

thm_2Elist_2EMEM_LUPDATE (TMd59D7q18h8cC9zaS8LU1SjSS42rY4qppw)

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Definition 1 We define `c_2Emin_2E_40` to be $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. p \text{ (ap } P \ x)) \text{ then (the } (\lambda x. x \in A \wedge p \text{ of type } \iota \Rightarrow \iota).$

Definition 2 We define `c_2Emin_2E_3D` to be $\lambda A. \lambda x \in A. \lambda y \in A. \text{inj_o } (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 3 We define `c_2Ebool_2ET` to be $(\text{ap } (\text{ap } (\text{c_2Emin_2E_3D } (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 4 We define `c_2Ebool_2EBOUNDED` to be $(\lambda V0v \in 2. \text{c_2Ebool_2ET}).$

Let `ty_2Elist_2Elist` : $\iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0. \text{nonempty } A0 \Rightarrow \text{nonempty } (\text{ty_2Elist_2Elist } A0) \quad (1)$$

Let `c_2Elist_2EHD` : $\iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. \text{nonempty } A_27a \Rightarrow \text{c_2Elist_2EHD } A_27a \in (A_27a^{(\text{ty_2Elist_2Elist } A_27a)}) \quad (2)$$

Let `ty_2Enum_2Enum` : ι be given. Assume the following.

$$\text{nonempty } \text{ty_2Enum_2Enum} \quad (3)$$

Let `c_2Elist_2EEL` : $\iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. \text{nonempty } A_27a \Rightarrow \text{c_2Elist_2EEL } A_27a \in ((A_27a^{(\text{ty_2Elist_2Elist } A_27a)})^{\text{ty_2Enum_2Enum}}) \quad (4)$$

Let `c_2Elist_2ELENGTH` : $\iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. \text{nonempty } A_27a \Rightarrow \text{c_2Elist_2ELENGTH } A_27a \in (\text{ty_2Enum_2Enum}^{(\text{ty_2Elist_2Elist } A_27a)}) \quad (5)$$

Definition 5 We define `c_2Ebool_2E_3F` to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (\text{ap } V0P \ (\text{ap } (\text{c_2Emin_2E_40 } A_27a \ P))))$

Let `c_2Elist_2ELIST_TO_SET` : $\iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. \text{nonempty } A_27a \Rightarrow \text{c_2Elist_2ELIST_TO_SET } A_27a \in ((2^{A_27a})^{(\text{ty_2Elist_2Elist } A_27a)}) \quad (6)$$

Let $c_2Elist_2ECONS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2ECONS\ A_27a \in (((ty_2Elist_2Elist\ A_27a)^{(ty_2Elist_2Elist\ A_27a)})^{A_27a}) \quad (7)$$

Let $c_2Elist_2ENIL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2ENIL\ A_27a \in (ty_2Elist_2Elist\ A_27a) \quad (8)$$

Let $c_2Elist_2ELUPDATE : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2ELUPDATE\ A_27a \in (((ty_2Elist_2Elist\ A_27a)^{(ty_2Elist_2Elist\ A_27a)})^{ty_2Enum_2Enum})^{A_27a} \quad (9)$$

Definition 6 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (ap\ (ap\ (c_2Emin_2E_3D\ (2^{A_27a}))$

Definition 7 We define c_2Ebool_2EF to be $(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 8 We define $c_2Epred_set_2EEMPTY$ to be $\lambda A_27a : \iota. (\lambda V0x \in A_27a.c_2Ebool_2EF)$.

Definition 9 We define c_2Ebool_2EIN to be $\lambda A_27a : \iota. (\lambda V0x \in A_27a. (\lambda V1f \in (2^{A_27a}). (ap\ V1f\ V0x)))$

Definition 10 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2. \lambda Q \in 2. inj_o\ (p\ P \Rightarrow p\ Q)$ of type ι .

Definition 11 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in 2. (c_2Ebool_2E_5C_2F\ V1t2\ V2t))))$

Definition 12 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in 2. (c_2Ebool_2E_2F_5C\ V1t2\ V2t))))$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \quad (10)$$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EABS_prod\ A_27a\ A_27b \in ((ty_2Epair_2Eprod\ A_27a\ A_27b)^{(2^{A_27b})^{A_27a}}) \quad (11)$$

Definition 13 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0x \in A_27a. \lambda V1y \in A_27b. (ap\ (c_2Emin_2E_3D_3D_3E\ (c_2Ebool_2EIN\ V0x\ V1y)))$

Let $c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epred_set_2EGSPEC\ A_27a\ A_27b \in ((2^{A_27a})^{(ty_2Epair_2Eprod\ A_27a\ 2)^{A_27b}}) \quad (12)$$

Definition 14 We define $c_2Epred_set_2EINSERT$ to be $\lambda A_27a : \iota. \lambda V0x \in A_27a. \lambda V1s \in (2^{A_27a}). (ap\ (c_2Emin_2E_3D_3D_3E\ (c_2Ebool_2EIN\ V0x\ V1s)))$

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \quad (13)$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \quad (14)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \quad (15)$$

Definition 15 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum. (ap\ c_2Enum_2EABS_num$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \quad (16)$$

Definition 16 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Definition 17 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2. (ap\ (ap\ c_2Emin_2E_3D_3D_3E\ V0t)\ c_2Ebool_2E$

Definition 18 We define $c_2Eprim_rec_2E_3C$ to be $\lambda V0m \in ty_2Enum_2Enum. \lambda V1n \in ty_2Enum_2Enum$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum. ((V0m = c_2Enum_2E0) \vee (\exists V1n \in ty_2Enum_2Enum. (V0m = (ap\ c_2Enum_2ESUC\ V1n)))))) \quad (17)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (p\ (ap\ (ap\ c_2Eprim_rec_2E_3C\ (ap\ c_2Enum_2ESUC\ V0m))\ (ap\ c_2Enum_2ESUC\ V1n))) \Leftrightarrow (p\ (ap\ (ap\ c_2Eprim_rec_2E_3C\ V0m)\ V1n)))))) \quad (18)$$

Assume the following.

$$(\forall V0P \in (2^{ty_2Enum_2Enum}). ((\exists V1n \in ty_2Enum_2Enum. (p\ (ap\ V0P\ V1n))) \Leftrightarrow ((p\ (ap\ V0P\ c_2Enum_2E0)) \vee (\exists V2m \in ty_2Enum_2Enum. (p\ (ap\ V0P\ (ap\ c_2Enum_2ESUC\ V2m)))))))) \quad (19)$$

Assume the following.

$$True \quad (20)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \quad (21)$$

Assume the following.

$$(\forall V0t \in 2. (False \Rightarrow (p\ V0t))) \quad (22)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0t \in 2. ((\exists V1x \in A_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \quad (23)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \wedge True) \Leftrightarrow \\ & (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge (((p\ V0t) \wedge False) \Leftrightarrow False) \wedge \\ & (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)))))) \end{aligned} \quad (24)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \vee (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \vee True) \Leftrightarrow True) \wedge \\ & (((False \vee (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \vee False) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \vee \\ & (p\ V0t)) \Leftrightarrow (p\ V0t)))))) \end{aligned} \quad (25)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Rightarrow True) \Leftrightarrow \\ & True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((\\ & (p\ V0t) \Rightarrow False) \Leftrightarrow \neg(p\ V0t)))))) \end{aligned} \quad (26)$$

Assume the following.

$$\begin{aligned} & ((\forall V0t \in 2. ((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t)) \wedge (((\neg True) \Leftrightarrow False) \wedge \\ & ((\neg False) \Leftrightarrow True)))) \end{aligned} \quad (27)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.(V0x = V0x)) \quad (28)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \quad (29)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (30)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow \\ & (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow \neg(p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow \neg(\\ & p\ V0t)))))) \end{aligned} \quad (31)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in (2^{A_27a}).(\forall V1Q \in \\ & 2.(((\forall V2x \in A_27a.(p\ (ap\ V0P\ V2x))) \wedge (p\ V1Q)) \Leftrightarrow (\forall V3x \in \\ & A_27a.((p\ (ap\ V0P\ V3x)) \wedge (p\ V1Q)))))) \end{aligned} \quad (32)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in (\\ 2^{A_27a}). ((p\ V0P) \wedge (\forall V2x \in A_27a. (p\ (ap\ V1Q\ V2x)))) \Leftrightarrow (\forall V3x \in \\ A_27a. ((p\ V0P) \wedge (p\ (ap\ V1Q\ V3x)))))) \end{aligned} \quad (33)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in (\\ 2^{A_27a}). ((\forall V2x \in A_27a. ((p\ V0P) \vee (p\ (ap\ V1Q\ V2x)))) \Leftrightarrow ((p \\ V0P) \vee (\forall V3x \in A_27a. (p\ (ap\ V1Q\ V3x)))))) \end{aligned} \quad (34)$$

Assume the following.

$$\begin{aligned} (\forall V0A \in 2. (\forall V1B \in 2. (\forall V2C \in 2. (((p\ V0A) \vee (\\ (p\ V1B) \wedge (p\ V2C)))) \Leftrightarrow (((p\ V0A) \vee (p\ V1B)) \wedge ((p\ V0A) \vee (p\ V2C)))))) \end{aligned} \quad (35)$$

Assume the following.

$$(\forall V0v \in 2. ((p\ (ap\ c_2Ebool_2EBOUNDED\ V0v)) \Leftrightarrow True)) \quad (36)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0h \in A_27a. (\forall V1t \in \\ (ty_2Elist_2Elist\ A_27a). ((ap\ (c_2Elist_2EHD\ A_27a)\ (ap\ (\\ c_2Elist_2ECONS\ A_27a)\ V0h)\ V1t)) = V0h))) \end{aligned} \quad (37)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (((ap\ (c_2Elist_2ELENGTH\ A_27a) \\ (c_2Elist_2ENIL\ A_27a)) = c_2Enum_2E0) \wedge (\forall V0h \in A_27a. (\\ \forall V1t \in (ty_2Elist_2Elist\ A_27a). ((ap\ (c_2Elist_2ELENGTH \\ A_27a)\ (ap\ (ap\ (c_2Elist_2ECONS\ A_27a)\ V0h)\ V1t)) = (ap\ c_2Enum_2ESUC \\ (ap\ (c_2Elist_2ELENGTH\ A_27a)\ V1t)))))) \end{aligned} \quad (38)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ \forall V0h \in A_27b. (\forall V1t \in (ty_2Elist_2Elist\ A_27b). ((\\ (ap\ (c_2Elist_2ELIST_TO_SET\ A_27a)\ (c_2Elist_2ENIL\ A_27a)) = \\ (c_2Epred_set_2EEMPTY\ A_27a) \wedge ((ap\ (c_2Elist_2ELIST_TO_SET \\ A_27b)\ (ap\ (ap\ (c_2Elist_2ECONS\ A_27b)\ V0h)\ V1t)) = (ap\ (ap\ (c_2Epred_set_2EINSERT \\ A_27b)\ V0h)\ (ap\ (c_2Elist_2ELIST_TO_SET\ A_27b)\ V1t)))))) \end{aligned} \quad (39)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in (2^{(ty_2Elist_2Elist\ A_27a)}). \\ (((p\ (ap\ V0P\ (c_2Elist_2ENIL\ A_27a))) \wedge (\forall V1t \in (ty_2Elist_2Elist \\ A_27a). ((p\ (ap\ V0P\ V1t)) \Rightarrow (\forall V2h \in A_27a. (p\ (ap\ V0P\ (ap\ (ap\ (\\ c_2Elist_2ECONS\ A_27a)\ V2h)\ V1t)))))) \Rightarrow (\forall V3l \in (ty_2Elist_2Elist \\ A_27a). (p\ (ap\ V0P\ V3l)))))) \end{aligned} \quad (40)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ \forall V0n \in ty_2Enum_2Enum. (\forall V1l \in A_27b. (\forall V2ls \in \\ (ty_2Elist_2Elist\ A_27b). ((ap\ (c_2Elist_2EEL\ A_27a)\ c_2Enum_2E0) = \\ (c_2Elist_2EHD\ A_27a) \wedge ((ap\ (ap\ (c_2Elist_2EEL\ A_27b)\ (ap\ c_2Enum_2ESUC \\ V0n))\ (ap\ (ap\ (c_2Elist_2ECONS\ A_27b)\ V1l)\ V2ls)) = (ap\ (ap\ (c_2Elist_2EEL \\ A_27b)\ V0n)\ V2ls)))))) \end{aligned} \quad (41)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0l \in (ty_2Elist_2Elist \\ A_27a). (\forall V1x \in A_27a. ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V1x) \\ (ap\ (c_2Elist_2ELIST_TO_SET\ A_27a)\ V0l))) \Leftrightarrow (\exists V2n \in ty_2Enum_2Enum. \\ ((p\ (ap\ (ap\ c_2Eprim_rec_2E_3C\ V2n)\ (ap\ (c_2Elist_2ELENGTH\ A_27a) \\ V0l))) \wedge (V1x = (ap\ (ap\ (c_2Elist_2EEL\ A_27a)\ V2n)\ V0l)))))) \end{aligned} \quad (42)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow ((\forall V0e \in A_27a. (\forall V1n \in \\ ty_2Enum_2Enum. ((ap\ (ap\ (ap\ (c_2Elist_2ELUPDATE\ A_27a)\ V0e)\ V1n) \\ (c_2Elist_2ENIL\ A_27a)) = (c_2Elist_2ENIL\ A_27a)))) \wedge ((\forall V2e \in \\ A_27a. (\forall V3x \in A_27a. (\forall V4l \in (ty_2Elist_2Elist\ A_27a). \\ ((ap\ (ap\ (ap\ (c_2Elist_2ELUPDATE\ A_27a)\ V2e)\ c_2Enum_2E0) (ap\ (\\ ap\ (c_2Elist_2ECONS\ A_27a)\ V3x)\ V4l)) = (ap\ (ap\ (c_2Elist_2ECONS \\ A_27a)\ V2e)\ V4l)))))) \wedge (\forall V5e \in A_27a. (\forall V6n \in ty_2Enum_2Enum. \\ (\forall V7x \in A_27a. (\forall V8l \in (ty_2Elist_2Elist\ A_27a). (\\ (ap\ (ap\ (ap\ (c_2Elist_2ELUPDATE\ A_27a)\ V5e)\ (ap\ c_2Enum_2ESUC\ V6n)) \\ (ap\ (ap\ (c_2Elist_2ECONS\ A_27a)\ V7x)\ V8l)) = (ap\ (ap\ (c_2Elist_2ECONS \\ A_27a)\ V7x)\ (ap\ (ap\ (ap\ (c_2Elist_2ELUPDATE\ A_27a)\ V5e)\ V6n)\ V8l))))))))) \end{aligned} \quad (43)$$

Assume the following.

$$(\forall V0n \in ty_2Enum_2Enum. (\neg((ap\ c_2Enum_2ESUC\ V0n) = c_2Enum_2E0))) \quad (44)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. (\neg(p\ (ap\ (ap \\ (c_2Ebool_2EIN\ A_27a)\ V0x)\ (c_2Epred_set_2EEMPTY\ A_27a)))))) \quad (45)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in \\ A_27a. (\forall V2s \in (2^{A_27a}). ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a) \\ V0x)\ (ap\ (ap\ (c_2Epred_set_2EINSERT\ A_27a)\ V1y)\ V2s))) \Leftrightarrow ((V0x = \\ V1y) \vee (p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V0x)\ V2s)))))) \end{aligned} \quad (46)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. ((ap\ c_2Enum_2ESUC\ V0m) = (ap\ c_2Enum_2ESUC\ V1n)) \Leftrightarrow (V0m = V1n))) \quad (47)$$

Assume the following.

$$(\forall V0n \in ty_2Enum_2Enum. (\neg(p\ (ap\ (ap\ c_2Eprim_rec_2E_3C\ V0n)\ c_2Enum_2E0)))) \quad (48)$$

Assume the following.

$$(\forall V0n \in ty_2Enum_2Enum. (p\ (ap\ (ap\ c_2Eprim_rec_2E_3C\ c_2Enum_2E0)\ (ap\ c_2Enum_2ESUC\ V0n)))) \quad (49)$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \quad (50)$$

Assume the following.

$$(\forall V0A \in 2. ((p\ V0A) \Rightarrow ((\neg(p\ V0A)) \Rightarrow False))) \quad (51)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p\ V0A) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow ((p\ V0A) \Rightarrow False) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False)))) \quad (52)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg((\neg(p\ V0A)) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow ((p\ V0A) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False)))) \quad (53)$$

Assume the following.

$$(\forall V0A \in 2. (((\neg(p\ V0A)) \Rightarrow False) \Rightarrow (((p\ V0A) \Rightarrow False) \Rightarrow False))) \quad (54)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p\ V0p) \Leftrightarrow ((p\ V1q) \Leftrightarrow (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee ((p\ V1q) \vee (p\ V2r))) \wedge (((p\ V0p) \vee ((\neg(p\ V2r)) \vee (\neg(p\ V1q)))) \wedge (((p\ V1q) \vee ((\neg(p\ V2r)) \vee (\neg(p\ V0p)))) \wedge ((p\ V2r) \vee ((\neg(p\ V1q)) \vee (\neg(p\ V0p)))))))))) \quad (55)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p\ V0p) \Leftrightarrow ((p\ V1q) \wedge (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee ((\neg(p\ V1q)) \vee (\neg(p\ V2r)))) \wedge (((p\ V1q) \vee (\neg(p\ V0p))) \wedge ((p\ V2r) \vee (\neg(p\ V0p)))))))) \quad (56)$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (\\
& (p V1q) \vee (p V2r))) \Leftrightarrow (((p V0p) \vee \neg(p V1q)) \wedge (((p V0p) \vee \neg(p V2r))) \wedge \\
& ((p V1q) \vee ((p V2r) \vee \neg(p V0p))))))))))
\end{aligned} \tag{57}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (\\
& (p V1q) \Rightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee \neg(p V2r))) \wedge ((\\
& \neg(p V1q) \vee ((p V2r) \vee \neg(p V0p))))))))))
\end{aligned} \tag{58}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (((p V0p) \Leftrightarrow \neg(p V1q)) \Leftrightarrow (((p V0p) \vee \\
& (p V1q)) \wedge (\neg(p V1q) \vee \neg(p V0p))))))
\end{aligned} \tag{59}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\neg((p V0p) \Rightarrow (p V1q)) \Rightarrow (p V0p))) \tag{60}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\neg((p V0p) \Rightarrow (p V1q)) \Rightarrow \neg(p V1q))) \tag{61}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\neg((p V0p) \vee (p V1q)) \Rightarrow \neg(p V0p))) \tag{62}$$

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Assume the following.

$$(\forall V0p \in 2. (\neg(\neg(p V0p)) \Rightarrow (p V0p))) \tag{64}$$

Theorem 1

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0l \in (ty_2Elist_2Elist \\
& A_27a). (\forall V1x \in A_27a. (\forall V2y \in A_27a. (\forall V3i \in \\
& ty_2Enum_2Enum. ((p (ap (ap (c_2Ebool_2EIN\ A_27a)\ V1x) (ap (c_2Elist_2ELIST_TO_SET \\
& A_27a) (ap (ap (ap (c_2Elist_2ELUPDATE\ A_27a)\ V2y)\ V3i)\ V0l)))) \Leftrightarrow \\
& (((p (ap (ap\ c_2Eprim_rec_2E_3C\ V3i) (ap (c_2Elist_2ELENGTH\ A_27a) \\
& V0l))) \wedge (V1x = V2y)) \vee (\exists V4j \in ty_2Enum_2Enum. ((p (ap (ap\ c_2Eprim_rec_2E_3C \\
& V4j) (ap (c_2Elist_2ELENGTH\ A_27a)\ V0l))) \wedge (\neg(V3i = V4j)) \wedge ((ap \\
& (ap (c_2Elist_2EEL\ A_27a)\ V4j)\ V0l) = V1x)))))))))
\end{aligned}$$