

thm_2Elist_2EMEM__dropWhile__IMP (TMXme- fyc31rgq3nvUuQ53HeZBVHnzDd9AKP)

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Definition 1 We define `c_2Emin_2E_3D` to be $\lambda A. \lambda x \in A. \lambda y \in A. inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define `c_2Ebool_2ET` to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define `c_2Ebool_2EIN` to be $\lambda A.27a : \iota. (\lambda V0x \in A.27a. (\lambda V1f \in (2^{A.27a}). (ap V1f V0x)))$

Definition 4 We define `c_2Emin_2E_3D_3D_3E` to be $\lambda P \in 2. \lambda Q \in 2. inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 5 We define `c_2Ebool_2E_21` to be $\lambda A.27a : \iota. (\lambda V0P \in (2^{A.27a}). (ap (ap (c_2Emin_2E_3D (2^{A.27a})) (\lambda V1t \in 2.V1t)) (\lambda V2t \in 2.V2t)))$

Definition 6 We define `c_2Ebool_2E_5C_2F` to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t))))$

Definition 7 We define `c_2Ebool_2E_2F_5C` to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t))))$

Let `ty_2Epair_2Eprod` : $\iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0. nonempty A0 \Rightarrow \forall A1. nonempty A1 \Rightarrow nonempty (ty_2Epair_2Eprod A0 A1) \tag{1}$$

Let `c_2Epair_2EABS__prod` : $\iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A.27a. nonempty A.27a \Rightarrow \forall A.27b. nonempty A.27b \Rightarrow c_2Epair_2EABS_prod A.27a A.27b \in ((ty_2Epair_2Eprod A.27a A.27b)^{(2^{A.27b})^{A.27a}}) \tag{2}$$

Definition 8 We define `c_2Epair_2E_2C` to be $\lambda A.27a : \iota. \lambda A.27b : \iota. \lambda V0x \in A.27a. \lambda V1y \in A.27b. (ap (c_2Epair_2EABS__prod A.27a A.27b) V0x V1y)$

Let `c_2Epred__set_2EGSPEC` : $\iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A.27a. nonempty A.27a \Rightarrow \forall A.27b. nonempty A.27b \Rightarrow c_2Epred_set_2EGSPEC A.27a A.27b \in ((2^{A.27a})^{(ty_2Epair_2Eprod A.27a 2)^{A.27b}}) \tag{3}$$

Definition 9 We define `c_2Epred__set_2EINSERT` to be $\lambda A.27a : \iota. \lambda V0x \in A.27a. \lambda V1s \in (2^{A.27a}). (ap (c_2Epair_2E_2C A.27a V0x V1s))$

Definition 10 We define c_2Ebool_2EF to be $(ap (c_2Ebool_2E_21\ 2) (\lambda V0t \in 2.V0t))$.

Let $ty_2Elist_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Elist_2Elist\ A0) \quad (4)$$

Let $c_2Elist_2ELIST_TO_SET : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2ELIST_TO_SET\ A_27a \in ((2^{A_27a})^{(ty_2Elist_2Elist\ A_27a)}) \quad (5)$$

Definition 11 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A$.if $(\exists x \in A.p (ap\ P\ x))$ then $(the\ (\lambda x.x \in A \wedge P\ x))$ of type $\iota \Rightarrow \iota$.

Definition 12 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a.$

Let $c_2Elist_2ECONS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2ECONS\ A_27a \in (((ty_2Elist_2Elist\ A_27a)^{(ty_2Elist_2Elist\ A_27a)})^{A_27a}) \quad (6)$$

Let $c_2Elist_2ENIL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2ENIL\ A_27a \in (ty_2Elist_2Elist\ A_27a) \quad (7)$$

Let $c_2Elist_2EdropWhile : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2EdropWhile\ A_27a \in ((ty_2Elist_2Elist\ A_27a)^{(ty_2Elist_2Elist\ A_27a)^{(2^{A_27a})}}) \quad (8)$$

Definition 13 We define $c_2Epred_set_2EEMPTY$ to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.c_2Ebool_2EF)$.

Definition 14 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap\ c_2Emin_2E_3D_3D_3E\ V0t)\ c_2Ebool_2EF))$

Assume the following.

$$True \quad (9)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \quad (10)$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p\ V0t))) \quad (11)$$

Assume the following.

$$(\forall V0t \in 2.((p\ V0t) \vee \neg(p\ V0t))) \quad (12)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \quad (13)$$

Assume the following.

$$(\forall V0t \in 2.(((True \vee (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \vee True) \Leftrightarrow True) \wedge (((False \vee (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \vee False) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \vee (p\ V0t)) \Leftrightarrow (p\ V0t)))))) \quad (14)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow False) \Leftrightarrow (\neg (p\ V0t)))))) \quad (15)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (16)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg (p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg (p\ V0t)))))) \quad (17)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0t1 \in A_27a.(\forall V1t2 \in A_27a.(((ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27a)\ c_2Ebool_2ET)\ V0t1)\ V1t2) = V0t1) \wedge ((ap\ (ap\ (c_2Ebool_2ECOND\ A_27a)\ c_2Ebool_2EF)\ V0t1)\ V1t2) = V1t2)))) \quad (18)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p\ V0t1) \Rightarrow ((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \quad (19)$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1x_27 \in 2.(\forall V2y \in 2.(\forall V3y_27 \in 2.(((p\ V0x) \Leftrightarrow (p\ V1x_27)) \wedge ((p\ V1x_27) \Rightarrow ((p\ V2y) \Leftrightarrow (p\ V3y_27)))))) \Rightarrow (((p\ V0x) \Rightarrow (p\ V2y)) \Leftrightarrow ((p\ V1x_27) \Rightarrow (p\ V3y_27)))))) \quad (20)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\forall V0h \in A_27b.(\forall V1t \in (ty_2Elist_2Elist\ A_27b).(((ap\ (c_2Elist_2ELIST_TO_SET\ A_27a)\ (c_2Elist_2ENIL\ A_27a)) = (c_2Epred_set_2EEMPTY\ A_27a)) \wedge ((ap\ (c_2Elist_2ELIST_TO_SET\ A_27b)\ (ap\ (ap\ (c_2Elist_2ECONS\ A_27b)\ V0h)\ V1t)) = (ap\ (ap\ (c_2Epred_set_2EINSERT\ A_27b)\ V0h)\ (ap\ (c_2Elist_2ELIST_TO_SET\ A_27b)\ V1t)))))) \quad (21)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0P \in (2^{(ty.2Elist.2Elist\ A.27a)}), \\ & (((p\ (ap\ V0P\ (c.2Elist.2ENIL\ A.27a))) \wedge (\forall V1t \in (ty.2Elist.2Elist \\ & \quad A.27a).(p\ (ap\ V0P\ V1t))) \Rightarrow (\forall V2h \in A.27a.(p\ (ap\ V0P\ (ap\ (ap\ (\\ & \quad c.2Elist.2ECONS\ A.27a\ V2h)\ V1t)))))) \Rightarrow (\forall V3l \in (ty.2Elist.2Elist \\ & \quad A.27a).(p\ (ap\ V0P\ V3l)))))) \end{aligned} \quad (22)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow ((\forall V0x \in A.27a.((p\ (ap\ (ap \\ & \quad (c.2Ebool.2EIN\ A.27a)\ V0x)\ (ap\ (c.2Elist.2ELIST_TO_SET\ A.27a) \\ & \quad (c.2Elist.2ENIL\ A.27a)))) \Leftrightarrow False)) \wedge (\forall V1x \in A.27a.(\forall V2h \in \\ & \quad A.27a.(\forall V3t \in (ty.2Elist.2Elist\ A.27a).((p\ (ap\ (ap\ (c.2Ebool.2EIN \\ & \quad A.27a)\ V1x)\ (ap\ (c.2Elist.2ELIST_TO_SET\ A.27a)\ (ap\ (ap\ (c.2Elist.2ECONS \\ & \quad A.27a)\ V2h)\ V3t)))) \Leftrightarrow ((V1x = V2h) \vee (p\ (ap\ (ap\ (c.2Ebool.2EIN\ A.27a) \\ & \quad V1x)\ (ap\ (c.2Elist.2ELIST_TO_SET\ A.27a)\ V3t)))))))))) \end{aligned} \quad (23)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow ((\forall V0P \in (2^{A.27a}).((ap\ (\\ & \quad ap\ (c.2Elist.2EdropWhile\ A.27a)\ V0P)\ (c.2Elist.2ENIL\ A.27a)) = \\ & \quad (c.2Elist.2ENIL\ A.27a))) \wedge (\forall V1P \in (2^{A.27a}).(\forall V2h \in \\ & \quad A.27a.(\forall V3t \in (ty.2Elist.2Elist\ A.27a).((ap\ (ap\ (c.2Elist.2EdropWhile \\ & \quad A.27a)\ V1P)\ (ap\ (ap\ (c.2Elist.2ECONS\ A.27a)\ V2h)\ V3t)) = (ap\ (ap\ (\\ & \quad ap\ (c.2Ebool.2ECOND\ (ty.2Elist.2Elist\ A.27a)\ (ap\ V1P\ V2h))\ (ap \\ & \quad (ap\ (c.2Elist.2EdropWhile\ A.27a)\ V1P)\ V3t))\ (ap\ (ap\ (c.2Elist.2ECONS \\ & \quad A.27a)\ V2h)\ V3t)))))) \end{aligned} \quad (24)$$

Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0x \in A.27a.(\neg(p\ (ap\ (ap \\ (c.2Ebool.2EIN\ A.27a)\ V0x)\ (c.2Epred_set.2EEMPTY\ A.27a)))))) \quad (25)$$

Theorem 1

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0P \in (2^{A.27a}).(\forall V1ls \in \\ & \quad (ty.2Elist.2Elist\ A.27a).(\forall V2x \in A.27a.((p\ (ap\ (ap\ (c.2Ebool.2EIN \\ & \quad A.27a)\ V2x)\ (ap\ (c.2Elist.2ELIST_TO_SET\ A.27a)\ (ap\ (ap\ (c.2Elist.2EdropWhile \\ & \quad A.27a)\ V0P)\ V1ls)))) \Rightarrow (p\ (ap\ (ap\ (c.2Ebool.2EIN\ A.27a)\ V2x)\ (ap\ (\\ & \quad c.2Elist.2ELIST_TO_SET\ A.27a)\ V1ls)))))) \end{aligned}$$