27c. \\ & nonempty\ A\_27c \Rightarrow (\forall V0f \in (A\_27c^{A\_27b}). (\forall V1g \in (A\_27b^{A\_27a}). \\ & ((ap\ (c\_2Elist\_2EMAP\ A\_27a\ A\_27c)\ (ap\ (ap\ (c\_2Ecombin\_2Eo\ A\_27a \\ & A\_27c\ A\_27b)\ V0f)\ V1g)) = (ap\ (ap\ (c\_2Ecombin\_2Eo\ (ty\_2Elist\_2Elist \\ & A\_27a)\ (ty\_2Elist\_2Elist\ A\_27c)\ (ty\_2Elist\_2Elist\ A\_27b))\ (ap \\ & (c\_2Elist\_2EMAP\ A\_27b\ A\_27c)\ V0f))\ (ap\ (c\_2Elist\_2EMAP\ A\_27a\ A\_27b) \\ & V1g)))))) \end{aligned}$$